



## Analysis of Power Spectrum for Linear Equalizer and Decision Feedback Equalizer in LTE SC-FDMA

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**Abstract**— Single carrier frequency division multiple access (SC-FDMA), is a single carrier technique that has recently received much attention as an alternative to orthogonal frequency division multiple access for 4G technology. SC-FDMA has been adopted for uplink transmission technique in 3GPP Long Term Evolution (LTE). For single-carrier systems with frequency domain equalization, decision feedback equalization (DFE) performs better than linear equalization and has much lower computational complexity than sequence maximum likelihood detection. The main challenge in DFE is the feedback symbol selection rule. In this paper we obtained the linearly equalized signal power spectrum for linear equalizer and decision feedback signal power spectrum for DFE.

**Keywords** -LTE uplink, Power spectrum, Linear equalizer, DFE.

### I. INTRODUCTION

#### Introduction to SC-FDMA:-

LTE has recently come up as a solution to modern wireless communications systems. It is vastly and rapidly used because of its promising high speed and ability to handle multi-data at same time. The modulation used in the uplink of LTE is SC-FDMA, also often referred to as Fourier spread FDMA. The reason for favoring SC-FDMA in the uplink over orthogonal frequency division multiple access (OFDMA) is its reduced peak-to-average power ratio (PAPR) compared to OFDMA. The principal design of a SC-FDMA communication system is shown in Figure (1).

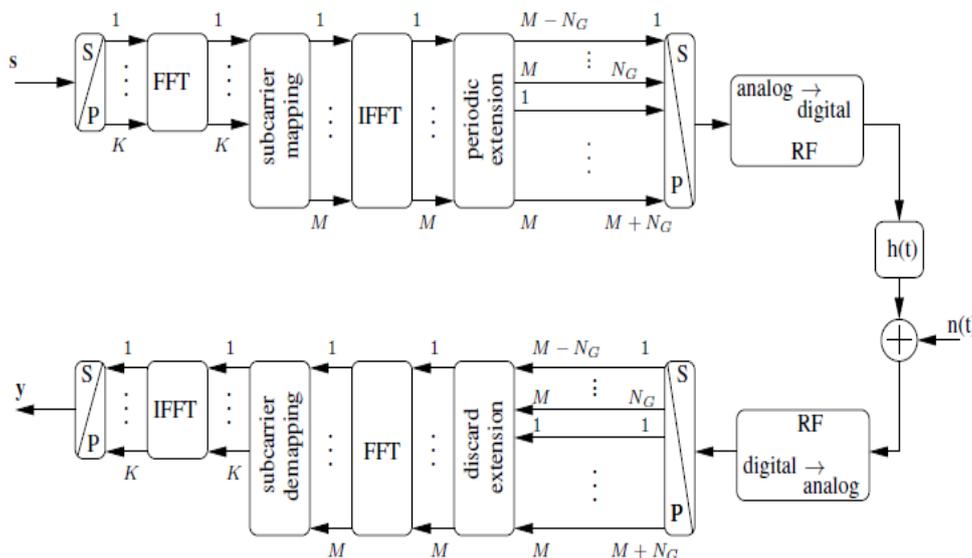


Figure 1: Signal processing for SC-FDMA [8].

Then the spread transmit symbols are mapped on the subcarriers of the OFDMA subsystem. This can be either performed localized (i.e. to a continuous block of subcarriers) or distributed (i.e. equally spaced over the subcarriers of the OFDMA system). Although the distributed mapping exhibits a lower PAPR than the localized mapping, it is favored over the distributed mapping due to an easier scheduling between the users and smaller vulnerability against frequency offsets. The mapped and spread symbols are transmitted over the channel using conventional OFDMA modulation. The conversion to the frequency domain is performed using an inverse FFT (IFFT), with the size of the IFFT depending on the subcarrier spacing and the total amount of bandwidth available in the system. After the transmission over the channel and the addition of white Gaussian noise, the received signal is demodulated by performing the inverse operations of the modulator. To keep the complexity for the simulations on a tolerable level, the maximum size of the IFFT in the transmitter is fixed to 512. Thus a maximum of 300 subcarriers are usable according to the specifications. Thus, OFDM based SC-FDMA is a multicarrier modulation technology, and can provide high spectral efficiency, low implementation

complexity, less vulnerability to echoes and non-linear distortion. Because of these advantages of SC-FDMA system, it is vastly used in various modern communication systems.

**Power spectrum:-**

The goal of spectral density estimation is to estimate the spectral density of a random signal from a sequence of time samples. Depending on what is known about the signal, estimation techniques can involve parametric or non-parametric approaches, and based on time-domain or frequency-domain analysis. The spectral density is usually estimated using Fourier transform methods (such as the Welch method). The spectrum analyzer measures the magnitude of the short-time Fourier transform (STFT) of an input signal. The power of a signal is distributed over the different frequencies. Quasi periodic signals give peaks at linear combinations of two or more irrationally related frequencies (often giving the appearance of a main sequence and sidebands); and chaotic dynamics give broad band components to the spectrum.

**Linear equalization**

A linear equalizer is assumed and implemented with a  $N = 2L+1$  tap transversal filter which follows the following equation.

$$H_{eq}(z) = \sum_{i=-L}^L w_i z^{-i} \tag{1}$$

Obviously, the greater the value of N with the filter, the better will be the system. But this will add to the complexity to filter, so this is an important consideration in designing the filter. Now with a known equalizer size N, the design must specify the tap weights  $w_i$  for a given frequency response. Also, the equalization algorithm must vary these weight values as the channel varies so as to maintain the uniformity of the channel. As it is known that, channel transmission is rated on the probability of error happening, so the equalizer would need to minimize this probability of error from occurring.

**Non-Linear equalization**

The Decision Feedback Equalizer (DFE) is a non-linear equalizer. The DFE is implemented with two digital non-linear transversal filters. The filters are digital because they operate on sequences. The transversal structure can be represented by several delay elements and weighted coefficients. The filters are non-linear as they have one or more delay elements. Figure (2) illustrates a simplified block diagram of the DFE. It contains the forward digital filter  $W(z)$  and the feedback filter  $V(z)$  in the feedback loop. Both the filters have transversal structures. Furthermore the equalizer contains a decision device.

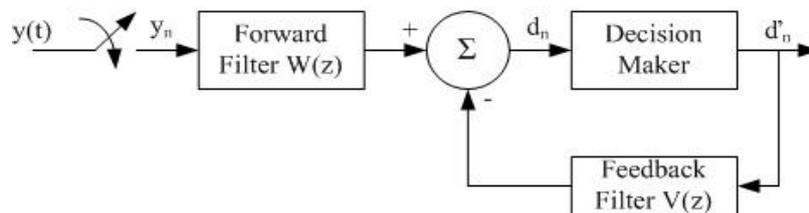


Figure 2: DFE function block [4].

Essentially the concept of the DFE is to estimate the ISI contribution of past decisions and with negative feedback, subtract it from the incoming data. If the filters can't accurately estimate the ISI contribution from past decisions or the past decisions are incorrectly detected then the equalizer will suffer from error propagation. The operation of the DFE can be modeled in terms of mathematics. Firstly the model is the same as in Figure (2) with the linear filter replaced by the DFE. Hence data  $d(t)$  passes through the pulse shaping filter  $p(t)$  and the ISI time variant channel with impulse response  $c(t)$  and additive white Gaussian noise  $v(t)$ . This is essentially  $w(t)$ , which is then passed through the matched filter  $g^*(-t)$ . The received signal  $y(t)$  is described by the expression below.  $v(t)$  passed through the matched filter yields  $n(t)$ .

$$y(t) = d(t) \otimes f(t) + n(t) \tag{2}$$

As the equalizer is implemented digitally, the received analog signal is sampled at a rate  $f_s=1/T_s$  before being processed by the equalizer. The expression below describes received sampled signal.

$$y_n = \sum_{k=0}^M f_k \cdot d_{n-k} + n(nT_s) \tag{3}$$

This is just the discrete convolution expression for the data  $d_{n-k}$  and the combined channel description  $f_k$ . M is the length of the combined channel response sequence  $f_k$ . Furthermore,  $y_n$  can be expressed in terms of the desired decision, the ISI decisions and the noise as shown in the expression below.

$$y_n = d_n \cdot f_0 + \sum_{k=1}^M f_k \cdot d_{n-k} + n_n \tag{4}$$

It is to be noticed that the middle term above is due to the ISI. The first term is the desired decision and if the channel is free from ISI then the received signal is just scaled by the combined channel response  $f_0$  with the added sampled channel noise  $n_n$ . The decision device basis its decision according to how similar the incoming data is to one of the constellations. The error between the constellation and the received data is used by an algorithm to update the filter tap weights. This process is called equalizer tracking. The filter coefficients estimate the channel response and attempt to minimize the error. Training sequence and process of Tracking Equalizer training and tracking are adaptive techniques used by the system to “learn” about the channel characteristics and update the system so as to reduce the imperfections caused by the channel. Equalizer training or adaptive equalization continuously estimates the channel and updates the coefficients of the equalizer accordingly. The training algorithm must be carried out before the channel changes drastically. There are several different algorithms used for this purpose which differ according to complexity, number of calculations or iterations and rate of convergence of the calculations. MMSE is the most complex algorithm used however the error convergence is quadratic. Least Mean Square is the simplest to implement however the error converges linearly hence it is slow.

**Using Equalizer to transmit without ISI**

Figure (3) shows that an input signal  $x_k$  is passed through a pulse shape filter  $g(t)$  and then transmitted over an ISI channel with impulse response  $c(t)$ .  $y(t)$  is the output of matched filter which then sampled into  $y(n)$ ,  $w(t)$  is the input of matched filter having impulse response  $g_m^*(-t)$ . The equivalent channel impulse response is then given by

$$h(t) = g(t) \otimes c(t) \tag{5}$$

The transmitted signal is then equal to:

$$x(t) \otimes g(t) \otimes c(t) = \sum_k x_k \delta(t - kT_s) \otimes g(t) \otimes c(t) \tag{6}$$

Where  $x(t)$  is the train of information symbols sent from the transmitter. The pulse shape filter  $g(t)$  is controlled by the designer but the channel  $c(t)$  is outside the designers control.

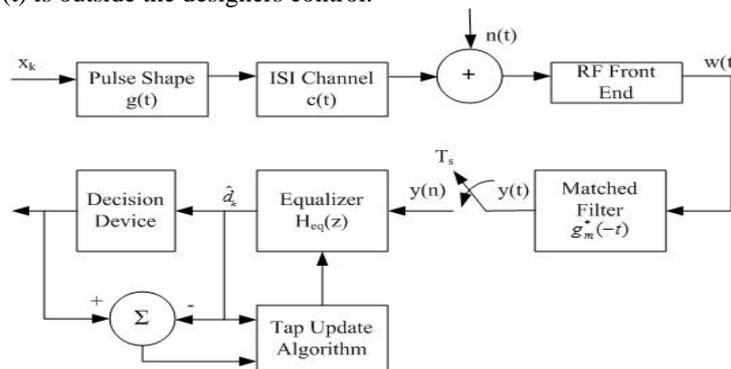


Figure 3: Model of complete end-to-end system [4].

At the receiver end additive white Gaussian noise (AWGN)  $n(t)$  is added to give a resulting signal of  $w(t)$  and then passed through a matched filter  $(-t)$ . A matched filter is used to maximize the SNR prior to sampling. The matched filter is modeled from the channel  $h(t)$  so ideally  $g_m(t)=h(t)$ . However, as the channel impulse response is time varying, so it is generally not possible to get  $g_m(t)=h(t)$ . Thus part of the challenge is to design  $g(t)$  to get good performance. Often the matched filter is simply modeled from the pulse shape filter which is optimal when  $c(t) = \delta(t)$  but sub-optimal when  $c(t) \neq \delta(t)$ . This problem can be reduced by sampling  $y(t)$  at a rate much faster than the symbol rate and designing the equalizer for this over sampled signal. This is known as Fractionally-spaced equalization (FSE).

**II. RESULTS**

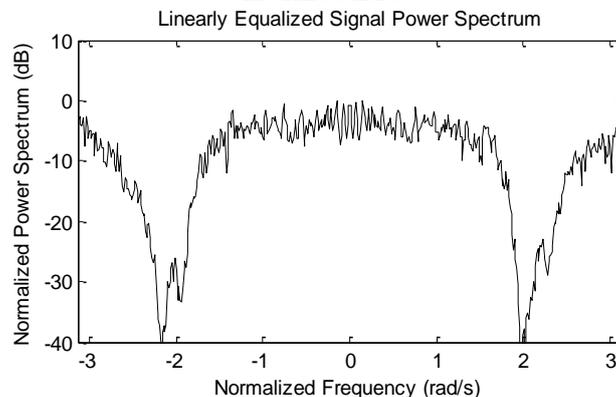


Figure4: Equalized signal Power spectrum for linear equalizer

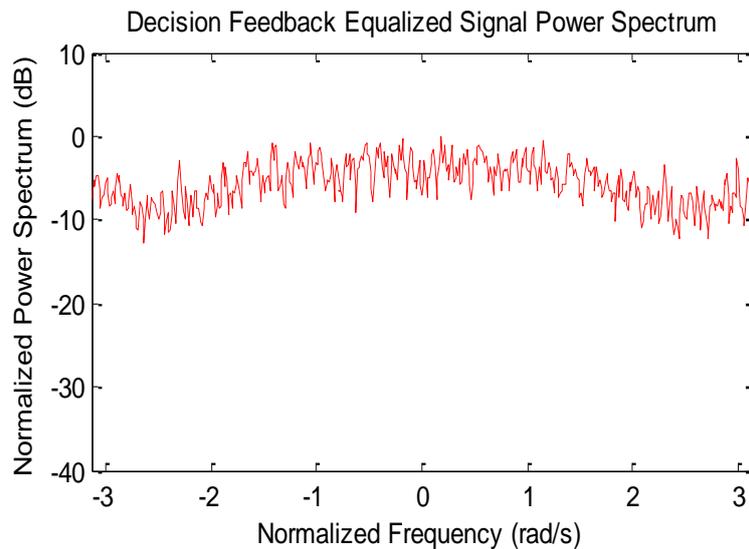


Figure 5: Equalized signal Power spectrum for Decision feedback equalizer

### III. CONCLUSION

In this paper we studied single carrier frequency division multiple access (SC-FDMA). The result of power spectrum for linear equalizer and Decision feedback equalizer in SC-FDMA is shown. The performance evaluation of both equalizers are based on power spectrum. The linearly equalized signal spectrum has a progressively deeper null. The DFE is much better able to mitigate the channel null than the linear equalizer as shown in the power spectrum plot. The Decision Feedback Equalizer (DFE) is a non-linear equalizer. The DFE is implemented with two digital non-linear transversal filters. We used a 31-tap linear equalizer and a DFE with 15 feed forward and 15 feedback taps. We used the recursive least squares (RLS) algorithm for the first block of data to ensure rapid tap convergence and use the least mean square (LMS) algorithm thereafter to ensure rapid execution speed.

### References

1. Adão Silva, José Assunção, Rui Dinis and Atilio Gameiro, "Performance evaluation of IB-DFE-based strategies for SC-FDMA systems", *EURASIP Journal on Wireless Communications and Networking*, vol. 2013:292, 2013.
2. Jiayi Zhang, Lie-Liang Yang and Lajos Hanzo, "Energy-Efficient Dynamic Resource Allocation for Opportunistic-Relaying-Assisted SC-FDMA Using Turbo-Equalizer-Aided Soft Decode-and-Forward", *IEEE Transactions On Vehicular Technology*, Vol. 62, No. 1, January 2013.
3. Atapattu, Lakmali Nadisha Kumari, "Novel channel tracking and equalization methods in MU-MIMO-OFDM systems", in *World of Wireless Mobile and Multimedia Networks, 2013 IEEE International Symposium on a*, 4-7 June 2013, Carlos III University of Madrid, Spain.
4. Jiayi Zhang, Lie-Liang Yang and Lajos Hanzo, "Frequency-Domain Turbo Equalization in Coded SC-FDMA Systems: EXIT Chart Analysis and Performance", *Vehicular Technology Conference, 2012, IEEE*, 3-6 Sept. 2012, Quebec City, QC.
5. Uyen Ly Dang, Michael A. Ruder, Wolfgang H. Gerstacker, and Robert Schober, "MMSE Beam forming for SC-FDMA Transmission over MIMO ISI Channels with Linear Equalization", *IEEE Globecom 2010 proceedings*, 2010.
6. Eva Peiker-Feil, Nicolas Schneckenburger, Werner G. Teich and Jürgen Lindner, "Improving SC-FDMA Performance by Time Domain Equalization for UTRA LTE Uplink", *Institute of Information Technology, University of Ulm, Ulm, Germany*, 2010.
7. O. Rousseam and G. Leus, "Gaussian Maximum-Likelihood Channel Estimation With Short Training Sequences", *IEEE Transactions on Wireless Communications*, Vol. 4, No. 6, November 2005.
8. M. Jiang, D. Yaun and Q. Yin, "A new adaptive equalization technology in time varying multipath channel", *IEEE International Symposium on Microwave, Antenna, Propagation and EMC Technologies for Wireless Communications Proceedings*, 2005.
9. G. Panci, P. Campisi, S. Colonnese and G. Scarano, "Blind Phase Recovery for QAM Constellations", *IEEE Signal Processing*, Vol.53, No.4, April 2005.
10. M. A. Diaz and R. L. Valcarce, "Diamond contour-based phase recovery for (cross)-QAM constellation", *IEEE Signal Processing*, 2005.
11. O. Rousseaux and G. Leus, "Gaussian Maximum-Likelihood Channel Estimation With Short Training Sequences", *IEEE Transactions on Wireless Communications*, Vol. 4, No. 6, November 2005.
12. M. Jiang, D. Yaun and Q. Yin, "A new adaptive equalization technology in time varying multipath channel", *IEEE International Symposium on Microwave, Antenna, Propagation and EMC Technologies for Wireless Communications Proceedings*, 2005.

13. G. Panci, P. Campisi, S. Colonnese and G. Scarano, "Blind Phase Recovery for QAM Constellations", IEEE Signal Processing, Vol.53, No.4, April 2005.
14. M. A. Diaz and R. L. Valcarce, "Diamond contour-based phase recovery for (cross)-QAM constellation", IEEE Signal Processing, 2005.
15. M. D. Paura, L. Sterle, "Widely linear MMSE equalizer for MIMO linear time-dispersive channel", Telecommunication, University di Napoli Federico II, Italy, Vol. 2, 2003.
16. S.Y. Yee and M. Sandell, "Iterative channel estimation with MIMO MMSE-turbo equalisation", Vehicular Technology Conference, 2003. VTC 2003-Fall. 2003 IEEE 58th, Vol. 2, 2003.
17. M. A. Diaz and R. L. Valcarce, "Diamond contour-based phase recovery for (cross)-QAM constellation", IEEE Signal Processing, 2005.
18. K. V. Cartwright and J. Kaminsky, "Blind Phase Recovery in Cross QAM Communication Systems with the Reduced Constellation Eighth Order Estimator (RCEO)", IEEE Globecom 2005.
19. Weihua Zhuang, "RLS Algorithm with Variable Forgetting Factor for Decision Feedback Equalizer over Time Variant Fading Channels" *Wireless Personal Communications* 8: 15–29, 1998.
20. Jovana Ilic and Thomas Strohmer" SPARSITY BASED ADAPTIVE THRESHOLDING FOR DFE IN SC-FDMA" 19th European Signal Processing Conference, August 29 - September 2, 2011