



Wavelet-based Multifractal Analysis of RR Time Series

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Abstract— In this paper are presented the current results of scientific research of the RR time series for healthy and unhealthy subjects. The RR intervals are obtained from 24-hour digital Holter ECG records of subjects. The used in the presented research work wavelet-based multifractal analysis of RR time series is provided by Wavelet Transform Modulus Maxima method. This method is based on wavelet analysis and multifractal formalism. The obtained results show that investigated RR signals for healthy subjects are with multifractal behavior and in pathological cases the signals are with monofractal behaviour. This non invasive method is suitable for diagnostic, forecast and prevention of the pathological statuses.

Keywords— Monofractal, Multifractal, Wavelet transform, Partition function, Singularity spectrum.

I. INTRODUCTION

Electrocardiography is noninvasive method for functional status analysis of the heart and cardiovascular activity. By the electrocardiogram (ECG) was measured the electrical activity of the heart. The research of variation of time intervals between heart beats (Heart Rate Variability -HRV) is a method for assessing of cardiological intervals. HRV is used to diagnose and estimate of alterations in heart rate by measuring the variation of RR intervals (time intervals between two consecutive heart beats). The RR time series extracted from human electrocardiogram is considered as a fractal stochastic process [1]. The RR time series are nonlinear and nonstationary signals, as the significant part of the information is coded in the dynamics of their fluctuation. Through implementation of the conventional analytical methods based on the statistical parameters as mean values, standard deviations and harmonically analysis of the energetic spectrum of the signals, part of the important signal characteristics are missed [2],[3].

From the recent scientific research worldwide follow that scale invariance of the fluctuations can be easy identified for healthy and pathological cases via implementation of methods for fractal, multifractal and wavelet-based multifractal analysis [4]. Through these methods is determine that fluctuation of the physiological signals possess hidden information in the form of self-similarity, scale structure, monofractality and multifractality [5]. The fractal, multifractal and wavelet-based multifractal analysis of the fluctuations is useful not only for getting the comprehensive information for physiological signals of the patients, but give a possibility for foresight, prognosis and prevention of the pathological statuses. The prevention in the medicine is important not only for the human, but for the community as whole.

One of the most popular tools in wavelet-based multifractal analysis is the Wavelet Transform Modulus Maxima (WTMM) method. This method is based on wavelet analysis that is called “mathematical microscope” because of its ability to maintain a good resolution at different scales [6]. The WTMM method is a powerful tool for the statistical description of nonstationary signals because the wavelet functions are localized in time and frequency.

The main objective of this paper is to investigate the fractal structure of RR intervals time series extracted from 24-hour Holter ECG signals of healthy and unhealthy subjects by applying WTMM method and application of the results to the analysis of experimental medical data.

II. SUBJECTS AND METHOD

A. Subjects

In this article are analyzed two kinds of signals: RR time series of the healthy and unhealthy subjects. These signals, consisting of about 100 000 data points, corresponding to 24-hour recordings of ECG RR time intervals of 16 healthy adults (8 men and 8 women aged 35 to 60 years) and 16 unhealthy adults (8 men and 8 women aged 35 to 60 years). The data are taken from the Department of Cardiology of Multiprofile District Hospital for Active Treatment "Dr. Stefan Cherkozov" AD, town of Veliko Tarnovo, Bulgaria.

B. Wavelet Transform Modulus Maxima

The physiological signals, such as RR time series, can be efficiently represented by decomposition at different frequencies. The conventional method for this approach is a Fourier analysis. This analysis works well for stationary time series, but not for nonstationary signals, when the frequency content changes over time. Scalograms based on the wavelet transform simultaneously provide both time and frequency information, which is important for investigated signal. The WTMM method is used for analysis the multifractal scaling properties of fractal signals. This method uses continuous wavelet transform to detect singularities of a signal. WTMM is based on Wavelet analysis (continuous wavelet transform, skeleton construction) and Multifractal formalism (partition function calculation, scaling exponential function estimation, multifractal spectrum estimation). The method consists in the following basic steps [7],[8]:

Step 1: Calculation of the Continuous Wavelet Transform (CWT)

A wavelet is simply a finite energy function with a zero mean value. The wavelet transform is defined by the continuous time correlation between the time series and the particular wavelet of scaling parameter τ and shift parameter α :

$$W(\tau, \alpha) = \int_{-\infty}^{+\infty} f(t) \Psi_{\tau, \alpha}(t) dt, \quad (1)$$

where the analyzing wavelet $\Psi_{\tau, \alpha}(t)$ is a zero average function with local support, centered around zero. The family of wavelet vectors is obtained by the translations and dilatations of the “mother” wavelet:

$$\Psi_{\tau, \alpha}(t) = \frac{1}{\sqrt{\alpha}} \Psi\left(\frac{t - \tau}{\alpha}\right). \quad (2)$$

The wavelet transform has a time frequency resolution which depends on the scale α .

Step 2: Calculation of the local maxima of the modulus of the CWT

The modulus maxima (largest wavelet transform coefficients) are found at each scale α as the suprema of the computed wavelet transforms such that:

$$\frac{\partial W(\tau, \alpha)}{\partial \tau} = 0. \quad (3)$$

Step 3: Calculation of the partition function $Z(q, \alpha)$ based on wavelets, where α is the dilatation and q is a scale factor

The originality of the WTMM method is in the calculation of the partition function $Z(q, \alpha)$ from the maxima lines. The space-scale partitioning given by the wavelet tiling or skeleton defines the particular partition function:

$$Z(q, \alpha) = \sum_{\tau, \alpha} \sup_{\alpha} |W(\tau, \alpha)|^q, \quad (4)$$

where α is the dilatation and q is a scale factor. This partition function effectively computes the moments of the absolute values of the wavelet resonance coefficients $W(\tau, \alpha)$.

Step 4: Calculation of the decay scaling exponent $\tau(q)$

The scaling exponent $\tau(q)$ is the Legendre Transform of the multifractal spectrum $f(\alpha)$ for self-similar time series and relates the fractal dimensions to the order q of the partition function $Z(q, \alpha)$. The slope in the double-logarithmic plot allows the computation of the decay scaling exponent. This slope can be obtained using linear regression:

$$\log_2 Z(q, \alpha) \approx \tau(q) \log_2 \alpha + C(q). \quad (5)$$

If scaling exponent is everywhere convex, that indicates multifractal behaviour of the signal. In case of monofractal behaviour, the scaling function is line.

Step 5: Estimation of the spectrum of singularities

Multifractal formalism uses multifractal spectrum for the detailed fractal analysis of the signal. Multifractal spectrum function shows the scope of all fractal measures. Multifractal spectrum function is calculated from scaling function via Legendre transform by formulas:

$$\alpha(q) = \frac{d\tau(q)}{dq}. \quad (6)$$

$$f(\alpha) = \min_{\alpha} (\alpha q - \tau(q)). \quad (7)$$

The spectrum width on multifractality degree is $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$, this quantity is a measure of the range of fractal exponents in the time series, so if $\Delta\alpha$ is large, the signal is multifractal.

III. RESULTS AND DISCUSSION

The analyzed data for described two kinds of signals are combined in two group averages with records for 16 healthy and 16 unhealthy subjects.

Fig.1 shows RR time series of the group averages for 18 healthy and 18 unhealthy subjects. The graphs of RR time data are highly nonstationary (the mean, variance and other vary in the time statistical parameters).

The wavelet decomposition of RR signals can be used to provide a visual representation of the fractal structure of investigated signals (Fig. 2). The wavelet coefficients are shown in their absolute values and coloured in accordance with colour bar. Dark colours correspond to lower absolute wavelet coefficient values. Light colours indicate higher absolute wavelet coefficient values corresponding to large heartbeat fluctuations. The wavelet analysis uncovers hierarchical scale invariance and reveals a self-similar fractal structure in the healthy subjects and a loss of this fractal structure in the unhealthy subjects.

Wavelet coefficient matrix allows skeleton function construction. Local maxima lines are constructed using Wavelet coefficient matrix, selecting local maxima points on each scale parameter. The scope of all local maxima lines builds the skeleton function. The branching structure of the skeleton in the scale plane indicates the hierarchical organization of the singularities. This is clearly illustrated in Fig. 3.

The partition functions of the investigated signals, corresponding with the RR time series for the group averages for 18 healthy and 18 unhealthy subjects are shown in the plots on Fig. 4. The slight deviation from linearity is observed for the very detailed resolution of the log-log plot only. The top curve correspond to $q=-10$ and the bottom curve to $q=10$. The scaling exponents $\tau(q)$ are obtained from the slope of the $Z(q, \alpha)$ curve.

On the Fig.5 are shown the variations of scaling exponents $\tau(q)$ over all the values of q for the group averages for 18 healthy and 18 unhealthy subjects. The constantly changing curvature of the $\tau(q)$ curves for the healthy subjects suggests multifractal behaviour. In contrast, $\tau(q)$ is straight line for unhealthy subjects, indicating monofractal behaviour.

The Figure 6 shows the multifractal spectrum of RR time series of the group averages for 18 healthy and 18 unhealthy subjects. The curve of healthy subject have multifractal behaviour, due to the wide range of local values of the Hölder exponent α ($\Delta\alpha = \alpha_{\max} - \alpha_{\min}$). The range of values of the Hölder exponent α for RR time series of healthy subject is $\Delta\alpha = 0.79177$, and for RR time series of unhealthy subject is $\Delta\alpha = 0.30351$. The interval $\Delta\alpha$, corresponding to the RR signal for unhealthy subject is more than two times lower than the signal for healthy subject. The values of α , $f(\alpha)$ and $\Delta\alpha$ for different values of q of RR series for the group averages for 18 healthy and 18 unhealthy subjects are reported in Table 1. The values of these parameters may be important to distinguish healthy from diseased patients.

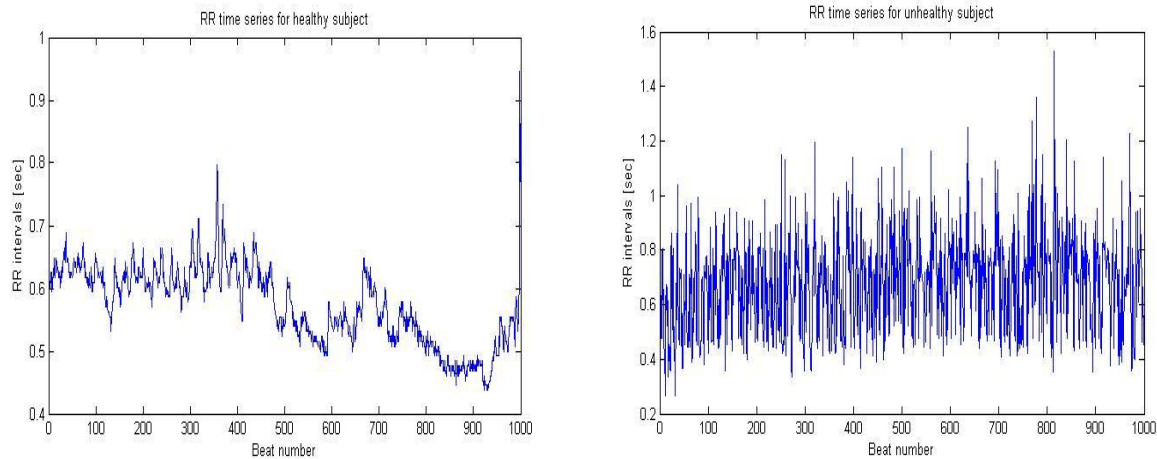


Fig. 1 RR interval time series of the group averages for 18 healthy and 18 unhealthy subjects

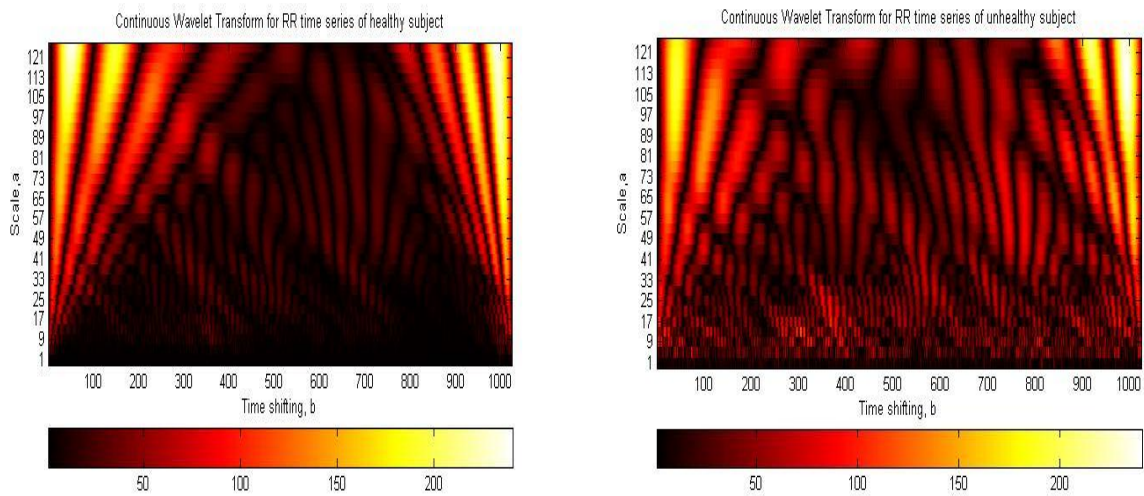


Fig. 2 Continuous wavelet transform of the group averages for RR time series for 18 healthy and 18 unhealthy subjects

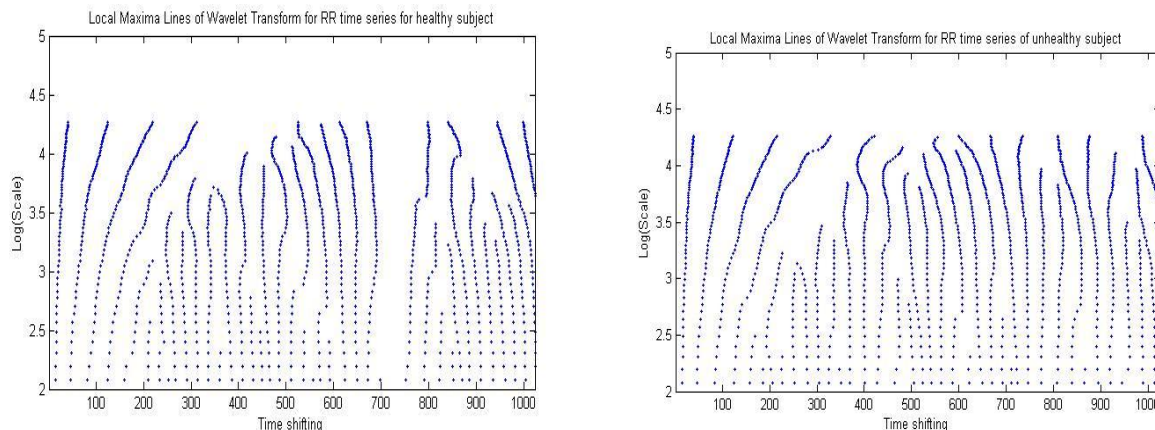


Fig. 3 Wavelet transform maxima lines of the group averages for RR time series for 18 healthy and 18 unhealthy subjects

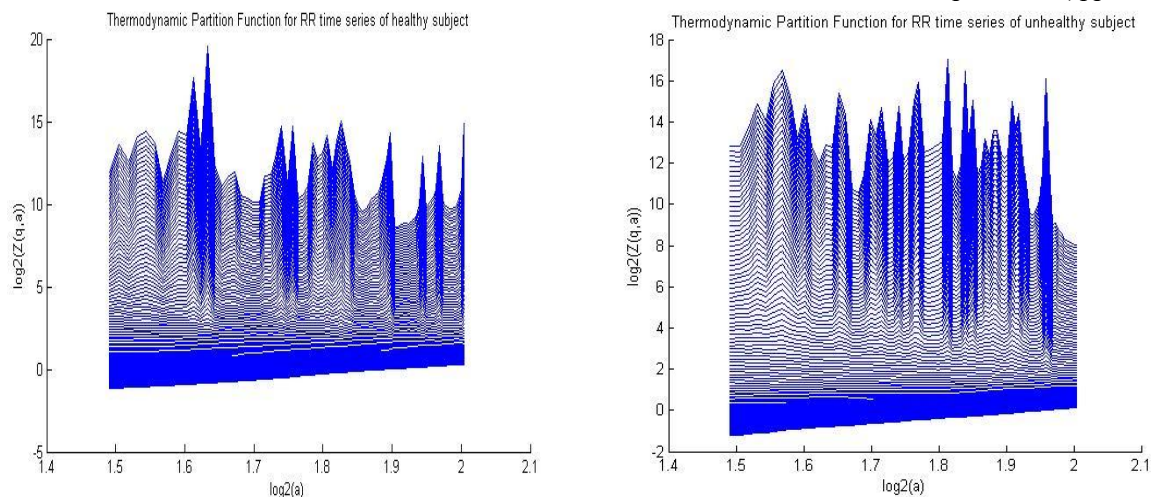


Fig. 4 Thermodynamic partition function of the group averages for RR time series for 18 healthy and 18 unhealthy subjects

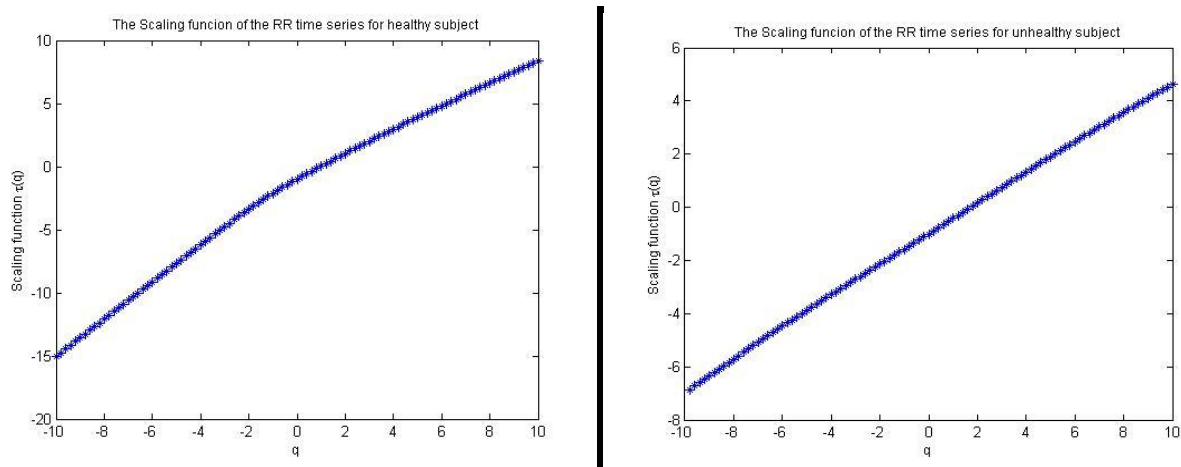


Fig. 5 Scaling exponent $\tau(q)$ of the group averages for RR time series for 18 healthy and 18 unhealthy subjects

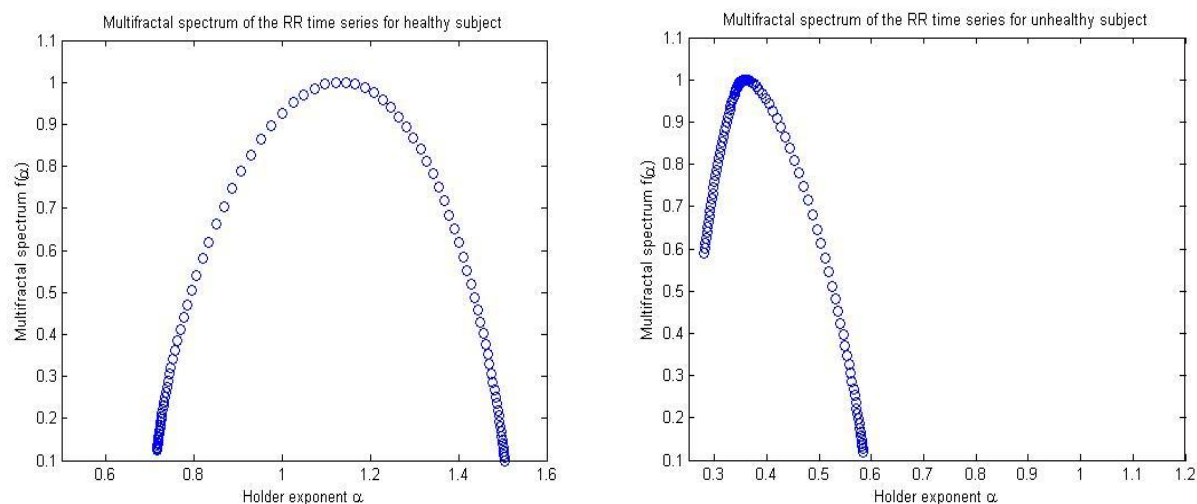


Fig. 6 The multifractal spectrum of the group averages for RR time series for 18 healthy and 18 unhealthy subjects

TABLE I The values of α , $f(\alpha)$ and $\Delta\alpha$ for different values of q of RR time series for healthy and unhealthy subjects

q	RR time series for healthy subject			RR time series for unhealthy subject		
	α	$f(\alpha)$	$\Delta\alpha$	α	$f(\alpha)$	$\Delta\alpha$
q=0	1.1214	1	0.79177	0.35752	1	0.30351
q=10	0.7167	0.12414		0.28008	0.59004	
q=-10	1.5085	0.068787		0.58359	0.12029	

IV. CONCLUSION

Physiological signals generated by the human body during the functioning the organs like heart, brain, lung, etc. are nonlinear and nonstationary with “hidden” or coded information in the dynamic of their fluctuation.

The WTMM method defines the complexity of investigated data through significant differences in behavior of RR time series for healthy and unhealthy subjects. On the base of the wavelet-based multifractal analysis the possible conclusion is that the investigated signals, corresponding to the healthy subjects are with multifractal behavior and in pathological cases the signals are monofractals. Applications of this analysis may lead to new diagnostics for patients at high risk of cardiac disease. The wavelet-based multifractal analysis of the RR time series is suitable non invasive method of diagnostics, forecast and prevention of the pathological statuses.

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