



## Multi-Objective Optimization Using Differential Evolution

Er. Anuj Kumar Parashar  
Computer Science & Engineering  
Agra College, Agra, India

Dr. BDK Patro  
Computer Science & Engineering  
FET, RBS College, Agra, India

Dr. K Srinivas  
Electrical Engineering  
DEI, Agra, India

**Abstract**—In most real world multi-objective optimization problems the objectives are conflicting and therefore, they do not lend themselves to a single solution but result in a set of non-dominating solutions. Several issues arise in Multi-objective Optimization. Firstly, the entire search space has to be searched (in order to find all the good non-dominated solutions) without getting stuck in local optima, secondly, the search should approach the global Pareto-optimal front as closely as possible, thirdly, the search should also ensure a good spread of solutions along the obtained Pareto-optimal front and fourthly, it should achieve convergence in a reasonable time. Therefore, the search algorithm has to be carefully designed to address the above mentioned issues. In this work, Elitist Multi-objective Differential Evolution (E-MODE) a new Multi-objective Optimization algorithm is designed and implemented for the solution of real parameter multi-objective function optimization.

**Keywords**—Differential Evolution, Optimization, multi-Objective

### I. INTRODUCTION

This Optimization is the process of finding solutions that minimize or maximize a set of objective functions subject to constraints. When multiple objectives are present, the optimization problem is called a multi-objective optimization (MO) problem. An MO problem can be formally defined as finding a vector of decision variables that satisfies constraints and optimizes a vector function whose elements represent the objective functions [1]. . By using suitably adapted stochastic optimization methods it is possible to reveal the trade-off surface of a multi-objective optimization problem in a single run.[2]. The fact that EAs search from population to population rather than from one individual solution to another, makes them very well suited to performing multi-objective optimization. It is easy to conceive of a population being evolved onto the trade-off surface by a suitably configured EA. In fact, with an appropriate archiving scheme in place, the only modification required to a single objective EA, in order to perform multi-objective optimization, is in the selection scheme. As with single objective EAs, a wide variety of multi-objective selection schemes have been devised. Three of the most widely used (and most easily implemented) will be described here.[3]

### II. DIFFERENTIAL EVOLUTION

Differential Evolution (DE) is a population-based and directed search method [4], [5]. Like many other evolutionary algorithms, it starts with an initial population vector, which is randomly generated when no a priori knowledge about the solution space is available. Let us assume that  $X_{i,G}$  ( $i = 1, 2, \dots, N_p$ ) are candidate solution vectors in the generation  $G$  ( $N_p$ : population size). Successive populations are generated by adding the weighted difference of two randomly selected vectors to a third randomly selected vector.

#### Mutation

For each vector  $X_{i,G}$  in generation  $G$  a mutant vector  $V_{i,G}$  is defined by

$$V_{i,G} = X_{a,G} + F(X_{b,G} - X_{c,G}),$$

where  $i = \{1, 2, \dots, N_p\}$  and  $a, b,$  and  $c$  are mutually different random integer indices selected from  $\{1, 2, \dots, N_p\}$ . Further,  $i, a, b,$  and  $c$  are different so that  $N_p \geq 4$  is required.  $F \in [0, 2]$  is a real constant which determines the amplification of the added differential variation of  $(X_{b,G} - X_{c,G})$ . Larger values for  $F$  result higher diversity in the generated population and lower values cause faster convergence.

#### Crossover

DE utilizes the crossover operation to increase the diversity of the population. It defines the following trial vector:

$$U_{i,G} = (U_{1i,G}, U_{2i,G}, \dots, U_{Di,G}),$$

where  $D$  is the problem dimension and

$$U_{ji,G} = \begin{cases} V_{ji,G} & \text{if } \text{randj}(0, 1) \leq Cr, \\ X_{ji,G} & \text{otherwise.} \end{cases}$$

$Cr \in (0, 1)$  is the predefined crossover rate constant, and  $\text{randj}(0, 1)$  is the  $j$ th valuation of uniform random number generator. Most popular values for  $Cr$  are in the range of  $(0.4, 1)$  [5].

### Selection

The approach that must decide which vector ( $U_{i,G}$  or  $X_{i,G}$ ) should be a member of next (new) generation,  $G + 1$ . For a maximization problem, the vector with the higher fitness value is chosen. There are other variants based on different mutation strategies [6].

### III. PROPOSED ALGORITHM

1. Initialize parameters  $t=0, NP=100$ . ( $N$  is number of individuals in population),  $C$  and  $F$ . Where  
 $C \in (0,1)$  is crossover constant  
 $F \in (0,2)$  is mutant constant
2. Initialize target population  $X^t$
3. Evaluate each individual  $i$  in the population and obtain the fitness values corresponding to the respective objective functions.
4. Obtain the best\_front\_so\_far from the current population.
5. Obtain the mutant population (a mutant individual,  $V_i^{t+1} = [v_{i1}^{t+1}, v_{i2}^{t+1}, \dots, v_{in}^{t+1}]$  is determined such that  $V_i^{t+1} = Ceil(X_{best}^t + F(X_{bi}^t - X_{ci}^t))$  where  $X_{best\_front\_individual}^t$  is an individual randomly selected from the best\_front\_so\_far so far and  $b_i$ , and  $c_i$  are two randomly chosen individuals from the population such that  $(b_i \neq c_i)$ .
6. Obtain the trial population (For each mutant individual,  $V_i^{t+1} = [v_{i1}^{t+1}, v_{i2}^{t+1}, \dots, v_{in}^{t+1}]$  an integer random number between 1 and  $n$ , i.e.,  $D_i \in (1,2,\dots,n)$ , is chosen, and a trial individual,  $U^{t+1} = [U_1^{t+1}, U_2^{t+1}, \dots, U_{NP}^{t+1}]$  is generated such that:
$$U_{ij}^{t+1} = \begin{cases} v_{ij}^{t+1}, & \text{if } r_{ij}^{t+1} \leq CR \text{ or } j=D_i \\ x_{ij}^t, & \text{otherwise} \end{cases}$$
where the index  $D$  refers to a randomly chosen dimension ( $j=1,2,\dots,n$ )
7. Evaluate trial population
8. Obtain the best\_front\_so\_far from the combined population of Parent population and trial population.
9. Selection procedure for evolving the new parent population  $P(t+1)$ 
  - i. If the number of solutions in the best\_front\_so\_far is equal to  $N$  then put all these solutions in the new population  $P(t+1)$ .
  - ii. If the number of solutions in the best\_front\_so\_far is greater than  $N$  then select  $N$  solutions from this set using Crowding Distance Metric.
  - iii. If the number of solutions in the best\_front\_so\_far is less than  $N$  then put all the solutions of this set in the new population  $P(t+1)$  and select remaining members of the new population using the following procedure:
    - (i) Mark those parents who have not made it to the new population  $P(t+1)$ .  
For each parent marked in step (i).The selection is based on the survival of the fittest among the trial population and target population such that:
$$X_i^{t+1} = \begin{cases} U_i^{t+1}, & \text{if } (f_k(U_i^{t+1}) \geq f_k(X_i^t)) \text{ for } k=1 \text{ to } m \\ X_i^t, & \text{otherwise} \end{cases}$$
10. Update the external archive of non-dominated solutions using the current best\_front\_so\_far.
11. Repeat steps 3 to 9 while termination condition not reached.

### IV. TEST FUNCTIONS

The performance of E-MODE was evaluated on six benchmark problems proposed by Zitzler et al. [7] namely ZDT1, ZDT2 and ZDT3.

#### ZDT1

$$F_1(X) = X_1$$

$$F_2(X) = G(X) [1 - \sqrt{X_1/G(X)}]$$

$$G(X) = 1 + 9 / (NP-1) \sum X_i$$

#### ZDT2

$$F_1(X) = X_1$$

$$F_2(X) = G(X) [1 - (X_1/G(X))^2]$$

$$G(X) = 1 + 9 / (NP-1) \sum X_i$$

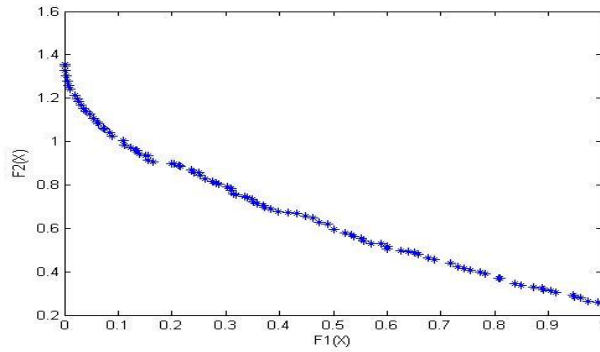
#### ZDT3

$$F_1(X) = X_1$$

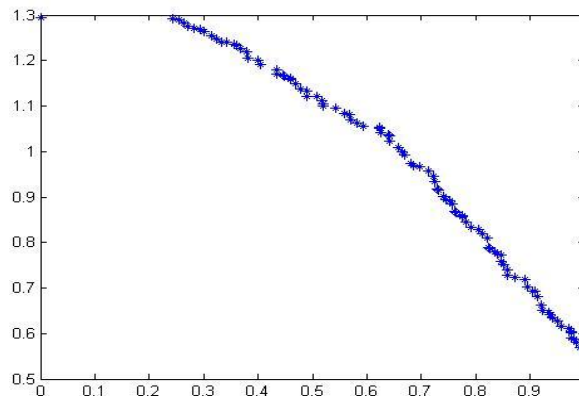
$$F_2(X) = G(X) [(1 - \sqrt{X_1/G(X)}) - (X_1/G(X)) \sin(10\pi X_1)]$$

$$G(X) = 1 + 9 / (NP-1) \sum X_i$$

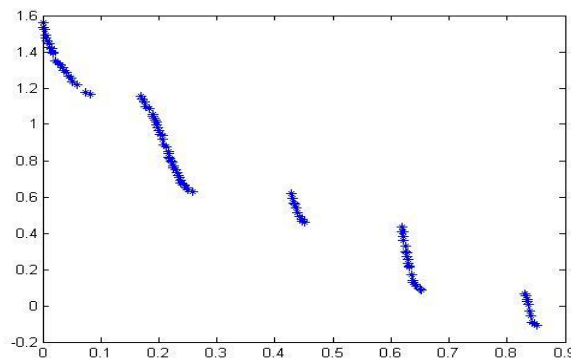
## V. RESULTS



Simulation Results of ZDT1



Simulation Results of ZDT2



Simulation Results of ZDT3

## VI. CONCLUSION AND FUTURE WORK

For the optimization of real-world problems it is important that the applied algorithm is capable of handling multiple objective functions and several constraints. In this work Differential Evolution algorithm is extended to a new version of Multi-objective Differential Evolution (E-Mode) based on a non-dominant sorting and crowding distance.

In this work, Elitist Multi-objective Differential Evolution (E-MODE) a new Multi-objective Optimization algorithm is designed and implemented for the solution of real parameter multi-objective function optimization. E-MODE extends Differential Evolution to deal with multi-objective optimization. It incorporates many important novel features like Pareto dominance, elitism, and crowded distance metric for ensuring a good spread of solutions and reducing the number of non-dominated solutions in the external archive of non-dominated solutions.. E-MODE algorithm is implemented in Matlab 7.0. The performance of the algorithm is tested on some standard multi-objective test functions. The obtained results indicate good performance of the algorithm in terms of obtaining quality solutions (global Pareto-optimal solutions with good spread along the Pareto-optimal curve), and fast convergence.

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