



A Multiple Thresholding Technique for Image Denoising

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Abstract— Currently Image processing is widely applied in various area of applications such as agriculture, health etc.. The problem which mostly arises in image processing is removal of noise generated by various sources. In this paper a robust technique is proposed for the removal of noise. This technique uses two level of thresholding. The technique combines the advantages of multi scale thresholding and bayes shrinkage technique. The noisy image is first pass through bayes shrink and the output of shrink is then applied to multiscale product threshold. The proposed method outperforms other methods both visually and in case of objective quality peak-signal-to-noise ratio (PSNR). Proposed method is verified for salt & pepper noise and additive white Gaussian noise.

Keywords— Image denoising, wavelet thresholding, Bayes threshold, Multiscale product threshold, PSNR, MSE, DWT, IDWT.

I. INTRODUCTION

In current scenario digital imaging is widely used in all fields related with our daily life because of increasing population need of advance imaging algorithms are needed. Generally a digital image corrupted by noise. There are different sources of noise in a digital image. For example, dark current noise is due to the thermally generated electrons at sensing sites; it is proportional to the exposure time and highly dependent on the sensor temperature. Shot noise is due to the quantum uncertainty in photoelectron generation; and it is characterized by Poisson distribution.

Amplifier noise and quantization noise occur during the conversion of the number of electrons generated to pixel intensities. The overall noise characteristics in an image depend on many factors, including sensor type; pixel dimensions, temperature, exposure time, and ISO speed [7]-[14]-[15]. Several image denoising algorithms have been proposed over the past few decades; among all these algorithms, wavelet thresholding is one of the most widely used techniques. In wavelet thresholding, a signal is decomposed into its approximation (low-frequency) and detail (high-frequency) subbands; since most of the image information is concentrated in a few large coefficients, the detail subbands are processed with hard or soft thresholding operations. The critical task in wavelet thresholding is the selection of threshold. Various threshold selection techniques have been proposed, for example, VisuShrink [2], SureShrink [3], and BayesShrink [4]. In the VisuShrink approach, a universal threshold that is a function of the noise variance and the number of samples is developed based on the minimax error measure. In the SureShrink approach, threshold value is optimal in terms of the Stein's unbiased risk estimator. The threshold value is selected in a Bayesian framework, through modeling the distribution of the wavelet coefficients as Gaussian, in BayesShrink approach. Several advancement have been proposed in thses shrinkage algorithms by considering interscale and intrascale correlations of the wavelet coefficients [8]-[10]-[16].

The paper is organized in the following behavior, in Section II wavelet transform is explained, in Section III multiscale product thresholding method is discussed. In section IV a new method for image denoising is proposed and compared in Section V based on performance metric and finally work in concluded in Section VI.

II. WAVELET THRESHOLDING

MULTISCALE decompositions have shown significant advantages in the representation of signals, and they are used extensively in image denoising, image compression and image segmentation [4]. The denoising of a natural image corrupted by Gaussian noise is a classic problem in signal processing. The wavelet transform has become a prominent tool image denoising due to its energy compaction property. According to it the wavelet transform yields a large number of small coefficients and a small number of large coefficients. Simple denoising algorithms that use the wavelet transform consist of three steps as.

- 1) Calculate the wavelet transform of the noisy signal.
- 2) Modify the noisy wavelet coefficients according to some rule.
- 3) Compute the inverse transform using the modified coefficients.

One of the most well-known rules for the second step is soft thresholding analyzed by Donoho [1]. Due to its effectiveness and simplicity, it is frequently used in the literature. The main idea is to subtract the threshold value from all coefficients larger than and to set all other coefficients to zero.

Let $f = \{f_{ij}, i, j = 1, 2, \dots, N\}$ denote the $M \times M$ matrix of the original image to be recovered and M is some integer power of 2. During transmission the x is corrupted by white Gaussian Noise n_{ij} with standard deviation σ . At the receiver side, the

noisy received signal $g_{ij} = f_{ij} + \sigma n_{ij}$ is obtained. The goal is to estimate the signal x from noisy observations g_{ij} such that Mean Squared error is minimum. Let W and W^{-1} denotes the two dimensional orthogonal discrete wavelet transform (DWT) matrix and its inverse respectively. $G = Wg$ represents the matrix of wavelet coefficients of g having four subbands (LL, LH, HL and HH). The sub-bands HL_1, HL_2, LH_1 are called *details*, where l is the scale varying from 1, 2, ..., J and J is the total number of decompositions. The size of the subband at scale l is $N/2^l \times N/2^l$. The subband LL_J is the low-resolution residue [1].

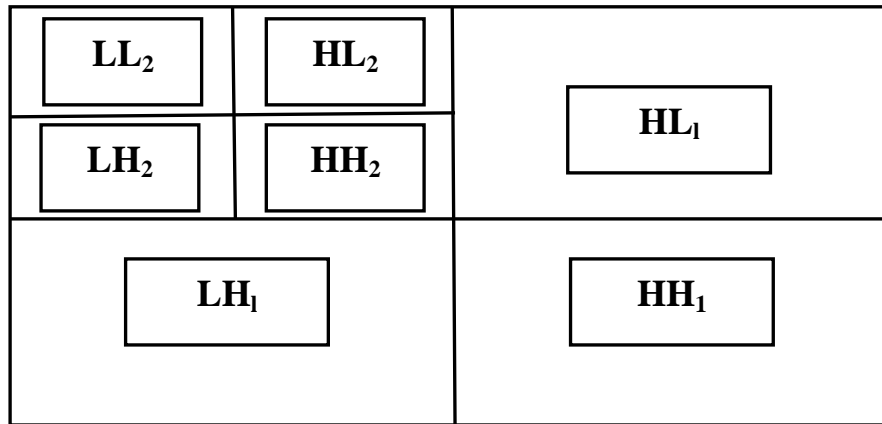


Figure 1: Wavelet Transform

The wavelet thresholding denoising method processes each coefficient of G from the detail subbands with a soft threshold function to obtain P . The denoised estimate is inverse transformed to $x = W^{-1}P$ [11]-[12].

III. MULTI SCALE PRODUCT THRESHOLD

All paragraphs must be indented. *Adaptive multiscale products thresholding* technique merges the merits of the thresholding technique and wavelet interscale dependencies. A significant wavelet coefficient $\widehat{W}_j^d f(x, y)$, where $d = x, y$ indicates x or y dimension, is identified if its corresponding multiscale products value $P_j^d f(x, y)$ is greater than an adaptive threshold $t_j^d(j)$. The algorithm as follows:

- 1) Compute the DWT of input image f up to J scales.
- 2) Calculate the multiscale products $P_j^d f(x, y)$ and preset the thresholds $t_j^d(j)$.

Then threshold the wavelet coefficients by

$$\widehat{W}_j^d f(x, y) = \begin{cases} W_j^d f(x, y) & P_j^d f(x, y) \geq t_j^d(j) \\ 0 & P_j^d f(x, y) < t_j^d(j) \end{cases}$$

$j = 1, \dots, J; d = x, y$ (1)

- 3) Recover the image from the thresholded wavelet coefficients $\widehat{W}_j^x f(x, y)$ and $\widehat{W}_j^y f(x, y)$.

Since a wavelet transform is a linear transform, the DWT of a noisy image $f = g + \varepsilon$ can be written as:

$$W_j^d f = W_j^d g + W_j^d \varepsilon \quad (2)$$

Where $W_j^d g$ is the DWT of original image and $W_j^d \varepsilon$ is the DWT of additive noise. For convenience, we denote

$$Z_j^d = P_j^d f = W_j^d f * W_{j+1}^d f \quad (3)$$

Due to the high dependencies existing between $W_j^d f$ and $W_{j+1}^d f$, the histograms of Z_j^d will have a heavy positive tail. A proper threshold $t_j^d(j)$ can be determined and imposed on Z_j^d to eliminate the highly noise corrupted pixels and identify the significant image structures. Suppose that the input image is Gaussian white noise and it is an ergodic stationary process. For the convenience of expression, we denote the DWT of ε by

$$U_j^*(x, y) = W_j^* \varepsilon(x, y) = \varepsilon * \varphi_j^*(x, y), \quad * = x, y$$

U_j^* is a Gaussian noise process with standard deviation

$$\sigma_j = \|\varphi_j\| \sigma$$

$$\|\varphi_j\| = \sqrt{\iint \varphi_j^2(x, y) dx dy}$$

$$p(u_j, u_{j+1}) = \frac{1}{2\pi\sigma_j\sigma_{j+1}\sqrt{1-\rho_{j+1,j}^2}} e^{-1/2(1-\rho_{j+1,j}^2)} [u_j^2/\sigma_j^2 - (2\rho_{j+1,j}u_ju_{j+1}/\sigma_j\sigma_{j+1}) + u_{j+1}^2/\sigma_{j+1}^2] \quad (4)$$

Where, $\rho_{j+1,j}$ is given by

$$\rho_{j+1,j} = \frac{\iint \psi_j(x, y) \cdot \psi_{j+1}(x, y) dx dy}{\sqrt{\iint \psi_j^2(x, y) dx dy} \cdot \sqrt{\iint \psi_{j+1}^2(x, y) dx dy}} \quad (5)$$

The scale products of U_j^d and U_{j+1}^d is:

$$V_j^d = U_j^d \cdot U_{j+1}^d$$

The pdf of $p(v_j)$ is given as:

$$p(v_j) = \frac{1}{\pi \Gamma(\frac{1}{2}) \sigma_j \sigma_{j+1} \sqrt{1 - \rho_{j+1,j}^2}} \cdot e^{(\rho_{j+1,j} v_j / (1 - \rho_{j+1,j}^2) \sigma_j \sigma_{j+1})} K_0 \left(\frac{|v_j|}{(1 - \rho_{j+1,j}^2) \sigma_j \sigma_{j+1}} \right) \quad (6)$$

The standard deviation of V_j^d is:

$$k_j = \sqrt{E[v_j^2]} = \sqrt{E[u_j^2 u_{j+1}^2]} \sqrt{1 + 2\rho_{j+1,j}^2 \cdot \sigma_j \sigma_{j+1}} \quad (7)$$

Values of probability

$\Pr_j(c) = P\{v_j \leq c \cdot k_j\}$, where c is the constant.

$$\mu_f^d(j) = E[Z_j^d], \mu_\varepsilon^d(j) = E[V_j^d]$$

$$\mu_g^d(j) = E[W_j^d \cdot g \cdot w_{j+1}^d]$$

Since the noise ε is independent of the noiseless image g , it can be derived that

$$\mu_g^d(j) = \mu_f^d(j) - \mu_\varepsilon^d(j) \quad (8)$$

$$\mu_\varepsilon^d(j) = \rho_{j+1,j} \sigma_j \sigma_{j+1} \quad (9)$$

The ratio $\mu_\varepsilon^d(j) / \mu_g^d(j)$ is the intensity of noise against signal in the multiscale products Z_j^d . This ratio can be used to adjust the threshold $t_j^d(j)$ imposed on Z_j^d . The multiscale products threshold is:

$$t_p^*(j) = 5k_j \left(1 + \frac{\mu_\varepsilon^d(j)}{\mu_g^d(j)} \right) \quad (10)$$

The adaptive threshold $t_j^d(j)$ is intuitive and effective. When noise is much stronger compared with the image, the ratio is high. Therefore, the threshold $t_j^d(j)$ becomes sufficiently large to suppress the overwhelming noise. When the image is dominative, the ratio is small and the threshold is at an appropriate level to preserve the image instantaneous features while removing noise [10].

IV. COMPARISON WITH EXISTING WORK

The proposed new denoising technique is compared with various existing technique based on PSNR (Peak signal to noise ratio) and MES (Mean square error). The Mean Square Error (MSE) and the Peak Signal to Noise Ratio (PSNR) are the two error metrics used to compare image quality. The MSE represents the cumulative squared error between the noisy and the original image, whereas PSNR represents a measure of the peak error. The lower the value of MSE, lower the error. To compute the PSNR, the block first calculates the mean-squared error using the equation:

$$MSE = \frac{1}{MN} \sum_{m=1}^M \sum_{n=1}^N (x(m, n) - \hat{x}(m, n))^2 \quad (11)$$

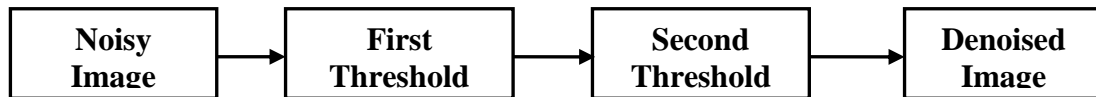


Figure 2: Block Diagram of Proposed Method

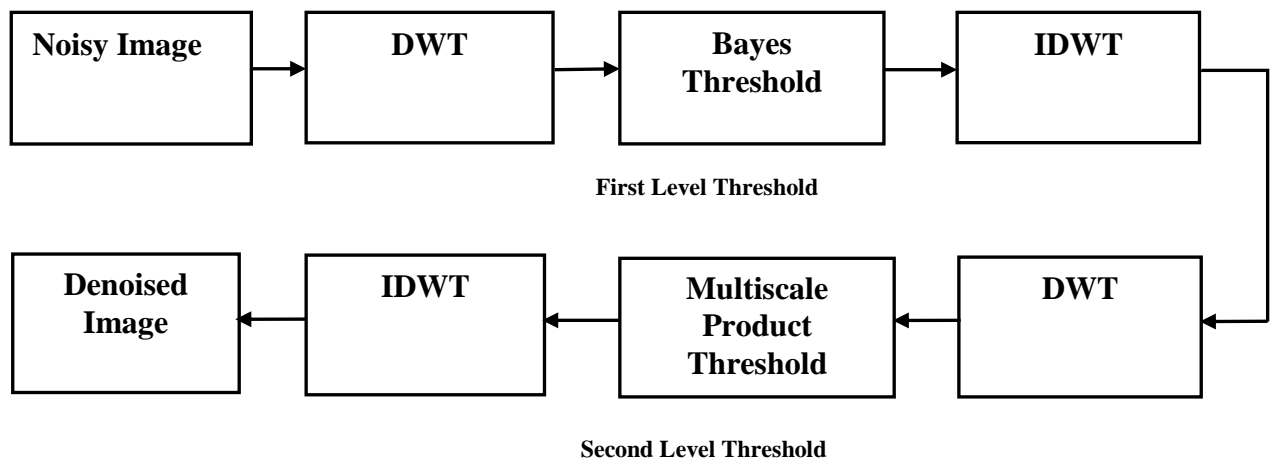


Figure 3: Detailed Block Diagram of Proposed Method

V. PROPOSED DENOISING METHOD

In this paper we proposed a multiple thresholding technique. First the received noisy image is applied to first thresholding method and output of first thresholding method is applied to second threshold in technique as shown in figure 2.

Figure 3 explains the proposed method in detail. As shown in first stage discrete wavelet transform of noisy image is taken and thresholded using Bayesian approach because Bayesian threshold has the better performance in comparison to normal and visu threshold technique. After applying threshold inverse discrete wavelet transform is taken. This is the end of first level thresholding. This output acts as input for next stage of thresholding.

In the previous equation, M and N are the number of rows and columns in the input images, respectively. Then the block computes the PSNR using the following equation:

$$PSNR = 10 \log \frac{R^2}{MSE} \quad (12)$$

In the equation (5.1), R is the maximum fluctuation in the input image. For example, if the input image has a double-precision floating-point data type, then R is 1. If it has an 8-bit unsigned integer data type, R is 255, etc. Table- 1 compares various techniques on the basis of their PSNR & MSE value for two types of noises salt & pepper noise and Gaussian noise. The value of noise density for which values are calculated here is $v=0.01$. Figure 3 shows the result for all the techniques. After the analysis of proposed scheme it is observed that it has better performance for Gaussian noise in comparison to salt & pepper noise. The result is simulated using matlab.

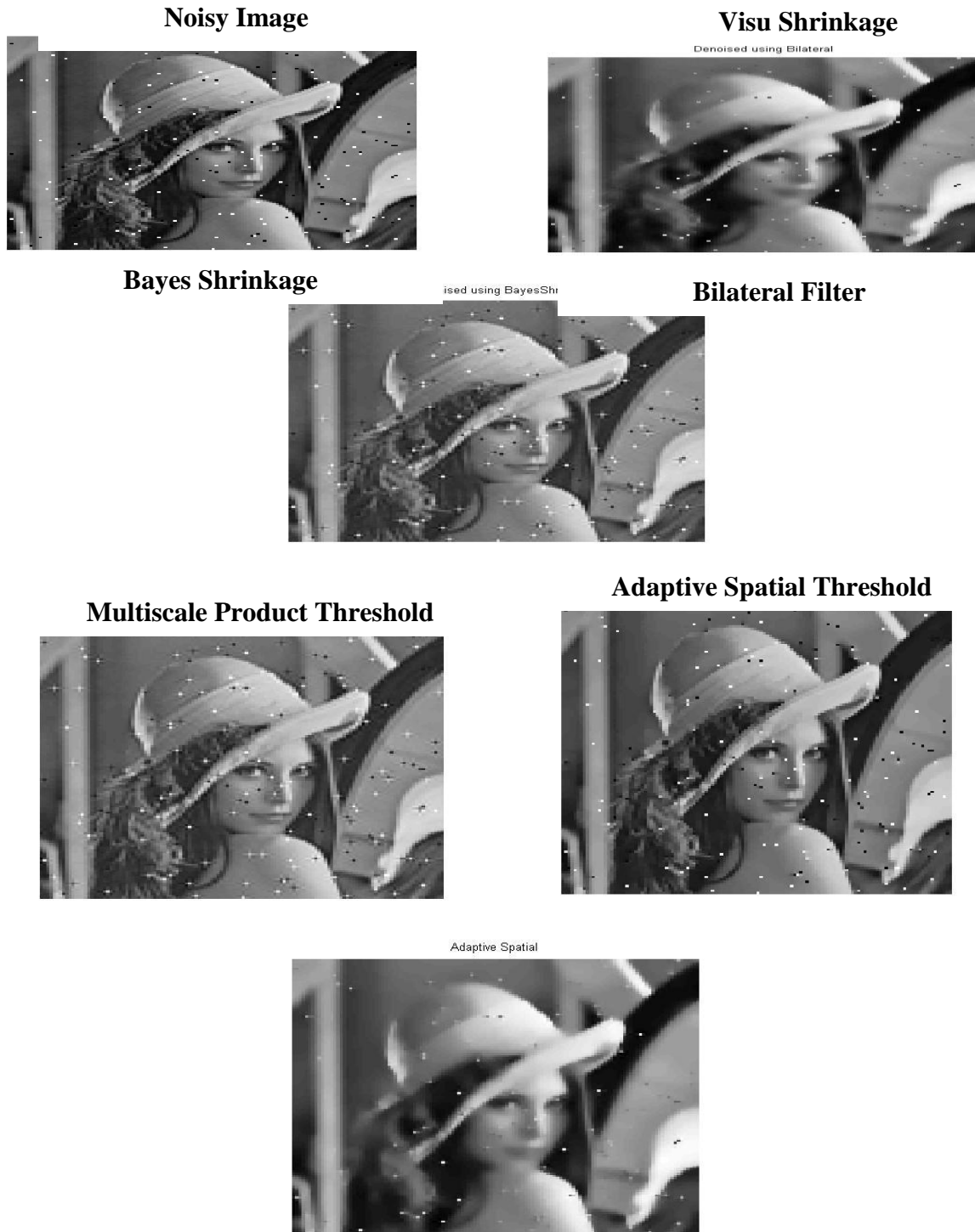


TABLE-1

COMPARISON WITH VARIOUS TECHNIQUES

Technique	Salt & Pepper Noise		Gaussian Noise	
	PSNR	MSE	PSNR	MSE
BayesShrink	74.57	0.0022	73.66	0.0028
VisuShrink	74.37	0.0023	70.55	0.0057
Bilateral	73.53	0.0028	73.71	0.0028
Multiscale Product Thresholdin	74.21	0.0024	69.67	0.0070
Adaptive Spatial Wavelet Thres	73.59	0.0028	73.54	0.0029
Proposed	75.14	0.0019	73.75	0.0027

Proposed



PROPOSED TECHNIQUE IMAGE

VI. CONCLUSION

This paper incorporates dual thresholding technique which combines Bayes Shrinkage thresholding and Adaptive Multiscale Products Thresholding. This method multiplies the adjacent wavelet subbands to strengthen the significant features in the image and then applies the thresholding to the multi scale products. The advantage of proposed technique is that it has not only improvement in image characteristics such as PSNR & MSE but also in image quality (Visual image). This technique is also less complex in terms of level of denoising in comparison to adaptive spatial multiscale technique. The proposed technique is verified for Gaussian noise and salt & pepper noise and observed that it has better performance for both but as the density of noise is increased performance for salt & pepper noise is less but for Gaussian noise it has significant performance.

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