



## Fingerprint Indexing using Low Order Delaunay Triangulation with Geometric Constraints

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**Abstract**— *Fingerprint Identification has been a great challenge due To its complex search of huge fingerprint databases. Fingerprint Indexing, a preprocessing phase before matching, aims to reduce the number of candidate hypotheses sent for further verification. Indexing large fingerprint databases eliminates the false positives and hastens the process of verification. This paper presents a method of indexing large databases which makes use of minutiae triangles and orientation images of a fingerprint as input. Low order Delaunay triangulation has been used here which significantly reduces the number of minutiae triangles. The different features of a minutiae triangle used here are the angles, handedness, type, circularity ratio and ratio of minimum and maximum sides to perimeter of triangle. This method tested on all the FVC 2004 databases which are markedly more difficult than previous versions of databases because of the induced perturbations. The algorithm has pruned the search space significantly over all the four databases.*

**Keywords** – *Fingerprint, Minutiae, Delaunay triangulation, Minutiae Triangle, Indexing*

### I. INTRODUCTION

Fingerprints are widely used biometrics for identification in civilian and commercial applications to identify an individual and verify their identity. Because of their uniqueness and immutability, fingerprints have been used for over a century, more recently becoming automated due to advancement in computing capabilities. Fingerprint identification is popular because of the inherent ease in acquisition, the numerous sources (ten fingers) available for collection, and their established use and collections by law enforcement and immigration.

Fingerprint-based identification systems [1] search through a large database of fingerprint entries for possible matches based on the given query print. Each entry has an attempt to match the query print with each fingerprint in the database would be computationally infeasible. Therefore, a filtering process is usually invoked in order to reduce the number of candidate hypotheses for matching operation. Filtering can be achieved by two different approaches: classification and indexing. Classification involves partitioning the database into multiple classes (such as Left Loop, Right Loop, Arch Tented Arch and Whorl) and comparing the query print with prints belonging to the class assigned to the query print. This method reduces the number of images to a certain extent but due to the small number of classes and the uneven distribution of fingerprints across these classes lead to high system penetration rate and false accept rate. In contrast, indexing assigns an index value to each fingerprint and, therefore the query print is compared with those prints in the database which have comparable index values. There have been several attempts to account for fingerprint indexing. R Cappelli et al. proposed an indexing approach which gets a reasonable performance and identification time [2]. R.S. Germain et al. used the triplets of minutiae in their indexing procedure [3]. J.D. Boer et al. improved them by combining of multiple features (orientation field, FingerCode and minutiae triplets) [4]. Bhanu and Tan [5] generated minutiae triplets and used angles, handedness, type, direction, and maximum side as the features for indexing. They also applied some constraints on minutiae selection to avoid spurious minutiae. The proposed approach extends the indexing framework based on minutiae triplets proposed by Liang et al. [6]. In our approach we include the two new indexing features: circularity ratio and ratio of minimum and maximum sides to perimeter of triangle [7] and two new geometric constraints: relative position of triangle with respect to core point, and orientation at image point of all the three vertices. We also demonstrate the efficacy of the indexing process by varying the number of impressions of each fingerprint in indexing and retrieval phase. Rest of the paper is organized as follows: Section 2 discusses our proposed approach in detail. Section 3 deals with the implementation details, results, and analysis of the results. Section 4 concludes the work.

### II. PROPOSED APPROACH

Our approach for fingerprint indexing composed of two stages, indexing and retrieval phase. Model database is constructed in indexing phase and query processing is done in the retrieval phase. During the indexing phase, fingerprints in the database are processed as mentioned in [9]. The minutiae from each fingerprint forms the input for the Delaunay triangulation. After the Delaunay triangulation, low order Delaunay triangles are obtained using [6]. Indexing features are then extracted from both Delaunay and low order Delaunay triangles and stored in the model database along with ID indicating the print from which these features are obtained. During the retrieval phase the same process is repeated as in

indexing phase. For every triangle generated in the query print, model database is searched for a matching triangle and the corresponding print in database is voted. This is repeated by all the triangles in the query print and a list of ID's in the descending order of number of matching triangles is generated. These are taken as potential correspondences for the query print and are further sent to verification.

A. Page Layout

A. Features obtained from a order 1-Delaunay and Delaunay minutiae triangle

The different distortion invariant features that form part of the index are: angles of triangles, handedness, triangle type, maximum side [5], ratio of minimum and maximum sides to perimeter of triangle [7], circularity ratio.

Angles of triangle @min, @med This include the mini mum angle @min and median angle @med of a triangle. Median angle of the triangle is obtained as @med=180 - (@min+@max), where @max is maximum angle of tri- angle. Angles are chosen because they are independent of translation and rotation of image as they are the ra- tio of distances. The labelling of triangle vertices is done in such a way that P1 is the label of vertex con taing @max, P2 is label containg the angle @med, P3 is the label containing the angle @min. This labeling is maintained in the implementation for every triangle.

2. Handedness of triangle H Let  $Z_i = x_i + jy_i$  be the complex number representing coordinates  $(x_i, y_i)$   $i = 1, 2, 3$

of three vertices of triangle. Define  $Z_21 = Z_2 - Z_1$  ;  $Z_1 Z_32 = Z_3 - Z_1$  ;  $Z_2, Z_13 = Z_1 - Z_3$  then triangle hand- edness is defined as  $H = \text{sign}(Z_21 \times Z_32)$  where X is crossproduct of Z21 and Z32 and  $\text{sign}(\cdot)$  represents sign function. The cross product of two complex num bers Z21 and Z32 is defined as  $(x_2 - x_1) * (y_3 - y_2) - (x_3 - x_2) * (y_2 - y_1)$ . As P1, P2, P3 are never collinear  $H = -1$  or

3. Triangle type T: This is calculated as  $T = 4 * t_p1 + 2 * t_p3$  where  $t_p = 1$  if minutiae at P is a bifurcation else  $t_p = 0$  if minutiae at P is a bifurcation else  $t_p = 0$  if minutiae at P is ridge ending minutiae  $0 \cdot T \cdot 7$ .

4. Maximum side Lmax The length of maximum side of the triangle  $L_i = \max(L_i)$   $i = 1, 2, 3$  where  $L_1 = |Z_21|$ ,  $L_2 = |Z_32|$ , and  $L_3 = |Z_13|$ .

5. Circularity ratio Cr It is the ratio of area of the triangle with the area of a circle that has the same perimeter as the triangle. It expresses the degree of compactness of a triangle. It is insensitive to orientation and scale changes. It is given by:

$$Cr = 4 \sqrt{A} / P^2$$

where A=area of triangle, P=perimeter of triangle

6. Ratio of minimum and maximum sides to prime ter of triangle  $r_1, r_2$  These are calculated as  $r_1 = L_{min} / (P)$ ,  $r_2 = L_{med} / (P)$  where  $L_{min}$  is length of minimum side,  $L_{med}$  is length of medium side, and P is perimeter of triangle.

B. Geometric constraints

Apart from the above features Geometric constraints are used to remove any false positives obtained by using only above features. They are

Relative position of triangle with respect to Core point pos This is used to identify the position of the triangle with respect to core point of the fingerprint. This is calculated as

$$\text{pos} = \text{atan2}((G_y - P_y), (G_x - P_x))$$

where  $G_x, G_y$  are x, y coordinates of Centroid G of triangle respectively and  $P_x, P_y$  are x, y coordinates of Core point P of fingerprint respectively. In the absence of core point relative position of triangle with respect to (0,0) vertex is measured

Orientation at image of a vertex  $o_1, o_2, o_3$  This is calculated from orientation image of the finger-print. This is calculated as: Let  $P_1(x_1, y_1), P_2(x_2, y_2), P_3(x_3, y_3)$  be the minutiae triangle. Let the line equa-tion of  $P_2P_3$  be of the form  $ax + by + c = 0$  then image of  $P_1$  with respect to the side  $P_2P_3$  given by P 1 is calcu-lated as:

$$-P_1x = -2 * a * (ax_1 + by_1 + c) / (a^2 + b^2) + x_1$$

$$1y = -2 * b * (ax_1 + by_1 + c) / (a^2 + b^2) + y_1$$

$$o_1 = \text{OI} [P_1x][P_1y]$$

similarly  $o_2, o_3$  are calculated as follows:

$$\frac{-P_2x}{2x} = -2 * a * (ax_2 + by_2 + c) / (a^2 + b^2) + x_2$$

$$\frac{-P_2y}{2y} = -2 * b * (ax_2 + by_2 + c) / (a^2 + b^2) + y_2$$

$$o_2 = \text{OI} [P_2x][P_2y]$$

$$P_3x = -2 * a * (ax_3 + by_3 + c) / (a^2 + b^2) + x_3$$

$$P_3y = -2 * b * (ax_3 + by_3 + c) / (a^2 + b^2) + y_3$$

$$o_3 = \text{OI} [P_3x][P_3y]$$

where OI =orientation image of fingerprint. If the image of a vertex falls behind the fingerprint area, orien tation measured at  $(x+10, y+10)$ , where  $(x, y)$  is the vertex for which the image point is behind fingerprint area

3. Relative Rotation  $\mu_{21}, \mu_{32}, \mu_{13}$  Let  $\mu_{21} = \text{angle}(Z_{21})$ ,  $\mu_{32} = \text{angle}(Z_{32})$ ,  $\mu_{13} = \text{angle}(Z_{13})$  where  $Z_{21}, Z_{32}, Z_{13}$  are defined earlier and  $\text{angle}(Z)$  is the phase angle.

1) Conditions for two triangles to match: Two triangles are said to be matched if they satisfy the following conditions  $j L_{max} - L_{max}^0_j <$

Tmax

$$j @min - @min j < T@$$

$$j @med - @med j < T@$$

$$H = H^0$$

$$T = T^0$$

$$j r_1 - r_1^0 j < R, j r_2 - r_2^0 j < R$$

$$j Cr j Cr_0 j < C$$

$$j_{pos-pos} j < @core$$

$$j o_i - o_i j < @img \text{ where } i=1,2,3$$

$$j \mu_{21} - \mu_{21} j < \mu$$

$$j \mu_{32} - \mu_{32} j < \mu, j \mu_{13} - \mu_{13} j < \mu$$

The constants  $T_{max}, T_{C,R,C}, R_{core}, R_{img}, \mu$  represent different thresholds

### III. INDEXING ALGORITHM

Algorithm for Indexing phase :

1. For i = 1 to N, do 2 and 3.
2. Compute local orientation and extract minutiae locations in Image Ii, using feature extraction procedure
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3. Perform Delaunization on the minutiae set and also generate order 1-Delaunay triangles using algorithm in [6].
4. For every triangle obtained compute @min, @med, Lmax, H, T along with Geometric constraints mentioned in 1) of section B.

TABLE I  
FVC 2004 DATABASES

Database	Sensor type	Image size	Resolution
DB1	Optical	640X480	500 dpi
DB2	Optical	328X364	500 dpi
DB3	Thermal sweeping	300X480	512 dpi
DB4	SfinGe 3.0	288X384	¼ 500 dpi

5. Store all these features and constraints against an ID of a fingerprint that generated the triangle. This completes the indexing phase.

Algorithm for Retrieval Phase

Here we have a query print and we search in database constructed in indexing phase for possible matches with query:

Compute local orientation and extract minutiae locations for query image I

Perform Delaunization on the query minutiae set and also generate order 1-Delaunay triangles using algorithm in [6]

For every triangle generated do 3 and 4.

Compute @min, @med, Lmax, H, T for the query triangle

5. Search the database by @min, @med, Lmax, H, T under conditions mentioned in 1) of section B
6. If the triangle also satisfies geometric constraints further mentioned in section 2.2 then take it as successful correspondence of query triangle.
7. Output the list of database images that matched with the query in descending order of number Of matched triangles.

The output of retrieval phase is the list of possible matches to the query print in the database

### IV. RESULTS AND DISCUSSION

This section presents the results of proposed indexing algorithm. Speed and accuracy are two key requirements for any identification system. To characterize the accuracy of system we used the FVC 2004 databases.

#### A. Databases

FVC (Fingerprint Verification Competition) is an international competition focussed on fingerprint verification software assessment. FVC 2004 has four databases namely DB1, DB2, DB3, DB4 [8]. Each database consists of 100 unique fingerprints with 8 impressions of each, making a total of 800 images per database. Each impression represents one kind of distortion possible (translated, rotated partial, occluded) thereby increasing the robustness of the algorithm. The details of the databases are given in the TABLE-1

#### B. Experiments

We have conducted five experiments on each of the four databases mentioned. Each image in the database is referred to as i j where i denotes a unique print ranging from 1 to 100 and j refers to the impression of that print which ranges from 1 to 8. We have varied the number of impressions of each fingerprint in indexing phase and retrieval phase over all the experiments. The detailed list of experiments is given in the Table 2.

TABLE 2  
DETAILS OF EXPERIMENTS

Experiment	Enrolled FPs	Queried FPs
E1	i 2 to i 8	i 1
E2	i 3 to i 8	i 1 to i 2
E3	i 4 to i 8	i 1 to i 3
E4	i 1 to i 4	i 5 to i 8
E5	i 5 to i 8	i 1 to i 4
E6	i 6 to i 8	i 1 to i 5

C. Performance measures

We measure the efficiency of the system in terms of two performance measures Penetration rate and Accuracy Penetration rate (P) This indicates the amount of database searched to index an image. It is calculated basing on the Number of candidate hypotheses output by the indexing algorithm. It is given by

$$P = \frac{\text{No. of candidate hypotheses output by the indexing}}{\text{Total images enrolled in the database}}$$

Accuracy (A) This indicates the fraction of query impressions that were indexed correctly.

D. Results on FVC 2004 Databases

The results of our approach are presented in the form of t tables that show the values of two performance measures mentioned in the preceeding section, for each experiment The penetration rate mentioned in the table for each experiment is obtained by taking the average of penetration rates for every query print. Accuracy is measured as percentage of queries for which corresponding identity is correctly retrieved from the enrolled database.

The TABLE-3 depicts the penetration rate and accuracies for different experiments on FVC 2004 databases

TABLE-3  
PENETRATION RATE AND ACCURACY VALUES ON DB1

Experiment	Penetration rate(%)	Accuracy(%)
DB1		
E1	29.41	95.00
E2	29.55	93.00
E3	31.13	90.30
E4	32.21	90.00
E5	32.21	89.00
E6	31.15	82.00
DB2		
E1	33.08	95.00
E2	34.29	96.00
E3	35.11	93.00
E4	35.00	92.00
E5	35.00	91.80
E6	35.11	88.00
DB3		
E1	32.80	99.00
E2	32.75	97.00
E3	31.78	94.70
E4	31.47	91.00
E5	31.31	90.00
E6	31.82	87.00
DB4		
E1	30.18	92.00
E2	30.21	90.00
E3	29.79	89.30
E4	29.62	87.30
E5	29.63	88.00
E6	29.71	82.60

There is a decrease in the accuracy from E1 to E6, be cause of the decreasing number of enrolled impressions. But the variations in the penetration rates were not so sig nificant. For retrieval, we used a fixed radius search( for DB1,DB3) of

one adjacent bins to find corresponding triangles in the query and the template. Whereas for DB2 and DB4 we used a fixed radius search of two adjacent bins to find corresponding triangles in the query and the template. Because the search includes two adjacent bins the penetration rates were more for DB2, DB4 than DB1, DB3 for corresponding experiments. For DB3, results of the first few experiments E1, E2, E3 have better results compared to results of corresponding experiments in other databases. This can be attributed to fact that there are more minutiae available for the prints in DB3. The results on all FVC2004 databases can be summarized as follows:

1. In all the databases maximum accuracy is achieved when the maximum number of impressions were enrolled in the database in indexing phase
  2. There are only slight variations in penetration rate in all experiments across all databases. However accuracy decreased at higher rate compared to it.
  3. In case of DB3 maximum accuracy of 99% is achieved at a penetration rate of 32.80%. This can be attributed to the fact that number of minutiae in all the images in DB3 is higher than other databases.
  4. In case of DB2 a minimum penetration rate of 14% achieved an accuracy of 88%.
- Overall results across all databases show that our approach achieves an average penetration rate of 30%, 35%, 31%, 29% on DB1, DB2, DB3, DB4 respectively

## V. CONCLUSION

A Minutiae triplet based indexing method is proposed in this work. Usage of Low order Delaunay triangulation yielded minutiae triangles that are more invariant to distortion, than Delaunay triangles. Besides, the overall computation time for Indexing and Retrieval has reduced significantly. Minutiae triplet features like minimum angle, median angle, handedness, and circularity ratio are invariant to distortion. Geometric constraints like relative position of triangle with respect to core point, relative rotation have the ability to remove any false correspondences obtained by minutiae triplet based features. Different experiments are conducted on FVC 2004 databases by varying the number of enrolled prints. There are no significant variations in the results even when the number of enrolled and query impressions are varied. We have used the same thresholds for all databases over all experiments. This shows that our thresholds are not sensitive to the change in the datasets. Results on FVC 2004 databases show that this approach reduces the search space significantly.

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