



Image Denoising Using PCA with LPG

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Abstract— In this paper a spatially adaptive Principle Component Analysis (PCA) is used with local Pixel Grouping (LPG) to remove white Gaussian noise from digital image. LPG-PCA also plays an important role in image compression and denoising. Such an LPG-PCA algorithm is used to preserve similar contents from the image and remove the noise by coefficient shrinkage in PCA domain. LPG procedure ensures that the sample blocks with matching contents are used in the local statistics calculation for PCA transform estimation, So that after coefficient shrinkage in PCA domain the image local structures can be well preserved by removing the noise. Proposed procedure is iterated three times for better removal of the noise from an image. It is observed that PSNR (peak signal to noise ratio) has been much improved in the third stage. 3 stage LPG-PCA analysis also succeeds to provide improved denoising performance to recover the local structure edges and principle components. Experimental results prove that we can obtain qualitative image structure preservation as compared to different state of the art image denoising algorithms.

Keywords— Image denoising; Block matching; LPG-PCA

I. INTRODUCTION

Noise is introduced at the time of transmission and acquisition process of real world and medical images. These images are often affected by additive noise, which reduces its visual quality. The goal of denoising technique is to enhance important features of the image and to reduce the noise level [2]. Various denoising methods are: spatial domain and frequency domain denoising methods [13], curvelet [3], wavelet [4], [5], sparse representation [6] and K-SVD [7], two stage LPG-PCA [1], [9], bilateral & shape adaptive [8], non local means and non local collaborating filtering [10]. The wavelet transform provides time – frequency representation of the original signal and decomposes it into multiple scales [11], thus providing an ideal tool to analyze and threshold at each scale [10]. Wavelet transform uses fixed wavelet basis with dialation and translation [4], [5]. Because of this reason there is some distortion and visual artifacts in the recovered image using wavelet. Whereas in non local measures it preserves edges as well as noise also which are undesirable components. As per the observations an efficient orthogonal data adaptive LPG-PCA denoising technique [9] used only two stages, whereas there is possibility of third stage also. In this paper LPG-PCA algorithm with three denoising iterations technique is introduced. PCA is a statistical technique which has found applications in pattern recognition and dimensionality reduction. PCA converts a set of observations of possibly correlated variables into a set of uncorrelated variables called principle components [1], [12] and [14]. In PCA domain the principle components are traced out in accordance with the largest variance of a data matrix. In the LPG procedure, only the pixels with similar gray level are modeled as a vector variable. The sample blocks are taken from the local window by grouping the similar pixels and the local structures. By using the LPG-PCA method the local statistics of the variables can be computed accurately. The local structures of an image have been well preserved after coefficient shrinkage in principle component analysis for noise reduction.

II. SYSTEM OVERVIEW OF PROPOSED METHOD

LPG-PCA procedure unrelates the noisy components from the original dataset. PCA technique has been applied after the data selection from the noisy data set using LPG algorithm, So that the main principle pixel components of an image have been well preserved by using LPG-PCA algorithm.

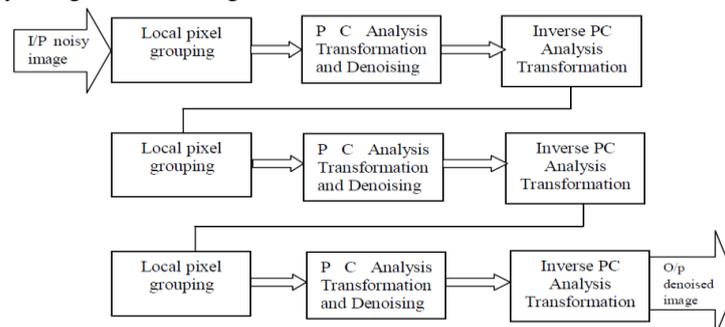


Fig. 1 Proposed process of LPG-PCA three stage denoising.

In the LPG-PCA algorithm pixel and its nearest intensity neighbors are modeled as vector variable and take an action to remove the noise on the vector instead of a single pixel. As shown in block diagram (fig-1), the proposed algorithm has three iterations. From the first and second stage output we received an initial approximation of the original image by eliminating most of the noise. All three iterations have the same procedure except for the extent of the noise level. Output of the second stage is improved from the first stage. But after third stage the LPG accuracy has been much improved and the final denoised output image is much clear. The proposed LPG-PCA algorithm guarantees that the principle components and edges are well preserved, whereas noise is suppressed significantly.

A. LPG (Local Pixel Grouping)

A number of grouping methods like K Means clustering, block matching and correlation based matching etc. can be employed for grouping the similar contents of a local window. But it is observed that the block matching method is simple as well as more suitable method among all. For grouping of same intensity pixels $K \times K$ variable moving window is taken from $L \times L$ training window within N_v , the noisy image. Maximum $(L-K+1)^2$ training blocks within $L \times L$ training window are taken. Let \vec{N}_0^v is represented as central column sample vector that contains $K \times K$ central block and other samples are represented as $\vec{N}_i^v, i=1, 2, \dots, (L-K+1)^2-1$.

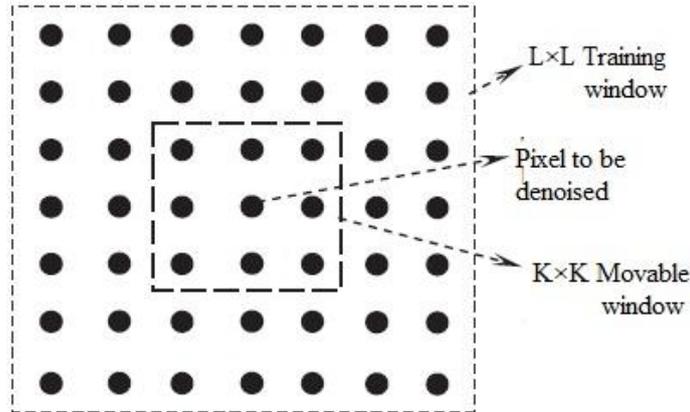


Fig 2. LPG-PCA Based denoising model

The noiseless samples of \vec{N}_0^v and \vec{N}_i^v can be easily calculated respectively

$$e_i = \frac{1}{m} \sum_{k=1}^m \vec{N}_0^v(k) - \vec{N}_i^v(k)^2 \tag{1}$$

$$e_i \approx \frac{1}{m} \sum_{k=1}^m \vec{N}_0^v(k) - \vec{N}_i^v(k)^2 + 2\sigma^2 \tag{2}$$

Under the assumption white noise v is uncorrelated with signal, if it satisfies the condition

$$e_i < T + 2\sigma^2 \tag{3}$$

Then \vec{N}_i^v can be taken as the sample vector of N_v . Here T is preset threshold value and the σ^2 shows the noise level of the corrupted image. σ related to the added noise in the original image, as the value of σ will be more means the image will be more corrupted by noise. \vec{N}_i^v Can be modeled as the sample vector of N_v and given as

$$N_v = [\vec{N}_0^v \vec{N}_1^v \dots \vec{N}_{n-1}^v] \tag{4}$$

Counterpart of N_v without noise which is estimated by denoising the central pixel within $K \times K$ block is represented as

$$N = [\vec{N}_1 \vec{N}_2 \dots \vec{N}_{n-1}] \tag{5}$$

Once N is estimated, consequently the central underlying pixel can be extracted by moving the $K \times K$ variable block over the whole $L \times L$ training window.

B. PCA (Principle Component Analysis)

PCA is spatially adaptive de-correlation technique. To obtain the principle components mathematically, let us consider $N = [\vec{N}_1 \vec{N}_2 \dots \vec{N}_m]^T$ an m component variable vector denoted by

$$N = \begin{bmatrix} N_1^1 & N_1^2 & \dots & N_1^n \\ N_2^1 & \vdots & \vdots & N_2^n \\ \vdots & \vdots & \vdots & \vdots \\ N_m^1 & \dots & \dots & N_m^n \end{bmatrix} \tag{6}$$

The sample matrix of N where $N_i^j, j=1, 2, \dots, n$ are sample variables of $N_i (i=1, 2, \dots, m)$. The i th position row of matrix N is known as the sample vector of N_i . The mean value of sample vector N_i can be calculated mathematically

$$\mu_i = \frac{1}{n} \sum_{j=1}^n N_i(j) \tag{7}$$

The centralized sample vector N_i

$$\bar{N}_i = N_i - \mu_i = [\bar{N}_i^1 \bar{N}_i^2 \dots \bar{N}_i^n] \quad (8)$$

co-variance matrix of the centralized data set is expressed as

$$\mathbf{\Omega} = \frac{1}{n} \bar{\mathbf{N}} \bar{\mathbf{N}}^T \quad (9)$$

Where $\bar{\mathbf{N}}$ is the centralized matrix of \mathbf{N} and is defined as

$$\bar{\mathbf{N}} = [\bar{N}_1^T \bar{N}_2^T \dots \bar{N}_m^T]^T \quad (10)$$

PCA domain is then used to find an orthogonal transformation matrix \mathbf{P} to decompose $\bar{\mathbf{N}}$. because the covariance matrix is symmetrical so it can be written as

$$\mathbf{\Omega} = \mathbf{\Phi} \mathbf{\Lambda} \mathbf{\Phi}^T \quad (11)$$

Where $\mathbf{\Phi} = [\Phi_1 \Phi_2 \dots \Phi_m]$ is an orthogonal Eigen vector matrix of size $m \times m$ and $\mathbf{\Lambda} = \text{dia} [\lambda_1 \lambda_2 \dots \lambda_m]$ is the diagonal Eigen value matrix. By setting orthogonal transformation matrix

$$\mathbf{P} = \mathbf{\Phi}^T \quad (12)$$

$\bar{\mathbf{N}}$ Can be decorrelated using

$$\bar{\mathbf{Z}} = \mathbf{P} \bar{\mathbf{N}} \quad \text{and} \quad \mathbf{\Lambda} = \frac{1}{n} \bar{\mathbf{Z}} \bar{\mathbf{Z}}^T$$

III. LPG-PCA: EIGENVECTORS OF COVARIANCE

In the implemented denoising algorithm we used combined LPG-PCA approach in image denoising by iterating this three times as discussed below

A. LPG-PCA Denoising

To denoise an underlying pixel a moving window of size $m=K \times K$ is centered on it, within the training window containing all the components. The captured image is corrupted by noise and is represented as

$$\mathbf{N}_v = \mathbf{N} + \mathbf{V}$$

Here \mathbf{N}_v is noisy image, \mathbf{N} is the original image and \mathbf{V} is noise respectively. The noisy vector of \mathbf{N} is represented by

$$\mathbf{N}_v = [N_1^v N_2^v \dots N_m^v]^T \quad \text{and} \quad \mathbf{N}_k^v = \mathbf{N}_k + \mathbf{V}_k \quad \text{where } k = 1, 2, \dots, m.$$

We take \mathbf{N}_k as noiseless and \mathbf{V}_k as noisy vectors to find out the estimation of \mathbf{N} from \mathbf{N}_v . For the LPG-PCA algorithm we have to find the covariance matrix. For this purpose we take an $L \times L$ training window. Accordingly there are total $(L-K+1)^2$ training samples for every component N_k^v of \mathbf{N}_v where $L > K$. The $(K \times K)$ training samples taken from \mathbf{N}_v are used for estimation of the co-variance matrix of \mathbf{N}_v .

$$\mathbf{\Omega}_{\bar{\mathbf{N}}_v} = \frac{1}{n} \bar{\mathbf{N}}_v \bar{\mathbf{N}}_v^T \quad (13)$$

$$\mathbf{\Omega}_{\bar{\mathbf{N}}_v} \approx \frac{1}{n} (\bar{\mathbf{N}} \bar{\mathbf{N}}^T + \mathbf{V} \mathbf{V}^T) = \mathbf{\Omega}_{\bar{\mathbf{N}}} + \mathbf{\Omega}_v \quad (14)$$

$$\text{Where } \mathbf{\Omega}_{\bar{\mathbf{N}}} = \frac{1}{n} \bar{\mathbf{N}} \bar{\mathbf{N}}^T \quad \text{and} \quad \mathbf{\Omega}_v = \frac{1}{n} \mathbf{V} \mathbf{V}^T$$

Because $\bar{\mathbf{N}} \mathbf{V}^T$ and $\mathbf{V} \bar{\mathbf{N}}^T$ is approximately Zero. The diagonal components of $m \times m$ matrix $\mathbf{\Omega}_v$ are taken by σ^2 . $\mathbf{\Omega}_v$ can be written as $\sigma^2 \mathbf{I}$. Here \mathbf{I} is an identity matrix and will be same for $\mathbf{\Omega}_{\bar{\mathbf{N}}}$ and $\mathbf{\Omega}_{\bar{\mathbf{N}}_v}$, thus $\mathbf{\Omega}_{\bar{\mathbf{N}}}$, $\mathbf{\Omega}_{\bar{\mathbf{N}}_v}$ and $\mathbf{\Omega}_v$ can be decomposed as

$$\mathbf{\Omega}_{\bar{\mathbf{N}}} = \mathbf{\Phi}_{\bar{\mathbf{N}}} \mathbf{\Lambda}_{\bar{\mathbf{N}}} \mathbf{\Phi}_{\bar{\mathbf{N}}}^T \quad (15)$$

$$\mathbf{\Omega}_v = \mathbf{\Phi}_{\bar{\mathbf{N}}} (\sigma^2 \mathbf{I}) \mathbf{\Phi}_{\bar{\mathbf{N}}}^T \quad (16)$$

$$\text{And } \mathbf{\Omega}_{\bar{\mathbf{N}}_v} = \mathbf{\Phi}_{\bar{\mathbf{N}}} \mathbf{\Lambda}_{\bar{\mathbf{N}}_v} \mathbf{\Phi}_{\bar{\mathbf{N}}_v}^T \quad (17)$$

$\mathbf{\Phi}_{\bar{\mathbf{N}}}$ and $\mathbf{\Lambda}_{\bar{\mathbf{N}}}$ are eigenvector and eigenvalue matrix respectively. We can find the PCA transformation matrix for $\bar{\mathbf{N}}$ as

$$\mathbf{P}_{\bar{\mathbf{N}}} = \mathbf{\Phi}_{\bar{\mathbf{N}}}^T \quad (18)$$

By applying $\mathbf{P}_{\bar{\mathbf{N}}}$ to dataset $\bar{\mathbf{N}}_v$ we get

$$\bar{\mathbf{Z}}_v = \mathbf{P}_{\bar{\mathbf{N}}} \bar{\mathbf{N}}_v = \mathbf{P}_{\bar{\mathbf{N}}} \bar{\mathbf{N}} + \mathbf{P}_{\bar{\mathbf{N}}} \mathbf{V} = \bar{\mathbf{Z}} + \mathbf{V}_z \quad (19)$$

$$\mathbf{\Omega}_{\bar{\mathbf{Z}}_v} = \frac{1}{n} \bar{\mathbf{Z}}_v^T \bar{\mathbf{Z}}_v = \mathbf{\Omega}_{\bar{\mathbf{Z}}} + \mathbf{\Omega}_{V_z} \quad (20)$$

From equation (20) Ω_{vz} is the covariance matrix of noise dataset and $\Omega_{\bar{z}}$ is covariance matrix of decomposed dataset \bar{z} . After that the noise is suppressed from \bar{z}_v , by taking decorrelated data set from covariance matrix, the most of the noise will be removed from the original dataset. Finally by transforming the complete dataset to the time domain, the denoised results are obtained.

B. Denoising in Third Stage Refinement

If the noise level in original dataset N_v is strong then covariance matrix is more noise corrupted. Also the strong noise in the original data tends to more LPG errors. Further denoising of the denoised output can give better results than earlier one in terms of noise reduction [12]. Since the noise level has been decreased in first and second iteration of the LPG-PCA denoising algorithm, we implemented the third iteration to enhance the denoising output results. There by LPG accuracy and covariance matrix estimates have been much improved. The noise level will be modified in the coming stages of LPG-PCA denoising algorithm although the PSNR (peak signal to noise ratio) value of the denoised output image have been much improved.

IV. SIMULATION RESULTS AND DISCUSSIONS

In this proposed method characteristics of LPG and PCA domain are utilized separately for image enhancement. LPG-PCA technique results in better smoothness of the background while keeping the principle components of image preserved.

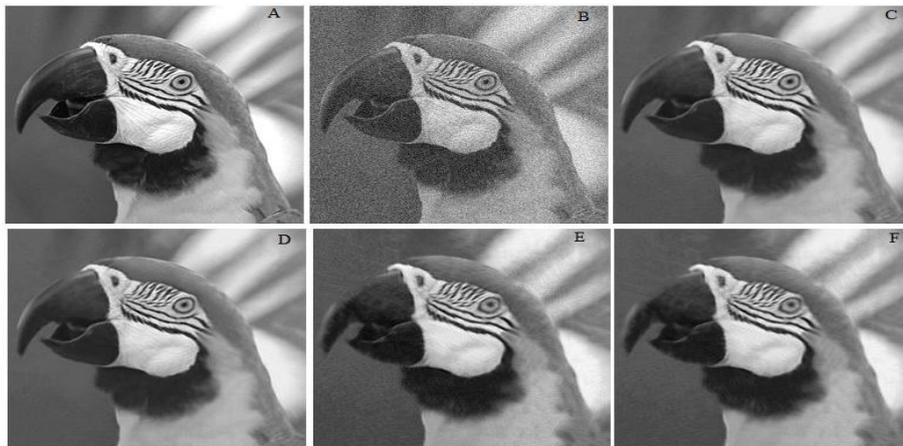


Fig 3. A) Original parrot.tif image (B) noisy image ($\sigma=20$) (C) wavelet transforms (D) K-SVD (E) 2 stages LPG-PCA (F) 3 stages LPG-PCA.

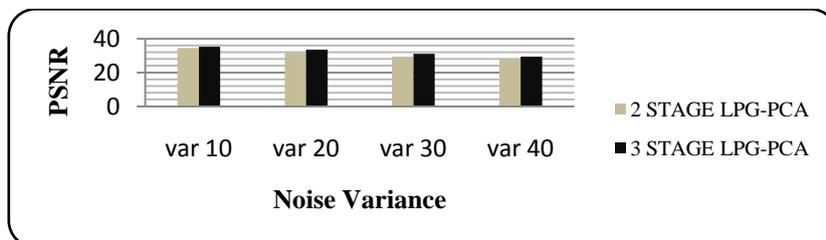


Fig 4. Graph of the comparison of 2-stage and 3-stage LPG-PCA algorithm output at different noise level for parrot.tif image.



Fig 5. A) Original lena.jpg image (B) noisy image ($\sigma=40$) (C) wavelet transforms (D) K-SVD (E) 2 stages LPG-PCA (F) 3 stages LPG-PCA.

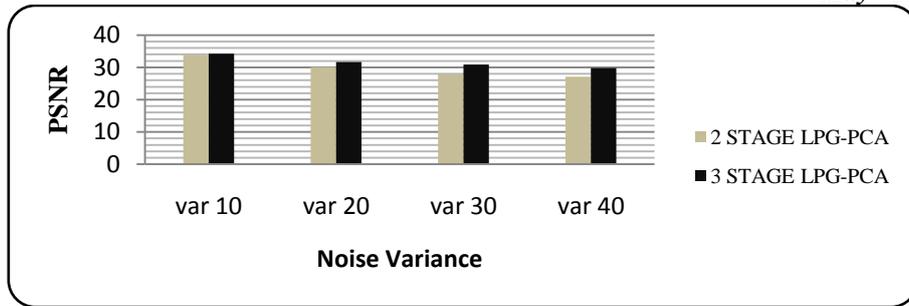


Fig 6. Graph of the comparison of 2-stage and 3-stage LPG-PCA algorithm output at different noise level for lena.jpg image.

Different kinds of test images have been considered to ensure the efficiency of the proposed algorithm. Figure 3 and 4 shows the output performance of parrot.tif and lena.jpg images by using different denoising techniques. Figure 5 and 6 shows the graphs between 2 stage and 3 stage LPG-PCA denoising results for parrot.tif and lena.jpg images at different noise levels.

TABLE I
EXPERIMENTAL RESULTS - PSNR VALUES OF THE DENOISED IMAGES

Methods		Wavelet Transform	K-SVD	2-Stage LPG-PCA Algorithm	3-Stage LPG-PCA Algorithm
Image	Noise Level				
Parrot	$\sigma = 10$	34.1	34.3	34.5	35.2
	$\sigma = 20$	30.6	30.8	31.7	33.6
	$\sigma = 30$	28.6	28.8	29.5	31.2
	$\sigma = 40$	27.2	27.5	28	29.4
Lena	$\sigma = 10$	33.2	33.5	33.9	34.3
	$\sigma = 20$	29.4	29.7	29.9	31.7
	$\sigma = 30$	27.5	27.8	27.8	30.8
	$\sigma = 40$	26.0	26.5	27.2	29.2
Barbara	$\sigma = 10$	31.6	32.3	32.5	33.0
	$\sigma = 20$	27.2	28.4	28.7	29.6
	$\sigma = 30$	25.0	26.3	27.0	28.7
	$\sigma = 40$	23.5	24.7	24.9	27.5
Cameraman	$\sigma = 10$	33.7	33.9	34.1	34.2
	$\sigma = 20$	29.6	29.9	30.1	30.9
	$\sigma = 30$	27.5	27.9	27.9	29.0
	$\sigma = 40$	26.0	26.5	26.4	27.6

As presented in the comparison results of different denoising algorithms such as wavelet, K-SVD, 2 stage LPG-PCA with the proposed 3 stage LPG-PCA denoising algorithm are shown in Table-1. Which shows that the 3 stage LPG-PCA algorithm gives the best PSNR results. In practice, the mathematical calculations are mainly dependent upon the variable block size i.e. $K \times K$. Within the local window, as the size of the $K \times K$ variable block will be small, the accuracy of LPG will be enhanced significantly. It has been found the PSNR values can improve from 0.1 to 1.5 db by the second stage and can further be improved from 0.2 to 2.0 db using the third stage. Simulation results of proposed 3 stage LPG-PCA algorithm, for real world images at different noise levels (σ is from 10 to 40) have been compared with different denoising algorithms like wavelet based denoising [5], K-SVD [7] and two stage LPG-PCA with the proposed three stage denoising algorithm. Table - 1 shows the comparison of different denoising techniques by taking different test images. For lower noise variance images second stage is sufficient to remove the noise. But, when the noise variance is high, then the results obtained from the second stage output are not satisfactory in the terms of peak signal to noise ratio. The proposed algorithm overcomes this drawback by using 3 stages LPG-PCA algorithm.

V. CONCLUSIONS

Hence it is concluded that for all types of images, the proposed algorithm outperforms other approaches used for comparison in terms of image denoising and it also gives the best PSNR value among all. The main novelty of proposed algorithm is that it is the advancement of PCA based denoising algorithm [1], [12]. Here, we used the three stage LPG-PCA denoising algorithm that is not only improves the noise reduction of an image, but also enhance the principle components preservation of all real world images like lena, Barbara, parrot and cameraman etc.

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