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Blind Identification of FIR using the third and fourth order Cumulants

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Abstract— ***Modern telecommunication systems require very high transmission rates, therefore, the problem of channels identification is a major challenge. The use of blind techniques allows an optimal compromise between a suitable bit rate and the quality of the retrieved information. In this research study, we are interested in learning the blind channel identification algorithms. We propose a new algorithm that combines the sequence three and four order cumulant to improve the channel estimation.***

Keywords— ***Transmission channel, Telecommunication systems, Blind identification, Higher order cumulants, Transmission channel with Gaussian noise.***

I. Introduction

The current progress in resolving systems has become more important in the telecommunication systems, especially the blind identification channel, including the requirement of modern telecommunications systems that seek to use very high transmission rates. In this context, the application of higher-order cumulants is an important technique commonly addressed by digital telecommunication systems. The objective of the use of blind identification is to have a good estimation of channel parameters, and therefore a good quality of retrieved information.

In this research study, we present tree algorithms for blind identification based on higher order cumulants [1]. The first objective is to present the new algorithm, which combines the cumulants of orders three and four, and then we present a comparative study of our algorithm with tree others existing algorithms [1], [2] and [3] to validate the level of estimation of a Gaussian white noise channel.

II. Problem Statement

The proposed identification for the non-minimum phase adjusted average model is represented by the following finite difference equation:

$$Y(k) = \sum_{j=0}^{a} h(j).X(k-j),\ h(0) = 0 \text{ (noiseless output)} \quad (1)$$

$$Z(k) = Y(k) + N(k) \text{ (noisy output)} \quad (2)$$

The problem is to determine [h(0), h(1),...,h(q)] from a statistical analysis of Z(k) (the channel response) received no information about the input signal X(k).

X(k) is an independent non-Gaussian component and identically distributed (i.i.d) zero mean excitation

N(k) is an independent white Gaussian noise of the input X(k). q is the order of the assumed known channel .

The cumulants for a Gaussian signal is zero, which justifies the use of statistical analysis using higher order cumulants.

III. Fundamental Relationships

A. General Equation

The Brillinger and Rosenblatt formula of identification of channels MA [4], under the above assumptions, can be written:

$$C_{m,Z}(\tau_1,...,\tau_{m-1}) = C_{m,Y}(\tau_1,...,\tau_{m-1}) = \gamma_{m,x}\sum h(i)h(i+\tau_1)...h(i+\tau_{m-1}) \quad (3)$$

For m = 2, the autocorrelation is:

$$C_{2,Z}(\tau) = C_{2,Y}(\tau) + C_{2,N}(\tau) \quad (4)$$

Where $C_{2,N}(\tau)$ is the autocorrelation of the noise skewing the results and $C_{2,Y}(\tau)$ is the autocorrelation of the noiseless signal given by:

$$C_{2,Y}(\tau) = \gamma_{2,x}\sum_{i=0}^{q} h(i)h(i+\tau),(\gamma_{2,x} = \sigma_x^2) \quad (5)$$

From (3) you can easily show [5] that the cumulants of order **m** and **n**, **m** > **n**, satisfy the following relationship:

$$\sum_{i=0}^{q} h(i) C_{m,Y}(i+\tau_1,...,i+\tau_{n-1},\tau_n,...,\tau_{m-1}) = \varepsilon_{m,n} \sum_{i=0}^{q} h(i) \left[\prod_{j=n}^{m-1} h(i+\tau_j) \right] C_{n,Y}(i+\tau_1,...,i+\tau_{n-1}) \quad (6)$$

Where $\varepsilon_{m,n} = \frac{\gamma_{m,x}}{\gamma_{n,x}}$.

This equation allows establishing relations between the autocorrelation and cumulants. It will also be the starting point for the algorithm that we propose.

B. Moments and cumulants estimation

1) Moments estimation

Let $X = x_{i=1..N}$ a random variable representing scalar **N** samples of a stationary signal.

The simplest estimator of order **k (**conventional estimator) is given by:

$$m_{k,x} = \frac{1}{N} \sum_{i=0}^{N} x(i) x(i+t_1) ... x(i+t_{k-1}) \quad (7)$$

2) Cumulants estimation

A detailed presentation of the theory of cumulants estimation can be found in [6], [7], [8]. As cumulants are expressed in terms of moments for centered **N** sample, the estimates of cumulants are obtained as follows:

$$\begin{aligned} &\hat{C}_{2,x}(t_1) = C_2(t_1) = m_2(t_1) \\ &\hat{C}_{3,x}(t_1,t_2) = m_3(t_1,t_2) \\ &\hat{C}_{4,x}(t_1,t_2,t_3) = m_4(t_1,t_2,t_3) - m_2(t_1)m_2(t_2-t_3) - m_2(t_2)m_2(t_1-t_3) - m_2(t_3)m_2(t_1-t_2) \end{aligned} \quad (8)$$

IV. APPROACH BASED CUMULANTS

Methods based only on cumulants ≥ 3 are more important when the processed signal is contaminated by additive Gaussian noise.

A. *Algorithm based on 4th Order Cumulant using equations q+1: ALG1*

The matrix form of the algorithm is given by ALG1 [1]:

$$\begin{pmatrix} C_{4,Y}(2q-1,2q-1,q-1) & \cdots & C_{4,Y}(q,q,0) \\ C_{4,Y}(2q-1,2q-1,q)-\varepsilon' & \cdots & C_{4,Y}(q,q,0) \\ 0 & \ddots & \vdots \\ \vdots & & \vdots \\ 0 & \cdots 0 & C_{4,Y}(q,q,q)-\varepsilon' \end{pmatrix} \begin{pmatrix} h(1) \\ \vdots \\ h(q) \end{pmatrix} = \begin{pmatrix} \varepsilon' - C_{4,Y}(2q,2q,q) \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad (9)$$

Where $\varepsilon' = \frac{C_{4,Y}(q,q,q) C_{4,Y}(q,0,0)}{C_{4,Y}(q,q,0)}$

Or in more a compact form:

$$M h_q = d \quad (10)$$

M is the matrix of size ((1 + q) x q) and the vector of dimension **d** ((q + 1) x 1) cumulants are compounds with the output, **Y** (k), the RIF system.

The elements of the vector q representing the parameter h (i) , i = 1, ..., q to be estimated in a blind manner, in the sense of least squares as follows:

$$h_q = (M^T M)^{-1} M^T d \quad (11)$$

B. *Algorithm based on 4th Order Cumulant using equations 2q +1: ALG2*

The matrix form of the algorithm is given by ALG2 [2]:

$$\begin{pmatrix} 0 & \cdots & 0 & C_{4,Y}(q,q,0) \\ \vdots & \ddots & & \vdots \\ 0 & & & \\ C_{4,Y}(q,q,0) & \cdots & & C_{4,Y}(q,q,q) \\ \vdots & \ddots & & 0 \\ & & & \vdots \\ C_{4,Y}(q,q,q) & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{h^2(q)} \\ \vdots \\ \frac{h^3(i)}{h^2(q)} \\ \vdots \\ \frac{h^3(q)}{h^2(q)} \end{pmatrix} = \begin{pmatrix} C_{4,Y}(0,0,-q) \\ \vdots \\ C_{4,Y}(0,0,0) \\ \vdots \\ C_{4,Y}(0,0,q) \end{pmatrix} \quad (12)$$

In a more compact form, the system of equations (12) can be written as follows:

$$Mb_q = d \quad (13)$$

With M and b_q are defined in the equation system (12). The solution in the sense of least squares, LS, of the equation system (13) is given by:

$$\hat{b}_q = (M^T M)^{-1} M^T d \quad (14)$$

This solution gives us an estimate of the quotient of parameters $h^3(i)$ and $h^3(q)$, by:

$$\hat{b}_q(i) = \left(\frac{h^3(i)}{h^3(q)}\right), i = 1,...,q \quad (15)$$

So, to estimate the parameters h(i) , i = 1,...,q we proceed as follows:

- The parameters $\hat{h}(i)$ for i = 1,...,q-1 are estimated from estimates of $\hat{b}_q(i)$ values using the following equation:

$$\hat{h}(i) = sign[\hat{b}_q(i)(\hat{b}(q))^2]\{abs(\hat{b}_q(i))(\hat{b}_q(q))^2\}^{1/3} \quad (16)$$

Where $sign(x) = \begin{cases} 1, & if \quad x > 0; \\ 0, & if \quad x = 0; \\ -1 & if \quad x < 0. \end{cases}$ and abs (x) = |x| indicates the absolute value of x.

- The parameter $\hat{h}(q)$ is estimated as follows:

$$\hat{h}(q) = \tfrac{1}{2} sign[\hat{b}_q(q)]\left\{abs(\hat{b}_q(q)) + \left(\frac{1}{\hat{b}_q(q)}\right)^{1/2}\right\} \quad (17)$$

C. *The Zhang algorithm*

Using equation (3), Zhang et al. [3] developed an equation, based on cumulants of order n, given by:

$$\sum_{i=0}^{q} h(i) C_{n,y}^{n-1}(i-t,q,...,0) = C_{n,y}(t,0,...,0) C_{n,y}^{n-3}(q,0,...,0) C_{n,y}(q,q,0,...,0) \quad (18)$$

For n = 4, from equation (18), we obtain the following equation:

$$\sum_{i=0}^{q} h(i) C_{4,y}^{3}(i-t,q,...,0) = C_{4,y}(t,0,...,0) C_{4,y}(q,0,...,0) C_{4,y}(q,q,0,...,0) \quad (19)$$

For t = -q, -q+1, ..., q, the system of equations (19) is solved according to the least square to estimate the parameters h(i) for i = 1,...q .The quality of the estimate can be measured by dividing the estimated parameters $\hat{h}(i)$ by the estimate of $\hat{h}(0)$ $\hat{h}(0)$ is close to 1 in the case of a good estimate.

IV. PROPOSED METHODS

In this section we propose to estimate the impulse responses $\theta = [h(0), h(1), ..., h(q)]$ FIR channel of order q using an algorithm that combines the three order cumulants and fourth order cumulants, as previously proposed as a hypothesis.

D. *General basic equation*

The relation (6) turns into an equation that links m and n such that m = n+1 follows:

$$\sum_{i=0}^{q} h(i) C_{n,y}(i+t_1,...,i+t_{n-1},t_n) = \varepsilon_{m,n} \sum_{i=0}^{q} h(i) h(i+t_n) C_{n,y}(i+t_1,...,i+t_{n-1}) \quad (20)$$

E. *Approach using cumulants of orders 3 and 4 together*

The above equation (20) becomes for m = 4 and n = 3:

$$\sum_{i=0}^{q} h(i) C_{4,y}(i+t_1, i+t_2, t_3) = \varepsilon_{4,3} \sum_{i=0}^{q} h(i) h(i+t_3) C_{3,y}(i+t_1, i+t_2) \quad (21)$$

We take $t_1 = t_2 = q$ and $t_3 = \tau$, the equation (21) becomes:

$$\sum_{i=0}^{q} h(i) C_{4,y}(i+q, i+q, \tau) = \varepsilon_{4,3} \sum_{i=0}^{q} h(i) h(i+\tau) C_{3,y}(i+q, i+q) \quad (22)$$

Knowing that $C_{4,y}(t_1,t_2,t_3) = C_{3,y}(t_1,t_2) = 0$, if $t_i > q$; the equation (22) becomes:

$$h(0) C_{4,y}(q,q,\tau) = \varepsilon_{4,3} h(0) h(\tau) C_{3,y}(q,q) \quad (23)$$

Therefore, the $h(\tau)$ are given by:

$$h(\tau) = \frac{C_{4,y}(q,q,\tau)}{\varepsilon_{4,3} C_{3,y}(q,q)} \quad (24)$$

Where $\varepsilon_{4,3}$ are given by (6):

$$\varepsilon_{4,3} = \frac{\gamma_{4,x}}{\gamma_{3,x}} \quad (25)$$

F. *Estimate of* $\varepsilon_{4,3}$

$\varepsilon_{4,3}$ is estimated from two approaches, assuming that h(0) = 1, noiseless channel or from Giannakis C(q,k) algorithm.

1) From h(0) = 0

If we assume that h(0) = 1, we obtain from equation (26):

$$\varepsilon_{4,3} = \frac{C_{4,y}(q,q,0)}{C_{3,y}(q,q)} \quad (26)$$

2) From Giannakis C(q,k) algorithm

According to the algorithm C (q, k) Giannakis [9], [10] $\gamma_{m,x}$ are given by:

$$\gamma_{m,x} = \frac{C_{m,y}^{2}(q,0,\dots,0)}{C_{m,y}(q,q,0\dots,0)} \quad (27)$$

So

$$\varepsilon_{4,3} = \frac{C_{4,y}^{2}(q,0,0) C_{3,y}(q,q)}{C_{4,y}(q,q,0) C_{3,y}^{2}(q,0)} \quad (28)$$

When the $\widehat{h}(i)$ are estimated on the divided by $\widehat{h}(0)$.

In the simulation we use this approach to correct the error of calculation of cumulants.

V. SIMULATION

To verify the performance of the proposed algorithms compared to ALG1, ALG2 and Zhang we use simulation test. The comparison is made using two examples one with non-Gaussian noise and the other with a Gaussian noise. In each case, the input excitation X(k) is random (i.i.d) zero mean. The Z(k), Y(k) and N(k) signals are connected by the equations (1) and (2). To assess the influence of noise, we define the signal to noise ratio (signal-to-noise ratio (SNR)) such as:

$$SNR = 10 \log\left(\frac{\sigma_y^2}{\sigma_N^2}\right) \quad (29)$$

With σ_D is the variance of D random distribution.

Similarly, to measure the accuracy of the estimated parameters compared with the real ones, we define the normalized mean square error, EQM (mean square error) for each iteration:

$$EQM = \sum_{i=0}^{q} \left(\frac{h(i) - \hat{h}(i)}{h(i)} \right)^2 \qquad (30)$$

The $\hat{h}(i)$, i = 1,...,q represent the parameters estimated in each iteration, and h(i), i = 1,...q represent the actual parameters of the model.

We represent EQM of the estimated parameters, with numbers relatively smaller sample (400, 800 and 1200) and for 100 iterations in the case without noise and in the case with noise with 10 dB of SNR. To compare the estimators for different levels of channel we normalize the value of EQM by dividing the order of the channel.

A. *Estimation with zero SNR*

The table below summarizes the estimated values for different samples sizes for zero SNR for a channel impulse response h(0) = 1 , h(1) = -0,85 et h(2) = 1.

TABLE I

TABLE ESTIMATES OF IMPULSE RESPONSES FOR ZERO SNR

Sample size N	**SNR = 0**				
	Algorithm	**h(0)**	**h(1)**	**h(2)**	**EQM**
400	ALG1	1,0000	-0,1941	0,2613	1,1411
	ALG2	1,0000	-0,2749	0,4501	0,2534
	Zhang	1,0000	0,1119	0,0086	2,2637
	ALGaT	1,0000	-0,8089	0,3706	0,1328
800	ALG1	1,0000	-0,2470	0,1547	1,2177
	ALG2	1,0000	-0,3189	0,4919	0,2162
	Zhang	1,0000	-0,0070	-0,0005	1,9844
	ALGaT	1,0000	-0,4746	0,9646	0,0654
1200	ALG1	1,0000	-0,3285	0,2645	0,9174
	ALG2	1,0000	-0,3390	0,6697	0,1569
	Zhang	1,0000	-0,0021	-0,0009	1,9969
	ALGaT	1,0000	-0,8804	0,4187	0,1131

According to the values summarized in Table I above we can see that the performance generally increases as a function of sample size and that the proposed algorithm ALGaT gives satisfactory results for different sample sizes.

The following figure (Fig.1) shows the curves of the magnitude (dB) for different algorithms for a sample size of about 800.

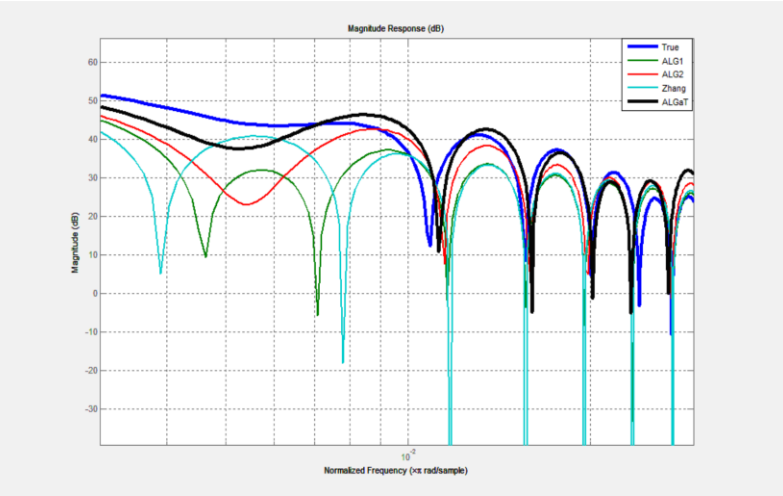

Fig. 1 The curves of the magnitudes responses for different algorithm using 800 sample size with out noise.

The curve of the magnitude response in dB, black color , given by ALGaT, perfectly follows the curve that represents the true real magnitude response curve, blue color, of the magnitude. However curves ALG1 and ALG2 are slightly different from the actual curve. But Zhang algorithm is far from perfect.

B. *Estimated with the presence of Gaussian noise*

The table below summarizes the estimated values for different samples sizes for a SNR = 10 dB for a channel impulse response h(1) = 1 , h(2) = -0,85 et h(3) = 1.

TABLE II
TABLE ESTIMATES OF IMPULSE RESPONSES FOR ZERO SNR

Sample size N	**SNR = 10 dB**				
	Algorithm	**h(0)**	**h(1)**	**h(2)**	**EQM**
400	ALG1	1,0000	-0,6128	0,6365	0,2100
	ALG2	1,0000	-0,6471	0,8994	0,0224
	Zhang	1,0000	-0,0049	0,0001	1,9883
	ALGaT	1,0000	-0,7613	0,8365	0,0125
800	ALG1	1,0000	-0,7110	0,8334	0,0545
	ALG2	1,0000	-0,6938	0,8651	0,0173
	Zhang	1,0000	0,0014	0,0007	2,0018
	ALGaT	1,0000	-0,7510	0,9292	0,0062
1200	ALG1	1,0000	-0,7753	0,8578	0,0280
	ALG2	1,0000	-0,8753	1,0492	0,0011
	Zhang	1,0000	0,0043	0,0124	1,9856
	ALGaT	1,0000	-0,8622	1,1428	0,0069

According to Table II above the various algorithms we can see that the performance increases with the size of the samples. Note the Zhang algorithm has poor performance. However the ALGaT algorithm shows good performance estimation compared to other algorithms.

The following figure (Fig.2) shows the curves of the magnitude (dB) for different algorithms for a sample size of about 800.

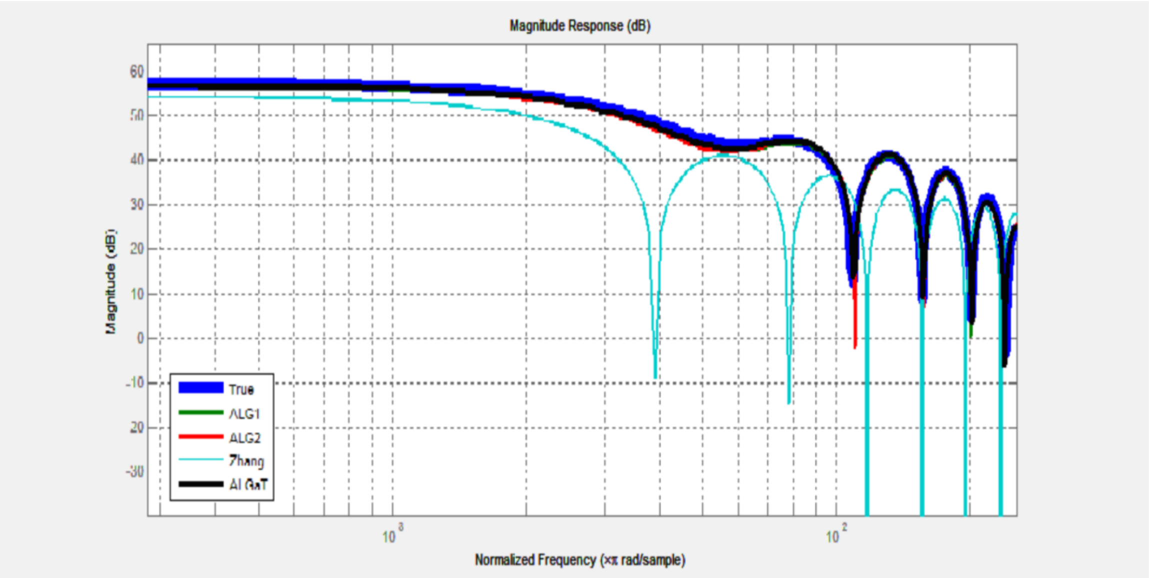

Fig. 2 The curves of the magnitudes responses for different algorithm using 800 sample size with a 10 dB SNR Gaussian noise.

The curve of the magnitude response in dB, black color , given by ALGaT, perfectly follows the curve that represents the true real magnitude response curve, blue color, of the magnitude.

VI. CONCLUSION

In this research study we propose an algorithm that combines the cumulants of order three and order four. Then we compare this algorithm to three other algorithms found in the literature, which depend only on the fourth order cumulants. However it depends on the square Zhang cumulants of order four. We notice that the proposed algorithm gives good results in the identification of the channel, then ALG1 and ALG2. However the Zhang algorithm is far from the best. This can be explained by the increasing error with the square of the cumulants.

We also observed that the various algorithms depend slightly on the size of the samples for size 400, 800 and 1200.

REFERENCES

[1] I.BADI, M.BOUTALLINE, E.ATIFY, S.SAFI, ''Blind identification of transmission channel with the method of higher-order cumulants'', International Journal of Advances in Science and Technology, vol 6 (3), 2013.

[2] Said Safi, Miloud Frikel, M. M'Saad, ''Higher order cumulants for identification and equalization of multicarrier spreading spectrum systems'', Journal of Telecommunications and Information Technology, pp: 74-84, 2011.

[3] X. D. Zhang, Y. S. Zhang,'' Fir system identification using higher order statistics alone'', IEEE Transactions on Signal Processing, 1994, vol 42 (12), pp:2854 – 2858.

[4] D. Brillinger, L. Rosenblatt, ''Computation and interpretation of kth order spectra'', in Spectral Analysis of Times Signals, New York: Wiley.

[5] A.Stogioglou, S.McLaughin, ''Ma parameter estimation and cumulant enhancement'', IEEE Trans Signal Processing, vol 44 (no 7), pp 1704 – 1717 July 1996.

[6] G.Giannakis, A. Swami,'' Higher order statistics'', Elsevier Science Publ, 1997.

[7] K. Abderrahim, R. B. Abdennour, G.Favier, M.Ksouri and F. Msahli, ''New results on fir system identification using cumulants'', APII-JES, 35(5), pp: 601-622, 2001.

[8] L. Srinivas, K. V. S. Hari, ''Fir system identification based on subspaces of a higher order cumulant matrix'', IEEE Transactions on Signal Processing, 44(6):1485-1491, 1996.

[9] G.Giannakis, '' On the identifiability of non-Gaussian arma models using cumulants'', IEEE Transactions on Automat vol 35.

[10] M'.BAKRIM, D.ABOUTAJDINE, ''Cumulant-based identification of non gaussian moving average signals'', Traitement du signal, vol 16 (3), pp: 175 – 186, 1999.