



An Enhanced Ant Colony Optimization Based Image Edge Detection

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Abstract—Edge detection is a fundamental problem in image analysis. Different evolutionary optimization techniques have been recently applied to this problem. Ant Colony Optimization is also an evolutionary optimization technique, which has been applied to this problem. But in this paper we present modifications in the previous implementation of ACO to further increase the clarity of detected edges in an image. Thus in this paper an improved ACO based algorithm for image edge detection has been presented. Series of simulation experiments demonstrate the feasibility, effectiveness and superior performance of the proposed approach as compared to basic ACO.

Keywords- Ant Colony Optimization (ACO); Edge Detection

I. INTRODUCTION TO ACO

Ant Colony optimization (ACO), are the groups of optimization algorithms i.e. inspired from the nature of ants exploring for food. The first ACO algorithm, called ant system was proposed by Dorigo et al. [1]. It is motivated by natural foraging behavior of ant species. Ants deposit pheromone on the ground to mark paths between a food source and their colony, which should be followed by other members of the colony. Over time, pheromone trails evaporate. The longer it takes for an ant to travel down the path and back again, the more time the pheromones have to evaporate. Shorter – and thus, favorable – paths get marched over faster and receive greater compensation for pheromone evaporation. Pheromone densities remain high on shorter paths because pheromone is laid down faster. This positive feedback mechanism eventually leads the ants to follow the shorter paths. It is this natural phenomenon that inspired the development of the ACO met heuristic.

The various evolutionary techniques have been recently applied to edge detection i.e. J.Tian, W.Yu and S.Xie, discussed ant colony optimization in 2006 [2], Peng Xiao, Jun Li and Jian-Ping, explained ant colony optimization algorithm for image extracting in 2010 [4], Javed Rahebi, Zahra Elmi, Ali Farzam Nia and Kamraan Shayan explained edge detection using ACO and Genetic algorithm in 2010 IEEE [5] and Om Prakash Verma and Madasu Hanmandlu discussed a Novel Fuzzy Ant System For Edge Detection in 2010 [6].

ACO has been applied in various applications [7] – [14]. In this paper, we are focusing on edge detection of a particular image. Edges in the image may be regarded as a boundary between two dissimilar regions. Edges may even contain other edges, when look closer. An edge is easy to detect since differences in pixel values between regions are relatively easy to calculate. For this we need to know the exact content of the picture i.e. intensity value of each pixel. The proposed approach exploits number of ants, which move on the image driven by the local variation of image intensity values, to establish a pheromone matrix, which represents the edge pixel location of the image. In any case there are some differences between our proposed method and others. First, our way uses different parameters as evaporation rate and pheromone information. Secondly, we had tried to use some more kernel functions for better calculation. Thirdly, the main emphasis is given to increase the normal eye visibility of detected edges.

This paper is organized as follows. In section II, a brief introduction is provided to present the fundamental concepts of ACO. Then, an ACO-based image edge detection approach is proposed in section III. Experimental results are presented in section IV. Finally, Section V concludes this paper.

II. Ant Colony Optimization

Ant Colony Optimization is a relatively new approach to problem solving that takes inspiration from the social behavior of the ants. ACO is a population based approach and is inspired by foraging behavior of ant species. In many ant species, ant walking to and from a food source deposit some substance on the ground called pheromone. Different ants adopt different

paths to reach the food source and deposit the pheromone based on the fact that higher concentration of pheromone is deposited on shorter paths and smaller concentration on longer paths. Here, the collective behavior of ants provides intelligent solution for finding the shortest path from the nest to the food source. If a single ant finds a shorter path and deposit higher concentration of pheromone on the way to food source then all the other ants gets attracted towards the higher concentration and hence following the shorter path.

Ant Colony Optimization is an iterative algorithm. At each iteration, a number of artificial ants are considered. Each of them builds a solution by walking from vertex to vertex on the graph with the constraint of not visiting any vertex that she has already visited in her walk. At the end of an iteration, on the basis of the quality of the solutions constructed by the ants, the pheromone values are modified in order to bias ants in future iterations to construct solutions similar to the best ones previously constructed.

Artificial ants are like real ants with some major differences: 1) Artificial ants have memory, 2) They aren't completely blind, 3) They live in a discrete time environment. However they have some adopted characteristics from the real ants, like 1) They probabilistically prefer path with a larger amount of pheromone, 2) Shorter path is true path, larger is the rate of growth in the pheromone concentration, 3) They communicate to each other by means of the amount of pheromone laid on each path.

Parameters defined in the procedure of ACO are as follow [2]:

- 1) *Pheromone matrix*: This matrix contains the value of pheromone intensity which attracts the ants to follow paths traversed by other ants. Pheromone matrix are updated twice, once after the movement of each ants and secondly after movement of all the ants.
- 2) *Probabilistic transition matrix*: The value of probability for the ant's movement from one pixel to another is stored in probability transition matrix. The procedure of ACO can be summarized as follows:
 - a) Let total K ants are applied to find the optimal solution in a space χ that consists of $M1 \times M2$ nodes.
 - b) Initialize the positions of total K ants, as well as the pheromone matrix $\tau(0)$.
 - c) For the construction-step index $n = 1 : N$ and for the ant index $k = 1 : K$. Consecutively, move the k^{th} ant for L steps, from the node i to the node j according to a probabilistic transition matrix $P(n,[])$

$$P_{i,j}^{(n)} = \frac{(\tau_{i,j}^{(n-1)})^\alpha (\eta_{i,j})^\beta}{\sum_{j \in \eta_i} (\tau_{i,j}^{(n-1)})^\alpha (\eta_{i,j})^\beta} \quad (1)$$

where $\tau_{i,j}^{(n-1)}$ is the pheromone information value and $\eta_{i,j}$ represents the heuristic information of the arc linking node i to the node j; the constants α and β represents the influence of pheromone information and heuristic information, respectively.

- d) Update the pheromone matrix $\tau(n)$ after the movement of each ant within each construction step.

$$\tau_{i,j}^{(n)} = \begin{cases} (1 - \rho) \cdot \tau_{i,j}^{(n-1)} + \rho \cdot \Delta_{i,j}^{(k)}, & \text{if } (i,j) \text{ belongs to the best tour} \\ \tau_{i,j}^{n-1}, & \text{otherwise} \end{cases} \quad (2)$$

Where ρ is the evaporation rate.

- e) Make the solution decision according to the final pheromone matrix $\tau(N)$, performed after the move of all K ants within each construction step; and the pheromone matrix is updated as

$$\tau^{(n)} = (1 - \varphi) \cdot \tau^{(n-1)} + \varphi \cdot \tau^{(0)} \quad (3)$$

Where φ is the pheromone decay coeff.

Note that the ant colony system performs two update operations (2) and (3) for updating the pheromone matrix, while the ant system only performs one operation (i.e. (3)).

III. The Proposed Enhanced ACO Based Image Edge Detection

The flowchart of the proposed enhanced ACO based method for edge detection is as shown in the fig. 1. Here the modification has been done in two steps i.e. construction and decision process, rest of the method is same as that of [2].

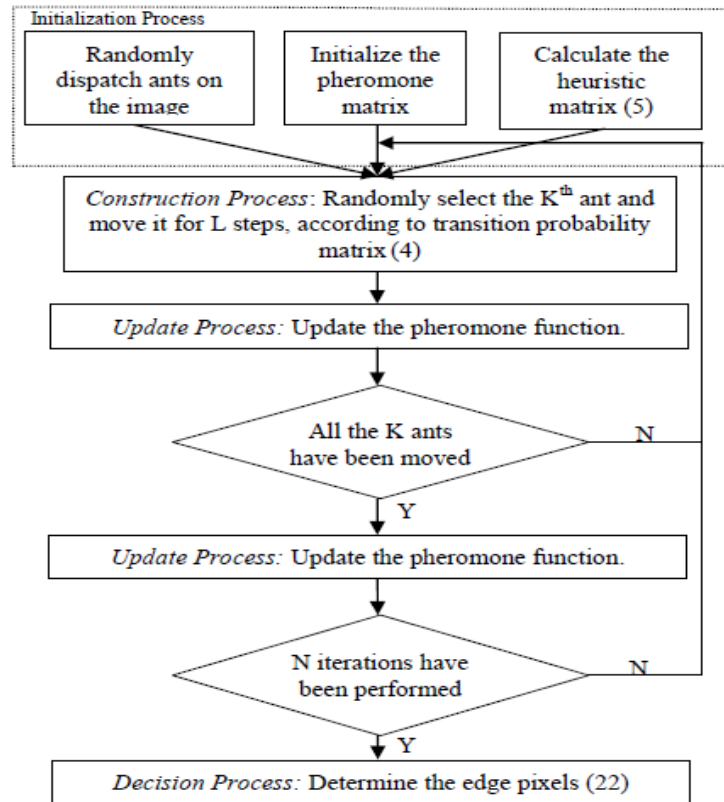


Fig. 1 Flow chart of implementation steps in Proposed ACO approach [2].

A. Modified Construction Process

In this step modification done is in formation of new clique and addition of two new kernel functions. In the construction process, selected ant will move on the image for L movement-steps & this process will be repeated in each construction step and till each ant moves on image. The selected ant moves from node (l,m) to its neighboring node (i,j) according to the transition probability that is defined as follows:

$$P_{(l,m),(i,j)}^{(n)} = \frac{(\tau_{i,j}^{(n-1)})^\alpha (\eta_{i,j})^\beta}{\sum_{(i,j) \in \Omega_{(l,m)}} (\tau_{i,j}^{(n-1)})^\alpha (\eta_{i,j})^\beta} \quad (4)$$

Where $\tau_{i,j}^{(n-1)}$ is the pheromone value of node (i,j) , $\Omega_{(l,m)}$ is the neighborhood nodes of the nodes (l,m) , $\eta_{i,j}$ represents the heuristic information at the node (i,j) . The constants α and β represent the influence of pheromone information and heuristic information respectively.

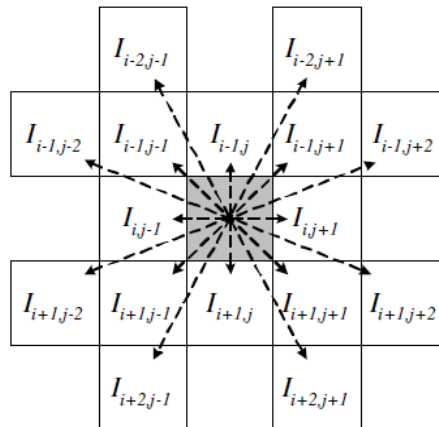


Fig.2. Original Clique as in [2]: A local configuration at the pixel position $I_{i,j}$ for computing the variation $V_c(I_{i,j})$ defined in (6). The pixel $I_{i,j}$ is marked as gray square.

There are two main issues in the construction process.

I. Determination of the heuristic information $\eta_{i,j}$ in (4).

In [2], it is proposed to be determined by the local statistics at pixel position (i,j) as

$$\eta_{i,j} = \frac{1}{Z} V_c(I_{i,j}) \quad (5)$$

Where $Z = \sum_{i=1}^{M_1} \sum_{j=1}^{M_2} V_c(I_{i,j})$, which is a normalization factor, $I_{i,j}$ is the intensity value of the pixel at the position (i,j) of the image \mathbf{I} , the function $V_c(I_{i,j})$ is a function of local group of pixels c (called the *clique*), and its value depends on variation of image's intensity values on the clique c (as shown in fig. 2).

Function $V_c(I_{i,j})$ as defined in [2] is given as follows :

$$V_c(I_{i,j}) = f(|I_{i-2,j-1} - I_{i+2,j+1}| + |I_{i-2,j+1} - I_{i+2,j-1}| + |I_{i-1,j-2} - I_{i+1,j+2}| + |I_{i-1,j+1} - I_{i+1,j-1}| + |I_{i-1,j} - I_{i+1,j}| + |I_{i-1,j+1} - I_{i+1,j-1}| + |I_{i-1,j+2} - I_{i-1,j-2}| + |I_{i,j-1} - I_{i,j+1}|) \quad (6)$$

Modified Function $V_c(I_{i,j})$ is given as follows :

$$V_c I(I_{i,j}) = f(|I_{i-2,j-2} - I_{i+2,j+2}| + |I_{i-2,j} - I_{i+2,j}| + |I_{i-2,j+2} - I_{i+2,j+2}| + |I_{j-1,j-1} - I_{i+1,j+1}| + |I_{i-1,j} - I_{i+1,j}| + |I_{i-1,j+1} - I_{i+1,j-1}| + |I_{i,j-2} - I_{i,j+2}| + |I_{i,j-1} - I_{i,j+1}|) \quad (7)$$

To determine the function $f(\cdot)$ in (6), four functions were used in the ACO approach used in [2].

$$f(x) = \lambda x \quad \text{for } x \geq 0 \quad (8)$$

$$f(x) = \lambda x^2, \quad \text{for } x \geq 0 \quad (9)$$

$$f(x) = \begin{cases} \sin\left(\frac{\pi x}{2\lambda}\right) & 0 \leq x \leq \lambda \\ 0 & \text{else} \end{cases} \quad (10)$$

$$f(x) = \begin{cases} \frac{\pi x \sin\left(\frac{\pi x}{\lambda}\right)}{\lambda} & 0 \leq x \leq \lambda \\ 0 & \text{else} \end{cases} \quad (11)$$

To determine the function $f(\cdot)$ in (7) six functions are used, first four functions were used in the ACO approach used in [2], in the proposed method two additional functions were used and are defined as

$$f(x) = \begin{cases} \frac{\pi x \exp\left(\frac{\pi x}{\lambda}\right)}{\lambda} & 0 \leq x \leq \lambda \\ 0 & \text{else} \end{cases} \quad (12)$$

$$f(x) = \begin{cases} \frac{\pi x \cos\left(\frac{\pi x}{\lambda}\right)}{\lambda} & 0 \leq x \leq \lambda \\ 0 & \text{else} \end{cases} \quad (13)$$

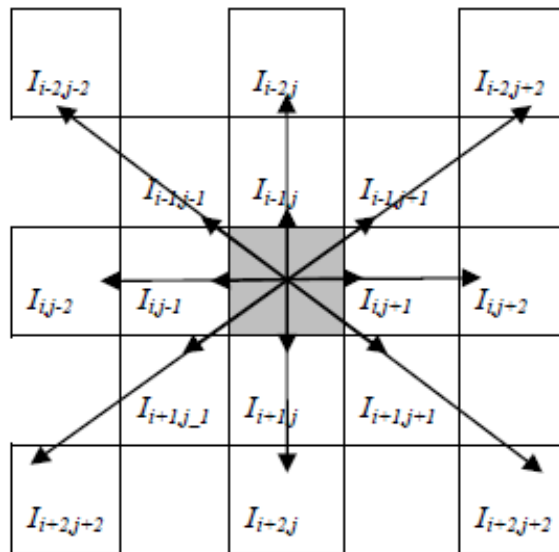


Fig.3. Modified Clique : A local configuration at the pixel position $I_{i,j}$ for computing the variation $V_c(I_{i,j})$ defined in (7). The pixel $I_{i,j}$ is marked as gray square.

The parameter λ in each of the above functions (8)-(13) adjusts the functions of the respective shapes as in fig. 5.

II. To calculate the authorized amount of ant movements (i.e. $\Omega_{(l,m)}$ in (4)) at the position (l,m) is. For this either the 4-connectivity neighborhood or the 8-connectivity neighborhood can be used, both of which are demonstrated in figure 4.

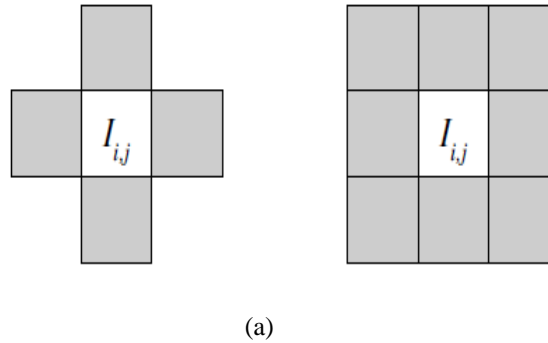


Fig.4. Various neighborhoods (marked as gray regions) of the pixel I_{ij} : (a) 4-connectivity Neighborhood (b) 8-connectivity neighborhood

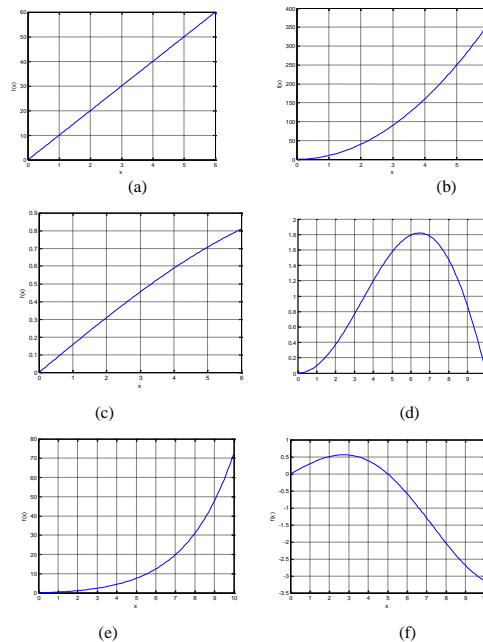


Fig.5. Various functions with the parameter $\lambda = 10$: (a) the function defined in (8); (b) the function defined in (9); (c) the function defined in (10); and (d) the function defined in (11). (e) the function defined in (12); and (f) the function defined in (13)

B. Decision Process

Here the modification is in the selection of new threshold, rest of the paper is same as that of [2]. Here pheromone matrix is used to classify each pixel either as an edge or a non-edge. The decision is made by applying a threshold T on the final pheromone. The threshold T is computed based on Otsu technique which is described in [3].

The initial threshold $T^{(0)}$ is selected as the mean value of the pheromone matrix. The entries of the pheromone matrix are classified in two parts as:

(a) Those entries of pheromone matrix whose value is less than $T^{(0)}$.

(b) Those entries of pheromone matrix whose value is larger than $T^{(0)}$.

Then the new threshold is calculated by taking the square of two mean values of each of the two categories and then taking their average. The complete process is repeated again and again until the threshold value becomes constant (in terms of user defined tolerance ϵ). The above iterative procedure can be summarized as follows;

Step 1. Threshold is initialized as

$$T^{(0)} = \frac{\sum_{i=1:M_1} \sum_{j=1:M_2} \tau_{ij}^N}{M_1 M_2}, \quad (14)$$

And set iteration index as $l=0$.

Step 2. Pheromone matrix $\tau^{(n)}$ is separated in two classes where first class contains the entries of τ whose value is below $T^{(l)}$ and other class in which values of τ are above $T^{(l)}$. Mean of the two categories (specified earlier) is calculated as

$$m_L^{(l)} = \frac{\sum_{i=1:M_1} \sum_{j=1:M_2} g_{T^{(l)}}^L(\tau_{ij}^{(N)})}{\sum_{i=1:M_1} \sum_{j=1:M_2} h_{T^{(l)}}^L(\tau_{ij}^{(N)})}, \quad (15)$$

$$m_U^{(l)} = \frac{\sum_{i=1:M_1} \sum_{j=1:M_2} g_{T^{(l)}}^U(\tau_{ij}^{(N)})}{\sum_{i=1:M_1} \sum_{j=1:M_2} h_{T^{(l)}}^U(\tau_{ij}^{(N)})}, \quad (16)$$

where

$$g_{T^{(l)}}^L(x) = \begin{cases} x, & \text{if } x \leq T^{(l)}; \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

$$h_{T^{(l)}}^L(x) = \begin{cases} x, & \text{if } x \leq T^{(l)}; \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

$$g_{T^{(l)}}^U(x) = \begin{cases} x, & \text{if } x \geq T^{(l)}; \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

$$h_{T^{(l)}}^U(x) = \begin{cases} x, & \text{if } x \geq T^{(l)}; \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

Step 3. New threshold is updated as

$$T^{(l)} = \frac{\left[\frac{(m_L^{(2)}) / (m_U^{(2)})}{2} \right]}{2} \quad (21)$$

Step 4. If $|T^{(l)} - T^{(n-1)}| > \epsilon$, then go to Step 2; otherwise, the iteration process is terminated and a binary decision is made on each pixel position (i, j) to determine whether it is edge (i.e., $E_{i,j} = 1$) or not (i.e., $E_{i,j} = 0$), based on the criterion

$$E_{i,j} = \begin{cases} 1, & \text{if } \tau_{ij}^N \geq T^{(l)}; \\ 0 & \text{otherwise} \end{cases} \quad (22)$$

IV. SIMULATION

To evaluate the approach, the experiments use four test images, cameraman, House, Lena, Pepper and which are shown in figure 6. Modified parameters of the proposed approach are show n, rest are defined in [2].

- $\alpha=1.1$: The factor of assessment related to information pheromone in the expression (4).
- $\beta=0.5$: The weighting factor of the heuristic information in (4).
- $\rho=0.5$: Evaporation amount defined in the [2].

Practical method for comparing results of the suggested method to identify method defined in [2] has been done. Also it is necessary to know how the heuristic matrix (5) is determined, for this various functions defined in (8)-(13) are individually substituted in (6), then in (7) and their resulted performance is presented. Fig. 7 to Fig. 21 shows the comparison between edge information of test images lena, camera, house and peppers w.r.t jing Tian's method and proposed approach.



Fig. 6. Original image lena



Fig. 7 .Various extracted edge information of test image lena w.r.t Jing Tian's method.

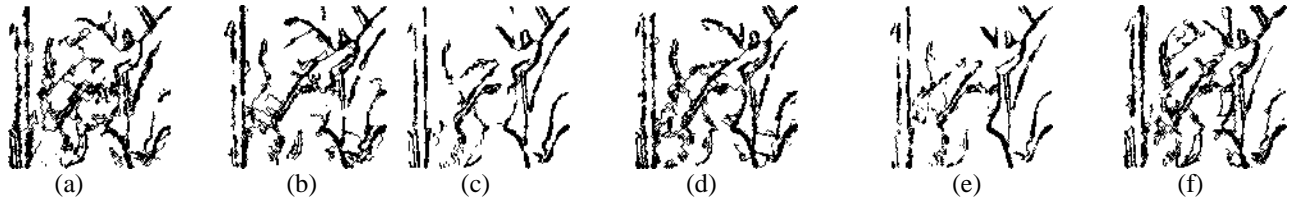


Fig.8.Various extracted edge information of test image lena w.r.t clique defined in (6), with modified threshold & modified parameters (a) the proposed approach with the incorporation of function defined in, (8); (b) the proposed approach with incorporation of function defined in (9); (c) the proposed approach with incorporation of function defined in (10), (d) the proposed approach with incorporation of function defined in (11), (e) the proposed approach with incorporation of function defined in (12); (f) the proposed approach with incorporation of function defined in (13).

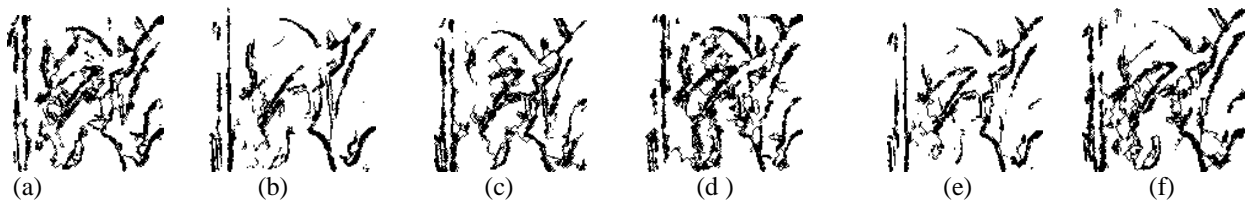


Fig.9.Various extracted edge information of test image lena w.r.t clique defined in (7), with modified threshold & modified parameters (a) the proposed approach with the incorporation of function defined in, (8); (b) the proposed approach with incorporation of function defined in (9); (c) the proposed approach with incorporation of function defined in (10), (d) the proposed approach with incorporation of function defined in (11), (e) the proposed approach with incorporation of function defined in (12); (f) the proposed approach with incorporation of function defined in (13).



Fig. 10. Original image camera



Fig. 11 .Various extracted edge information of test image camera w.r.t Jing Tian's method

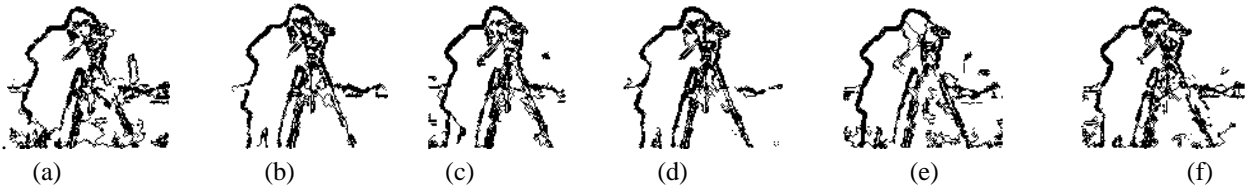


Fig.12.Various extracted edge information of test image camera w.r.t clique defined in (6), with modified threshold & modified parameters (a) the proposed approach with the incorporation of function defined in, (8); (b) the proposed approach with incorporation of function defined in (9); (c) the proposed approach with incorporation of function defined in (10), (d) the proposed approach with incorporation of function defined in (11), (e) the proposed approach with incorporation of function defined in (12); (f) the proposed approach with incorporation of function defined in (13).



Fig.13. Various extracted edge information of test image camera w.r.t clique defined in (7), with modified threshold & modified parameters (a) the proposed approach with the incorporation of function defined in, (8); (b) the proposed approach with incorporation of function defined in (9); (c) the proposed approach with incorporation of function defined in (10), (d) the proposed approach with incorporation of function defined in (11), (e) the proposed approach with incorporation of function defined in (12); (f) the proposed approach with incorporation of function defined in (13).



Fig. 14. Original image house



Fig. 15 .Various extracted edge information of test image house w.r.t Jing Tian's method.



Fig.16. Various extracted edge information of test image house w.r.t clique defined in (6), with modified threshold & modified parameters (a) the proposed approach with the incorporation of function defined in, (8); (b) the proposed approach with incorporation of function defined in (9); (c) the proposed approach with incorporation of function defined in (10), (d) the proposed approach with incorporation of function defined in (11), (e) the proposed approach with incorporation of function defined in (12); (f) the proposed approach with incorporation of function defined in (13).



Fig.17. Various extracted edge information of test image house w.r.t clique defined in (7), with modified threshold & modified parameters (a) the proposed approach with the incorporation of function defined in, (8); (b) the proposed approach with incorporation of function defined in (9); (c) the proposed approach with incorporation of function defined in (10), (d) the proposed approach with incorporation of function defined in (11), (e) the proposed approach with incorporation of function defined in (12); (f) the proposed approach with incorporation of function defined in (13).



Fig. 18. Original image peppers



Fig. 19 .Various extracted edge information of test image peppers w.r.t Jing Tian's method



Fig.20.Various extracted edge information of test image peppers w.r.t clique defined in (6), with modified threshold & modified parameters (a) the proposed approach with the incorporation of function defined in, (8); (b) the proposed approach with incorporation of function defined in (9); (c) the proposed approach with incorporation of function defined in (10), (d) the proposed approach with incorporation of function defined in (11), (e) the proposed approach with incorporation of function defined in (12); (f) the proposed approach with incorporation of function defined in (13)

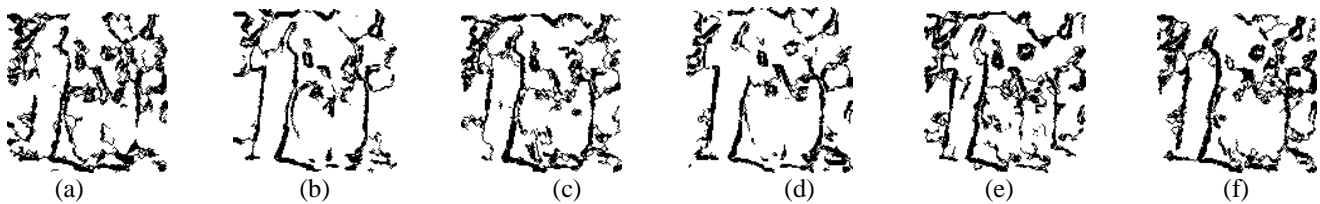


Fig.21.Various extracted edge information of test image peppers w.r.t clique defined in (7), with modified threshold & modified parameters (a) the proposed approach with the incorporation of function defined in, (8); (b) the proposed approach with incorporation of function defined in (9); (c) the proposed approach with incorporation of function defined in (10), (d) the proposed approach with incorporation of function defined in (11), (e) the proposed approach with incorporation of function defined in (12); (f) the proposed approach with incorporation of function defined in (13).

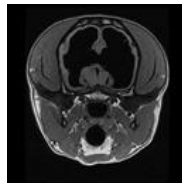


Fig. 22. Original medical image



Fig. 23 .Various extracted edge information of test image peppers w.r.t Jing Tian's method



Fig.24.Various extracted edge information of test medical image w.r.t clique defined in (6), with modified threshold & modified parameters (a) the proposed approach with the incorporation of function defined in, (8); (b) the proposed approach with incorporation of function defined in (9); (c) the proposed approach with incorporation of function defined in (10), (d) the proposed approach with incorporation of function defined in (11), (e) the proposed approach with incorporation of function defined in (12); (f) the proposed approach with incorporation of function defined in (13).

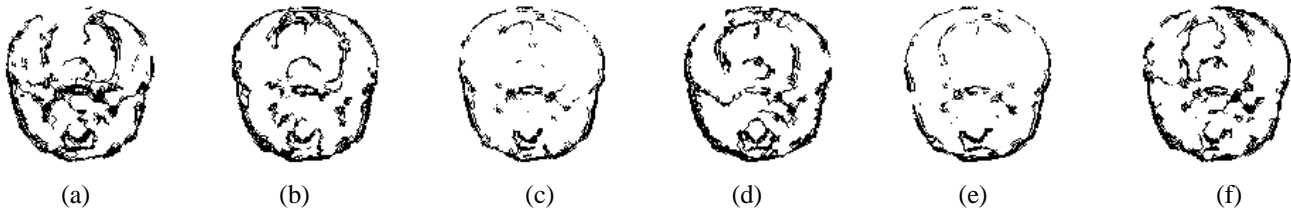


Fig.25. Various extracted edge information of test image peppers w.r.t clique defined in (7), with modified threshold & modified parameters (a) the proposed approach with the incorporation of function defined in, (8); (b) the proposed approach with incorporation of function defined in (9); (c) the proposed approach with incorporation of function defined in (10), (d) the proposed approach with incorporation of function defined in (11), (e) the proposed approach with incorporation of function defined in (12); (f) the proposed approach with incorporation of function defined in (13).

The proposed ACO approach is implemented using the *Matlab* programming language and run on a laptop with AMD Athlon DUAL-CORE QL-60 1.90 GHz CPU and a 2 GB RAM.

Table.(ii). SHOWS THE COMPARISON OF PROPOSED METHOD WITH CLIQUE DEFINED IN (7) WITH THE METHOD DEFINED IN [2]. JING TIAN'S METHOD (JT), PROPOSED APPROACH (PA)

	Number of connected components in an edge(mx)								M. Image	
	Lena (mx)		Camera		House		Pepper			
	PA	JT	PA	JT	PA	JT	PA	JT	PA	JT
Function in (7)	18	5	41	6	23	1	19	3	22	8
Function in (8)	9	1	30	6	17	1	13	1	7	2
Function in (9)	33	1	24	13	36	3	35	4	19	5
Function in (10)	7	1	34	2	12	1	11	2	4	2
Function in (11)	26	--	31	--	27	--	14	--	19	--
Function in (12)	32	--	33	--	28	--	31	--	23	--

V. CONCLUSIONS

Edge detection plays a vital role in image processing applications, which is important part in computer vision. In this paper an improved ACO based image edge detection approach has been successfully developed. Experimental results clearly show the superior performance of proposed approach with suitable parameters. The continuation research should concentrate on to reduce the computational time of the proposed approach and also on more effective way to calculate heuristic information.

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