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Expectation Maximization Algorithm for Image Segmentation

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Abstract: Image pre-processing and registration are discussed, as well as methods of validation. In this paper, we present a new multiresolution algorithm that extends the well-known Expectation Maximization (EM) algorithm for image segmentation. The conventional EM algorithm has prevailed many other segmentation algorithms because of its simplicity and performance. However, it is found to be highly sensitive to noise. To overcome the drawbacks of the EM algorithm we propose a multiresolution algorithm which proved more accurate segmentation than the EM algorithm.

Keywords: - Gaussian Mixture model, Maximum likelihood estimator, Multiresolution analysis

I. INTRODUCTION

Magnetic resonance imaging (MRI) represents the intensity variation of radio waves generated by biological systems when exposed to radio frequency pulses [1][2]. A Magnetic resonance image (MRI) of the human brain is divided into three regions other than the background, white matter (WM), gray matter (GM), and cerebrospinal fluid (CSF) or vasculature [2]. Because most brain structures are anatomically defined by boundaries of these tissues classes, a method to segment tissues into these categories is an important step in quantitative morphology of the brain. An accurate segmentation technique may facilitate detection of various pathological conditions affecting brain parenchyma, radiotherapy treatment and planning, surgical planning and simulations, and three-dimensional (3-D) visualization of brain matter for diagnosis and abnormality detection [2]. Image segmentation is to divide the image into disjoint homogenous regions or classes, where all the pixels in the same class must have some common characteristics. According to the nature of the image the approach of segmentation may be either region-based approaches [3][4], or pixel-based approaches, where the segmentation is done according to the pixels features, such as pixel intensity [5][6][7][8]. A will known approach of image segmentation based on pixel intensity is the Expectation Maximization (EM) algorithm [9][10], which is used to estimate the parameters of different classes in the image. To overcome the drawbacks of the EM algorithm, we propose a
multiresolution algorithm, the Gaussian Multiresolution EM algorithm, GMEM, which proved high reliability and performance under different noise levels, and in the same time it keeps the advantages of the conventional EM algorithm.

II. IMAGE SEGMENTATION

Image segmentation is one of the most important stages in artificial vision systems. It is the first step in almost every pattern recognition process. In some context other terms like *object isolation* or *object extraction* are used.
Image segmentation is computationally the division of an image into disjoint homogeneous regions or classes. All the pixels in the same class must have some common characteristics. The conventional segmentation procedure starts by transforming the original image into a feature space in order to find the boundaries between the different classes. It is followed by a mapping step, which assigns a label to each pixel such that all the pixels of the same features will have the same class.

III. STATISTICAL METHODS.

The Expectation Maximization (EM) algorithm was developed and employed independently by several different researchers until Dempsters et al. [DLR97]
Brought their ideas together, proved convergence, and coined the term “EM algorithm”. Since that seminal work hundreds of papers employing the EM algorithm in many areas have been published. Generally, the EM algorithm produces Maximum Likelihood (ML) estimates of parameters when there is a many-to-one mapping to the distribution governing the observation. The EM algorithm is used widely in the image segmentation field and it produces very good results especially with a limited
noise level. The image is considered as a Gaussian mixture model. Each class is represented as a Gaussian model and the pixel intensity is assumed
as an observed value of this model. The EM is used for finding the unknown parameters of the mixture model. A set of observed data $X = \{xi \mid i = 1, ...,N\}$ can be modeled as to be generated from a mixture of random processes $X1,X2, ...,XK$, with joint probability distribution $f(X1,X2, ...,XK)$, where K is the number of classes or distribution functions present in

the mixture. It is usually assumed that these processes represent independent identically distributed random variables. Then one can write:

$$f(X_1, X_2, ..., X_K) = \prod_{k=1}^{K} f(x, \theta_k)$$

where *f*(*x*, _*k*) 8*k* = 1, 2, ...,*K* is the probability distribution function of the random variable *Xk*, and _*k* = {*μk*, _*k*} stands for the parameters that define the distribution *k*.

ф = {*p*1*, ..., pK, μ*1*, ..., μK,* _1, ..., _*K*}

is called the parameter vector of the mixture, where *pk* are the mixing proportions

(0 _ *pk* _ 1, 8*k* = 1, ...,*K*, and∑ *k pk* = 1).

The EM algorithm consists of two major steps: an expectation step (Estep), followed by a maximization step (M-step). The expectation step is to estimate a new mapping (pixel-class membership function) with respect to the unknown underlying variables, using the current estimate of the parameters and conditioned upon the observations. The maximization step then provides a new estimate of the parameters. These steps iterate until convergence is achieved [TM96].

THE E-STEP:

Compute the expected value of *zi,k* using the current estimate of the parameter vector ф was introduced in [Sae97]:

$$z_{i,k}^{(t)} = \frac{p_k^{(t)} G(x_i \mid \theta_k^{(t)})}{f(x_i \mid \Phi^{(t)})}$$

Where *zi,k* is the probability of *xi* belonging to class *k*, where 1 ≤ *i* ≤*N*, 1 ≤ *k* ≤ *K* and *xi* is the intensity value of the pixel *i*. It should be referenced afterwards as the pixel *xi*.

zi,k satisfies the conditions:

$$i)\ 0 \leq z_{i,k} \leq 1$$
$$ii)\ \sum_k z_{i,k} = 1$$
$$iii)\ \sum_i z_{i,k} > 0.$$

$G(x_i \mid \theta_k^{(t)})$ is the probability of pixel x_i given it is a member of class k.

p_k is the class proportional in the model $\sum_k p_k = 1$.

The $f(x_i \mid \Phi)$ is the total probability function that is defined as:

$$f(x_i \mid \Phi) = \sum_{k=1}^{K} p_k G(x_i \mid \theta_k^{(t)})$$

The superscript (*t*) means the iteration number *t*.

THE M-STEP:

Use the data from the expectation step as if it were actually measured data and compute the mixture parameters as introduced in [Sae97]:

$$\mu_k^{(t+1)} = \frac{\sum_{i=1}^{N} z_{i,k}^{(t)} x_i}{\sum_{i=1}^{N} z_{i,k}^{(t)}}$$

$$\sigma_k^{2(t+1)} = \frac{\sum_{i=1}^{N} z_{i,k}^{(t)} (x_i - \mu^{(t+1)})^2}{\sum_{i=1}^{N} z_{i,k}^{(t)}}$$

$$p_k^{(t)} = \frac{\sum_{i=1}^{N} z_{i,k}^{(t)}}{N}$$

The EM algorithm starts with an initial guess ф(0) of the parameters of the distributions and the proportions of the distributions in the image. It iterates until a conversion of the parameter vector ф is achieved. Fig. 1. Shows its flowchart.

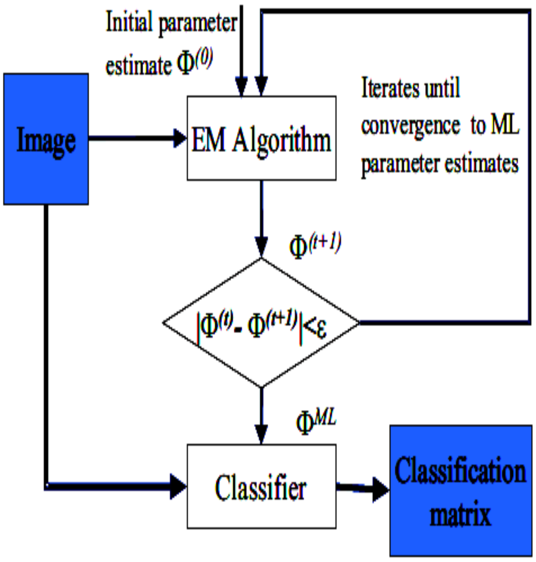

Figure Block diagram of the EM algorithm. I: input image. S: segmented image.

The EM algorithm is always followed by a classification step. The EM is producing the missing parameters in _, which are then used by a classifier
Which is defined as?

$$k_i = \underset{k}{\operatorname{argmax}}(G(x_i \mid \theta_k^{(t)}))$$

It assigns a class membership to a pixel *i* depending on its intensity *xi* to the class whose parameter vector maximizes the Gaussian density function. The value of this membership function is placed in a new matrix called *classification matrix*. It is a matrix that has the

same size as the image and the same dimensions. The values of the matrix elements represent the classes of the pixels of the corresponding image. The EM algorithm is used in different image segmentation problems, such as medical images, natural scene images, and texture images. The authors in [CD00] presented an enhancement segmentation of texture images by the EM algorithm. The basic idea behind their algorithm is to minimize the expected value of the number of misclassified pixels by EM estimates using the Maximization of the Posterior Marginal's (MPM) of the classification. After each iteration of the EM algorithm the MPM uses the estimated parameters to maximize the conditional probability of the classification of a certain pixel given its observed value.

EM ALGORITHM

In this paper we propose a new image segmentation algorithm, namely; Gaussian Multiresolution EM algorithm, GMEM, which is based on the EM algorithm and the multiresolution analysis of images. It keeps the advantages of the simplicity of the EM algorithm and in the same time overcome its drawbacks by taking into consideration the spatial correlation between pixels in the classification step. We mean by the term "spatial correlation", that the neighboring pixels are spatially correlated because they have a high probability of belonging to the same class. We think that utilizing the spatial correlation between pixels is the solution key to overcome the drawbacks of the EM algorithm. Therefore, we propose to modify the EM algorithm so that it takes in its consideration the effect of the neighbor pixels when classifying the current pixel, by utilizing the multiresolution technique

IV. THE MULTIRESOLUTION ANALYSIS.

The multiresolution-based image segmentation techniques, which have emerged as a powerful method for producing high-quality segmentation of images [5][8], are combined here with the EM algorithm to overcome the EM drawbacks and in the same time to take its advantages. The Multiresolution analysis is based on the aspect that "all the spaces are scaled versions of one space" [14], where successive coarser and coarser approximations to the original image are obtained. This is interpreted as representing the image by different levels of resolution. Each level contains information about different features of the image. Finer resolution, i.e., higher level, shows more details, while coarser resolution, i.e., lower level, shows the approximation of the image and only strong features can be detected.

Working with the image in multiresolution enables us to work with the pixel as well as its neighbors, which makes the spatial correlation between pixels easy to implement. In this work we have generated two successive scales of the image, namely, parent and grandparent images. We used an approximation filter, in particular, a Gaussian filter, to generate such low-resolution images. The Gaussian filter is a low pass filter used to utilize the low frequency components of neighboring pixels [15]. We used the Gaussian filter in a manner similar to a moving window, where a standard Gaussian filter of size n x n is created and in the same time the original image is divided into parts each of which has the same size as the filter size. The filter is then applied to each part of the image separately. This can be interpreted as a windowed convolution where the window size is the same as the filter size. and also this agree with the concept of the distinct block operation [16], where the input image is processed a block at a time. That is, the image is divided into rectangular blocks, and some operation is performed on each block individually to determine the values of the pixels in the corresponding block of the output image, the operation in our case is the Gaussian filter. Each time we apply the filter on a part of the image the result is placed as a pixel value in a new image in a similar location to that where it was obtained. Later we use this new image as the parent of the original image. In the following we illustrate this in more details. In Fig. 1, the original image I_0 at scale J=0, say of size 9x9 is divided into parts each part of size 3x3, then a Gaussian filter of size 3x3 is applied to the first part of the image $I_0(1:3,1:3)$ the result of the windowed convolution, say a_{11}, is placed in location (1,1) in the new image, I_1. This step is repeated to each part of the image which generate a sequence of coefficients, $a_{11}..a_{13}$, $a_{21}..a_{23}$, and $a_{31}..a_{33}$, these coefficients are placed in the new image by the same order as they obtained. The new created image isof size 3x3 represents a lower-resolution approximation of the original image and acts as a parent image, at scale J=1, of the original image I_0, at scale J=0, where each nine neighboring pixels in I_0 are used to generate one pixel in I_1. By the same way, we used a Gaussian filter of size 5x5 to create the grandparent image I_2 from I_0 at scale J=2. Generally, the distinct block operation may require image padding, since the image is divided into blocks. These blocks will not always fit exactly over the image.

THE GMEM ALGORITHM

The GMEM algorithm can be summarized in the following steps and as depicted in the flowchart shown in Fig. 2. 1-Start with an image I_o as input and generates its parent I_1 and grandparent I_2 using the Gaussian moving windows of sizes 3x3 and 5x5, respectively.
2-Apply the conventional EM algorithm for image segmentation on the images I_o, the parent I_1, and the grandparent I_2. The outputs of this step are the classification matrices C_0, C_1, and C_2, respectively.
3- Reclassify the original image I. using the weights specified previously to generate the final classification matrix C. That represents the classification of the image I_0 after taking into account the spatial correlation between pixels.
4-Assign colors or labels to each class and generates the segmented image S.

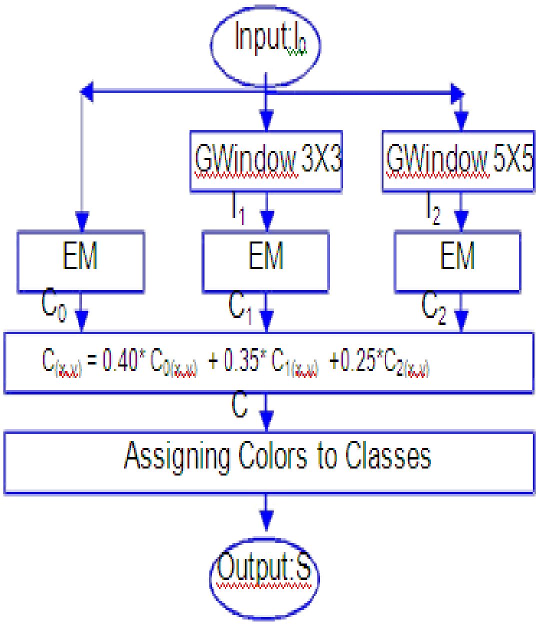

Fig: 3 The GMEM flowcharts, the input is the image to be segmented, Io and the segmented images S.

THE DRAWBACKS OF THE EM ALGORITHM

Although the EM algorithm is used in MRI of human brain segmentation, as well as image Segmentation in general, it fails to utilize the strong spatial correlation between neighboring pixels. For example, if a pixel, i, all its surrounding pixels, neighbor pixels, are being classified to belong to the same class say K_a, but if it has an intensity closer to the mean of another class, say K_b, the classifier would incorrectly classify this pixel to belong to class K_b.

This drawback is due to that the EM is based on the GMM which assumes that all the pixels distributions are identical and independent; however it has an advantage that it reduces the computational complexity of the segmentation task by allowing the use of the well-characterized Gaussian density function (Saeed, 1998).

V. CONCLUSION

The multiresolution analysis enables the algorithms to utilize the spatial correlation between neighboring pixels. The GMEM algorithm uses the Gaussian filter and the distinct block operation to generate low resolution images from the original image, where two images generated at two successive scales, the parent and the grandparent images. The proposed algorithm has been tested using both synthetic data and manually segmented magnetic resonance images (MRI). Moreover, performance analysis between this algorithm and the conventional EM algorithm has been presented. We found that the accuracy of the segmentation done by the proposed algorithm increased significantly over that of the conventional EM algorithm. In case of the synthetic data, about 15% increase in the segmentation overall accuracy (OA) is obtained for high noised images (variance = 400). In case of the real MR images with ground truth and added high Gaussian noise, an increase in the segmentation OA of about 9% is obtained. These results show that our new multiresolution algorithm provides superior segmentations over the one- scale image segmentation algorithms. The drawback found in the GMEM algorithm is that, when it is applied to pixels laying on the boundaries between classes or on edges, it generates many misclassified pixels, and this is because the parent and grandparent images contain only low frequencies and hence the edges are rarely appear in these images. Much of the error occurred because we used the classification of the parent and grandparent images to reclassify the pixels near the edges.

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