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Optimal Design of Digital IIR Band Pass Filter Using Practical Swarm Optimization

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Abstract- In this paper, a soft computing technique is proposed to solve the problem of designing an optimal digital infinite-impulse response (IIR) filter. This paper develops a nature inspired optimization methodology for the robust and stable design of digital IIR BP filter. Partical Swarm Optimization (PSO) technique is applied to design the optimal digital IIR filter in order to avoid local minima as the error surface of IIR filters is basically multi-modal and nonlinear. PSO is a global search method, which utilizes a set of particals that move through the whole search space to find the global minimum of our objective function. The goal of the optimization process is to find the optimal filter coefficients approximate closely to the desired filter response. To minimizing the Lp-norm approximation error and the ripple magnitudes of both pass band and stop band, a multi-criterion combination is employed to obtain the optimal IIR filter as our design criterion. The coefficients that are required essentially to design a digital IIR BP filter are calculated by using the above mentioned PSO algorithm. The magnitude response and the stability of the filter are determined by using Matlab with satisfying results. The pole-zero plot of the filter, described the stability of the predesigned filter. It has been demonstrated that the proposed approach can optimize the digital IIR filter in terms of minimizing the magnitude errors and minimizing the ripple magnitudes of both pass band and stop band.

Index Terms— Digital Infinite-Impulse Response (IIR) Filter, Partical Swarm Optimization (PSO) algorithm, L_1-Norm Error, L_2-Norm Error, Stability.

I. INTRODUCTION

Digital filter designing is always an important issue in digital signal processing. Digital Signal Processing (DSP) is an area of science and engineering that has developed its importance to both theoretically and technologically rapidly over the last few decades. DSP is the processing of signals by digital means. Basically, digital signal processor is an integrated circuits, that are designed for high-speed data manipulations and are used in various applications like audio, video communications, image processing, data-acquisition and other data-control applications. A digital filter uses a digital processor to perform numerical calculations and manipulations on the sampled values of the signal. DSP have advantages like low sensitivity to component tolerances and temperature change, high noise immunity. Digital filter is one of the most powerful tool of digital signal processing.

Filters are used to remove the unwanted portion of signal such as noise, (which could be generated due to unavoidable obstacles of the environment) and to extract the useful portion of the signal, from the input signal. In signal processing systems, filters are widely employed in applications like noise reduction, radars, channel equalization, speech synthesis, biomedical signal processing etc. Generally, on the basis of design and filtering, filters may be analog or digital in nature. Analog filters have electronic active and passive elements such as resistor, capacitor etc. A digital filter is a system which passes some desired discrete timed signal more than others (noise) to reduce or enhance the certain features of that signal. They can be used to pass signals according to the specified frequency in the pass-band and reject the frequency other than the pass-band specification in the stop-band. Nowadays, digital filters are used to perform almost all filtering tasks, previously which were performed by analog filters. Thus, digital filters are replacing the traditional role of analog filters almost in every application. The basic filter may be out of these four types: LP (Low-Pass), HP (High-Pass), BP (Band-Pass), and BS (Band-Stop).

But on the basis of impulse response, digital filters may be classified as IIR (Infinite Impulse Response) or FIR (Finite Impulse Response). An FIR digital filter has impulse response of finite duration. The current output of this filter is calculated solely from the present and past input values. Hence, these filters are said to be non-recursive. They have strictly exact linear phase and arbitrary amplitude-frequency characteristics. On the other hand, an IIR filter is one whose impulse response is continues i.e infinite and are termed as recursive filters. The current output of such type of filters, depends upon the present and past input values as well as on the previous output values. The concept of feedback is involved in IIR filters. Mostly, FIR filters are always highly stable but IIR filters are always unstable so there is not any issue of stability for designing the filter. However, a sharp narrow frequency response for transition-band can be easily realized with IIR filters. The key practical advantage of utilizing IIR filters is that IIR system is much more efficient to implement than FIR system. They are capable of yielding very accurate responses than FIR filters, when they are properly designed.

For the design of digital IIR filter, there are mainly two approaches: one is transformation approach and other is optimization approach. In first transformation approach, the analog IIR filters are designed first, and then these analog filters are transformed to the digital IIR filter at a given set of prescribed specifications. Generally a bilinear transformation method is adopted in the transformation approach. But the digital IIR filter designed by using the transformation approach is not good in performance like stability etc. So the optimization approach is preferred. As in the optimization approach, various optimization methods have been proposed to obtain optimal filter design specifications, where the stability, *Lp*-norm error and ripple magnitudes (tolerances) of both the pass band and stop band of the filter are usually used as criteria to measure the desired performance of the digital IIR filter. Optimization is the methodology of making a system as fully perfect or effective as such as possible. Optimization is a process of finding the condition that gives the maximum or minimum value of a function f(x). According to the type of the objective functions and search space, optimization methods are mainly of three types: 1) Direct search methods:- Direct search methods are best known as unconstrained optimization techniques as they don't explicitly use the derivatives of the function; 2) Gradient based methods:- In these methods, all the problems having derivatives are optimized. This method is applicable to continuous problem; 3) Nature inspired methods:- Nature inspired algorithms takes inspirations from the nature to optimize the problems. However, as the error surface of digital IIR filters are generally multimodal and nonlinear in nature, the various conventional gradient-based design methods are easily get trapped in the local minima of the error surface of the digital filter. So, to deal with the designing of digital IIR filter, it is very necessary to develop an efficient optimization algorithm approach.

For the design of a digital IIR filter, we must strictly impose the following constraints in order to meet the overall design criteria as in a satisfactory fashion: 1) determination of the lowest order of the filter, as low as possible; 2) stability of the filter i.e. the poles must lie inside the unit circle; and 3) fulfilment of the prescribed tolerance settings conditions that are determined by minimizing the (tolerances) ripple magnitudes errors of both pass band and stop band. But in the process of optimization, these constraints always pose a great difficulty to satisfies the final digital filter design as to obtain the desired filter performance. The goal of the optimization is to find the optimal coefficients of the digital IIR filter, approximately closely to the desired response of the filter. Therefore, in the literature, there are various search methods that address the optimization problem under different conditions such as Real Coded Genetic Algorithm (RGCA) [5], Hierarchical Genetic Algorithms (HGA) [8], Heuristic Search Method [9], Genetic Algorithm (GA) [10], Hybrid Taguchi Genetic Algorithm (HTGA) [11], Improved Immune Algorithm (IIA) [13], and many more for the design of digital IIR filters. There results were good but research beings to improve results by applying various optimization techniques for obtaining better results than previous ones and for minimizing the computational time that were quite large in previous methods.

Therefore, in this paper, a robust and stable optimization approach to solve the digital IIR filter design problem is applied which is named as the Partical Swarm Optimization (PSO) and to test the optimization procedure, the proposed PSO algorithm is implemented in Matlab and optimized results are found. Basically, a particle swarm optimization is a heuristic global optimization technique that was put forward in 1995. It is developed from swarm intelligence and is based on the research study of the behaviour of fish schooling and bird flocking movements. The basic PSO algorithm has various advantages; 1) PSO is based on the swarm intelligence. It may be applied to both engineering use and scientific research; 2) With 'speed' knowledge of particles, search can be carried out easily in PSO as only the most optimist particle can transmit information to the other particles in the search space; 3) Speed of the optimization is fast as compared to other techniques; 4) PSO have very simple calculation and can be easily evaluated.

The remainder of this paper is organized as follows. Section II describes the IIR filter design problem statement. The PSO algorithm for designing the optimal digital IIR filter is described in Section III. In Section IV, the performance of the proposed PSO algorithm has been evaluated and there results are compared with the design results of Kaur Ranjit *et al.* [5], Tang *et al.* [8], Tasi *et al.* [11] for the design of digital IIR BP filter. At last, the conclusions and discussions are outlined in Section V.

II. IIR FILTER DESIGN PROBLEM

A digital filter design problem involves the determination of a various set of the coefficients of the filter which meet the desired performance specifications like as pass-band width and their gain, stop-band width and its attenuation, frequencies at the band edges and peak ripple in the pass band and stop-band that are tolerable. The design of the digital IIR filter is described by the difference equation that is stated below:

$$y(n) = \sum_{k=0}^{M} a_k x(n-k) - \sum_{k=1}^{N} b_k y(n-k) \qquad (1)$$

where a_k and b_k are the coefficient of the filter. x(n) and y(n) are input and output of the filter. M and N are the number of filter coefficients a_k and b_k , with N ≥ M.

The equivalent transfer function of IIR filter is described as

$$H(z) = \frac{\sum_{k=0}^{M} a_k \; z^{-k}}{1 + \sum_{k=1}^{N} b_k z^{-k}} \qquad (2)$$

To design digital filter the important task of the designer is to find a set of filter coefficients, which meet specified desired performance response. A most used way of realizing IIR filters is to cascade various first-order and second-order filter sections simultaneously together to avoid instability. The structure of cascading type digital IIR filter, is stated below having, $H(\omega, x)$ transfer function.

$$H(\omega,x) = \mathrm{A}\prod_{i=1}^{M}\frac{1+a_{1i}e^{-j\omega}}{1+b_{1i}e^{-j\omega}} \times A\left(\prod_{k=1}^{N}\frac{1+r_{1k}e^{-j\omega}+r_{2k}e^{-2j\omega}}{1+s_{1k}e^{-j\omega}+s_{2k}e^{-2j\omega}}\right) \quad (3)$$

Here

$$x = [a_{11}, b_{11}, \ldots, a_{1M}, b_{1M}, r_{11}, r_{21}, s_{11}, s_{21}, \ldots, r_{1N}, r_{2N}, s_{1N}, s_{2N}, A]$$

and it denotes the coefficients of the filter with number of coefficients = 2M + 4N + 1 and A is the gain of the filter. This equation have digital IIR filter with same degree to both numerators and denominators.

The performances of the digital IIR filter can be calculated by using the Lp-norm approximation error of magnitude response and the ripple magnitudes of both pass-band and stop-band. The digital IIR filter is designed by optimizing the filter coefficients so that the Lp-norm approximation error function for magnitude is to be minimized. The absolute L_1-norm error, $e_1(x)$ of magnitude response is stated as below:

$$e_1(x) = \sum_{i=0}^{k}\left|H_d(\omega_i) - |H(\omega_i,x)|\right| \quad (4)$$

and the squared L_2-norm error, $e_2(x)$ of magnitude response is stated as below:

$$e_2(x) = \sum_{i=0}^{k}\left(\left|H_d(\omega_i) - |H(\omega_i,x)|\right|\right)^2 \quad (5)$$

Where,

$H_d(\omega_i)$ is desired magnitude response of IIR filter

$H(\omega_i, x)$ is obtained magnitude response of IIR filter.

$$H_d(\omega_i) = \begin{cases}1, & for\ \omega_i\ \epsilon\ passband\\ 0, & for\ \omega_i\ \epsilon\ stopband\end{cases} \quad (6)$$

$\delta_1(x)$ and $\delta_2(x)$ are the ripple magnitudes of pass-band and stop-band, respectively which are to be minimized, $\delta_1(x)$ and $\delta_2(x)$ are defined respectively as:

$$\delta_1(x) = max_{\omega_i}\{|H(\omega_i,x)|\} - min_{\omega_i}\{|H(\omega_i,x)|\} \qquad for\ \omega_i\ \epsilon\ passband \quad (7)$$

and

$$\delta_2(x) = max_{\omega_i}\{|H(\omega_i,x)|\} \qquad for\ \omega_i\ \epsilon\ stopband \quad (8)$$

The multi-criterion constrained optimization problem which is obtained by aggregating the all objectives and stability constraints is stated below as:

$$\begin{aligned}&\text{Minimize} \quad f_1(x) = e_1(x)\\ &\text{Minimize} \quad f_2(x) = e_2(x)\end{aligned} \quad (9)$$

Subject to the following stability constraints

$$\begin{aligned}&1 + b_{1i} \geq 0\ (i = 1,2,\ldots,M)\\ &1 - b_{1i} \geq 0\ (i = 1,2,\ldots,M)\\ &1 - s_{2k} \geq 0\ (k = 1,2,\ldots,N)\\ &1 + s_{1k} + s_{2k} \geq 0\ (k = 1,2,\ldots,N)\\ &1 - s_{1k} + s_{2k} \geq 0\ (k = 1,2,\ldots,N)\end{aligned} \quad (10)$$

For designing digital IIR filter, in the multi-criterion constrained optimization problem a single non-inferior point can be generated by solving following:

$$\min f(x) = \sum_{j=1}^{2} w_j f_j\,(x) \quad (11)$$

Where,

$f(x)$ is objective function

w_j are the weights having non-negative real numbers.

The design of causal recursive IIR filter requires the inclusion of stability constraints. Therefore, the stability constraints, which are obtained by applying the Jury method [14] on the coefficients of the digital IIR filter, must have been included in the optimization process.

III. PARTICLE SWARM OPTIMIZATION (PSO)

Particle Swarm Optimization (PSO) is a robust, flexible and population based stochastic optimization technique that is based on the intelligence and movement of swarms. Also, PSO has capability to handle non-differential objective function and larger search space. A group of particles are randomly set into motion through this search space. At each iteration, the "fitness" value of them and their neighbours are observed and by moving towards them, they "emulate" successful neighbours, whose current position has a better solution to the desired problem than their own position. PSO approach applies the concept of social interaction to solve the various problems.

PSO was originally invented by James Kennedy and Russell Eberhart after being inspired by the study of the behaviour of bird flocking by biologist Frank Heppner. It is related to natural-inspired problem solving techniques. It uses a number of particles (agents) that constitute a swarm (group) moving in the search space that are looking for the best solution. Each particle is treated as an independent point in search space which adjusts its “flying” (position) according to their own’s flying experience and also according to the flying experience of other particles. Each particle in the search space keeps track of its coordinates, which are associated with the fitness value that has been achieved by that particle so far. This value is known as personal best, pbest. Also, there is an another best value that have the knowledge of the global or their neighbourhoods’ best, which is the best value that is obtained so far by any particle in the neighborhood of that particle in the search space. This value is known as global best, gbest.

As stated above, PSO simulates the social behaviour of bird flocking and fish schooling. Suppose a scenario in which a number of birds are randomly searching food in a particular area and there is only one piece of meal in the area that is being searched and all the birds do not have any information about the food. So now the best strategy to find the food is to follow the bird that is nearest to the food. PSO learned from these types of scenarios and used them to solve optimization problems. In a PSO approach, each ‘bird’ in the whole search space and is called as "particle". All of the particles in the search space have some fitness values which are evaluated by the fitness function which is to be optimized and have some velocities by which the flying of the particles are directed in that search space.

A PSO approach is firstly, initialized with a number of random particles (solutions, birds) and then search begins for the optima by updating the generations. In every iteration, every particle is updated his values by the personal best (pbest) value that has achieved so far. Then the global best (gbest) value is tracked by the PSO, is obtained so far by any particle in the population. Particles of the swarm fly through the search space and also have their memory of their own best position which is known as (lbest) local best.

After finding the ‘pbest’ and ‘gbest’ values, the particle updates its velocity and positions with the help of two equations that are given below:-

v[] = w * v[] + c_1* rand() * (pbest[] - present[]) + c_2* rand() * (gbest[] - present[]) (12)

present[] = present[] + v[] (13)

where,
w is weight with w_{max} = 0.4 & w_{min} = 0.1,
v[] is the velocity of a partical,
present[] is the current partical value,
pbest[] and gbest[] are pbest and gbest which are stated before.
rand () is a random number that have value between (0,1).
c_1, c_2 are learning factors usually c_1& $c_2 = 2$.

Algorithm: Partical Swarm Optimization

```
For each particle
    Initialize particle
END
Do
  For each particle
  Calculate the fitness value
   If the fitness value is better than the best fitness value (pbest) in the history
      set the current value as the new pbest
    End
 Choose the particle with the best fitness value of all the particles as the gbest
    For each particle
        Calculate particle velocity according to equation (12)
        Update particle position according to equation (13)
    End
While maximum iterations or minimum error criteria is not attained
```


On every dimension, the particle's velocities are clamped to Vmax which is a maximum velocity. If by the applied sum of accelerations, it would causes the velocity to exceed maximum velocity i.e Vmax on that dimension, which is specified by the user, then the value of velocity on that dimension must be limited to Vmax value.

IV. DIGITAL IIR BAND-PASS FILTER DESIGN AND COMPARISONS

A Partical Swarm Optimization (PSO) algorithm approach is applied to design the digital IIR BP filter. For the purpose of comparison, the lowest order of the digital IIR BP filter is set exactly same as that given by Kaur Ranjit *et al.* [5], Tang *et al.* [8], Tasi *et al.* [11] for the BP filter. Therefore, the order of the digital IIR BP filter is not a variable, is a fixed number. The purpose of designing digital IIR BP filter is to minimize the $f(x)$ function stated in equation (11) with the stability constraints given by equation (10) under the prescribed designing conditions for BPF, that are given below in Table I.

TABLE I
PRESCRIBED DESIGN CONDITIONS ON BP FILTER

Filter type	**Pass-band**	**Stop-band**	**Maximum $\lvert \mathbf{H}(\boldsymbol{\omega}, \boldsymbol{x})\rvert$**
Band-Pass(BP)	$0.4\pi \le \omega \le 0.6\pi$	$0 \le \omega \le 0.25\pi$ $0.75 \le \omega \le \pi$	1

For designing a digital IIR BP filter 201 equally spaced points are set within the described frequency range. In the proposed PSO approach, for the designing of digital IIR BP filter the criteria of minimizing the L_1-norm and L_2-norm approximation errors are considered. For designing, the designer of the filter must need to adjust the weights of function according to the filter specifications. The computational results obtained by the proposed PSO approach are presented and compared with the results obtained by [5], [8], [11] are shown in Table II. Magnitude error versus iterations graph is drawn in figure 1 and the frequency response and pole-zero plot of digital IIR BP filter using PSO approach is drawn in figure 2. The designed digital IIR BP filter model obtained by the PSO approach is given as below.

$$H_{BP}(z) = 0.021955\left(\frac{(z^2 - 0.064799z - 0.778450)(z^2 + 0.222208z - 1.013286)}{(z^2 - 0.582334z + 0.808129)(z^2 + 0.007919z + 0.610140)}\right) \times \left(\frac{z^2 - 0.033806z - 0.912367}{z^2 + 0.594650z + 0.805044}\right) \quad (14)$$

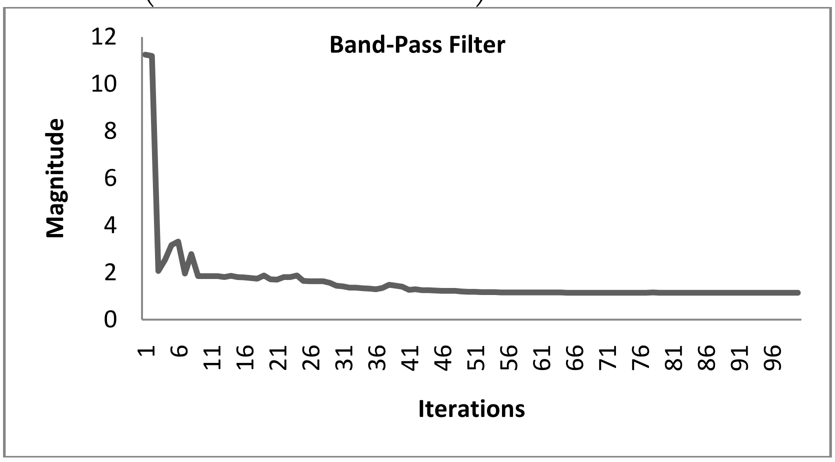

Fig. 1 Magnitude error versus iterations graph of band-pass filter using PSO approach

Table II: Design results for digital IIR Band-Pass Filter

Method	**L_1-norm error**	**L_2-norm error**	**Pass-band performance (Ripple magnitude)**	**Stop-band performance (Ripple magnitude)**
PSO Approach	1.1395	0.1423	$0.9667 \le \lvert H(e^{j\omega})\rvert \le 1.0229$ (0.0562)	$\lvert H(e^{j\omega})\rvert \le 0.0362$ (0.0362)
RCGA Approach[5]	1.4062	0.1961	$0.9862 \le \lvert H(e^{j\omega})\rvert \le 1.0050$ (0.0187)	$\lvert H(e^{j\omega})\rvert \le 0.0598$ (0.0598)
HTGA Approach[11]	1.9418	0.2350	$0.9760 \le \lvert H(e^{j\omega})\rvert \le 1.0000$ (0.0234)	$\lvert H(e^{j\omega})\rvert \le 0.0711$ (0.0711)
HGA Approach[8]	5.2165	0.6949	$0.8956 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.1044)	$\lvert H(e^{j\omega})\rvert \le 0.1772$ (0.1772)

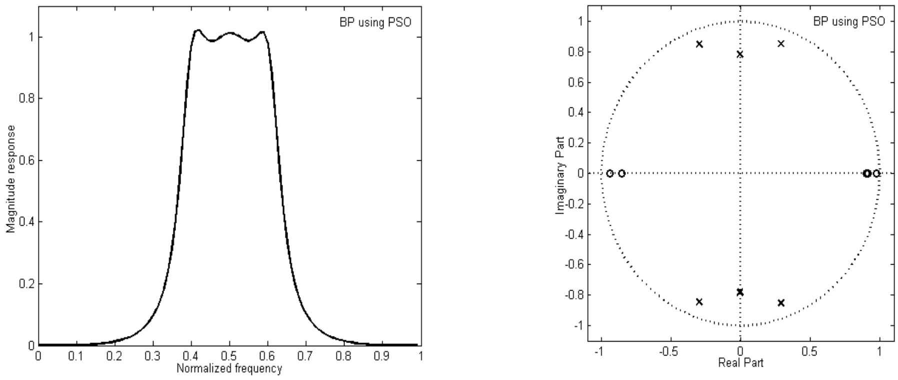

Fig. 2 Frequency response and pole-zero plot of band pass filter using PSO approach

The maximum value, minimum value and average value of magnitude error along with the standard deviation after 20 runs is shown in Table III.

Table III: Maximum, Minimum, Average value of L_1 and Standard deviation

Maximum value	**Minimum value**	**Average value**	**Standard deviation**
5.910318	1.133590	1.540444	0.99104

As shown in above Table III , the value of standard deviation is less than one; which assured the robustness of the designed system.

V. CONCLUSION

The applied PSO approach possesses the merits of global exploration, fast convergence, and robustness. The proposed PSO approach is executed to solve the multi criterion optimization problem of designing digital IIR BPF. The simulation results show that: 1) the results obtained by PSO approach in terms of magnitude error and ripple magnitudes in pass-band and stop-band are better or at least comparable to the results obtained by [5], [8] and [11]. More over the pole-zero plot as shown in figure 2 for the proposed digital IIR BP filter depicts the stability of the designed filter as all its poles lies inside the unit circle.; 2) the proposed PSO approach allows each filter to be independently designed, whether it may be any out of LP, HP, BP, BS; and 3) the proposed PSO approach is very much flexible, especially when the multiple criteria, complicated constraints, and design requirements are involved. Thus it is observed that the proposed technique for the design of digital IIR BP filter is capable of giving higher performance than others. It is concluded that Particle swarm optimization algorithm is a global search optimization technique for the design of IIR band-pass digital filter, and the benefits of PSO for designing digital filter have been studied.

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