Volume 4, Issue 12, December 2014 ISSN: 2277 128X
International Journal of Advanced Research in Computer Science and Software Engineering
Research Paper
Available online at: www.ijarcsse.com

New Nonlinear Quick Technique for Automatic Stability Augmentation System by Continuation and Nonlinear "Pilot Induced Oscillations" Analysis

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Abstract*— Since the Wright brothers Pilot Induced Oscillations (PIO) has been noted in airplanes. The PIO classification according types and causes leads to distinguish three categories. First Category: they are primarily oscillations of the linear system pilot-plane. Second Category: the case where the model of the pilot is linear whereas that of the plane is quasi linear. Third Category: it remains badly defined. However, it's based on transitions in the non-linear model of the system aircraft-pilot. Many works touched on the subject but no one of them treats with the third category of PIO and it's on the latter that the present study will focus. For this purpose, we fist, use a new quick and automatic continuation non linear method to increase the stability of the aircraft. Second, and after verify that we have exactly the desired system, we analyse by continuation and bifurcation the couple aircraft-pilot. It was illustrated that starting from some pilot gain value both stable and unstable limit cycles are potentially possible. They are with limited amplitude but rapidly go to chaos or diverge.

Keywords— Stability Augmentation System, Pilot induced Oscillation, Limit Cycle, Bifurcation and Continuation.

I. INTRODUCTION

During the last decades, active flight control technology has drastically changed the way of aircrafts design. Flight control systems with mechanical linkages have been replaced by full authority, 'fly-by-wire', digital control systems. As a consequence, the flying qualities of modern aircraft are largely determined by a set of control laws in the heart of a computer system. These systems are used to damp and stabilize high-frequency rotational modes of the aircraft, making it easier for pilots to control the aircraft. Common types of SAS are roll dampers, pitch dampers, and yaw dampers. If an augmentation system is intended to control a rotational mode and to provide the pilot with a particular type of response to the control inputs, it is known as a Control Augmentation System (CAS) [1].

The development of flight control systems is a costly and time-consuming process. Indeed, extensive simulations, ground and flight tests are needed in order to validate flight control systems and to satisfy the stringent requirements of high reliability and performance. For most aircraft flying today the control laws have been developed using predominantly classical single loop frequency response and root locus design techniques. These methods have been used successfully for both single and multi-loop control problems as they were the only methods available for many years [2, 3].

Classical control techniques are limited however in the sense that they offer no way to incorporate allowances for the assumptions made about the aircraft model. If the resulting controller does not perform well when tested in a more realistic aircraft environment, the designer is forced to go back and adjust control gains or possibly redesign the entire control law. This tuning process becomes cumbersome and time-consuming for any sort of complex, multiple loop control scheme.

In all the control conception process no focus is done over the pilot dynamic, so even the aircraft dynamic is stable it's not sure that this stability can be kept when the pilot is in the loop. In fact, the dynamic interaction between the pilot and the aircraft can lead to sustained oscillations which may have very large-amplitude around all symmetry axis of the aircraft. These oscillations, also known as PIO, can lead to the deterioration of handling qualities, loss of stability and destruction of the aircraft. Indeed, many flight accidents such as those of YF22 and Boeing 777 [4, 5] have been attributed to PIO problems. PIO often occur during events of elevated gain that require pilot's tight control, such as takeoff, landing, aerial refueling and training flights. However, the pilot should not be blamed for the resulting oscillations: the problem is caused by an anomaly in the interaction between the pilot and the aircraft.

In order to analyze completely nonlinear PIO, after a brief description of PIO phenomena and bifurcation theory respectively in section II and III, we give in section VI a design of very quick, good and automatic non stability augmentation system. Then in section V, we developed tools based on BT that allow us to analyze systematically complete nonlinear Pilot-Vehicle-System (PVS).

II. PILOT INDUCED OSCILLATIONS

In 1997, a classification of PIO into three principle categories, depending on the degree of nonlinearity in the event has been proposed [6]: Category I groups essentially oscillations of the linear PVS. In category II, the pilot's model is linear whereas the one of the plane is quasi-linear. Category III remains not well defined. However, it is based on the nonlinear model of the PVS. Since then, several researchers have suggested that there may be other Categories that are distinct from the three defined above [7]. They are due to structural modes and their interactions with the pilot. PIO's analysis is relatively a difficult task; it consists in explaining, predicting and eliminating these phenomena.

Studies done on the topic have varied between analyzing the pilot's behavior before, during and after oscillations [8, 9] and considering the phenomenon as a limit cycle [10]. Other analysis tools are based on geometric presentations as in the frequency domain 'Neal Smith (NS) criterion [11]' which use the pilot task bandwidth and the closed loop resonance to decide about Pilot Rating (PR) and PIO tendency. The position of Open-Loop Onset Point (OLOP) on Nichols Diagram was also proposed to predict PIO tendency [12]. In [13], the Describing Function (DF) approach was used to calculate the frequency and amplitude of possible oscillations due to multiple nonlinearities. This approach is limited however by restrictions on the types and positions of nonlinearities that can be considered.

In [10], Bifurcation Theory (BT) was used to illustrate the jump phenomenon in PVS. This was applied on a simple PVS model and illustrated only stable limit cycles. In [14], by using BT and extended one, we have show new results of PIO phenomenon particularly caused by multiples nonlinearities. From those results, there was unstable limit cycles situated in the jumps regions. Two types of analysis were presented; one is considering aircraft-pilot as unforced system and the second as forced one. The present study is dedicated to analyze completely nonlinear PIO, we have considered these phenomena as limit cycles and used numerical BT-based parametric analysis for prediction and stability of possible oscillations that may occur as some parameters are varied.

III. BIFURCATION THEORY

It is well known that predicting the asymptotic behavior of non linear parametric differential equations can be done by bifurcation theory. Several efficient numerical procedures are available and also many studies state that bifurcation analysis can be used to predict complex phenomena. A dynamic system is generally represented by a multivariable parametric differential equation as given by equation 1.

$$\dot{x} = f(x, p) \tag{1}$$

where f is a *smooth* function, $x \in \Re^n$ (state vector) and $p \in \Re^m$ (parameter vector).

The equilibrium solutions of (1), depend on *(p)* and are given by the solution of the equation $f(x, p) = 0$. As the parameter (p_i) varies, the implicit function theorem states that these equilibrium are given by smooth functions of *(p)*. Each such equilibrium path is called a branch of equilibrium of (1). The stability of that equilibrium is decided by the sign of the real parts of eigenvalues.

In some cases, there is a parameter value (p_c) and equilibrium point (x_0) at which the Jacobian matrix $D_x(x_0, p_c) \in \Re^{n\times n}$ has a zero eigenvalue. At this point, there is many solutions of the equation $f(x, p) = 0$, so several branches may join at this point, (x_0, p_c), which is called bifurcation point. A graph of solutions *x(p)* vs the bifurcation parameter p is called a bifurcation diagram. In other cases, there is a parameter value (p_h) and equilibrium point (x_h) at which the Jacobian matrix $D_x(x_h, p_h) \in \Re^{n\times n}$ has a pair of simple, purely imaginary eigenvalues $\pm iw_c$, and no other purely imaginary eigenvalues. Such cases correspond to the Hopf bifurcation and mean that at this point (x_h, p_h), there is a bifurcation to a periodic solution. The limit cycles are decided to be stable when all floquet multipliers are less then one and unstable otherwise.

IV. NONLINEAR AUTOMATIC SAS DESIGN AND REALISATION

Practically, the nonlinear model of a plane is linearised at several points of equilibrium and a linear controller is pre-programmed to obtain the desired system answers for each point. This approach remains very complex and aberrant, reason for which, we propose a continuation approach to find the gains ensuring the model of the desired aircraft dynamic.

The principle idea of using continuation is to consider the system in closed loop and the feedback gains as additional variables. It's to remind that our system is given by n equations (2) and if we consider extra variables it's necessary to add extra equations. Those extra equations (3) can be added by considering the manoeuvrability qualities witch give us the dumping and frequency values of some states variables of the desired system.

$$\dot{X} = f(X, U) \tag{2}$$

where $X=(M, \alpha, q, \theta)^T$, $U=(\eta, \delta_e)^T$, $f=(f_M, f_\alpha\ f_q, f_\theta)^T$

and $\dot{X}$ is the derivative of *X*. More details are given in Annexe 1.

$$\lambda_{1,2} = \frac{(\Delta_{11}+\Delta_{22}) \pm j\sqrt{(\Delta_{11}+\Delta_{22})^2 - 4(\Delta_{11}\Delta_{22} - \Delta_{21}\Delta_{12})}}{2} \tag{3}$$

For short period dynamic, we consider variables (α, q) and we have

$$\Delta_{11} = \left.\frac{\partial f_\alpha(X, -k_\alpha\alpha - k_q q)}{\partial \alpha}\right|_{X=X_0}, \Delta_{12} = \left.\frac{\partial f_\alpha(X, -k_\alpha\alpha - k_q q)}{\partial q}\right|_{X=X_0}, \Delta_{21} = \left.\frac{\partial f_q(X, -k_\alpha\alpha - k_q q)}{\partial \alpha}\right|_{X=X_0}, \Delta_{22} = \left.\frac{\partial f_q(X, -k_\alpha\alpha - k_q q)}{\partial q}\right|_{X=X_0}$$

X_0 is the equilibrium point and (k_α, k_q) are feedback gains of (α, q).

Table 1 gives the desired values of$\lambda_{1,2}$ for the shot period dynamic.

TABLE I DESIRED DUMPING AND NATURAL PULSATION FOR SHORT PERIOD DYNAMIC

	Short Period
Dumping ξ	ξ_{sp}=.5
Naturel pulsation w	w_{sp}=.73
Equivalent eigen values	$\lambda_{1,2}$=-.365±.i63

Moreover, we want to keep the plane under certain conditions of equilibrium: (path angle $\gamma=\alpha-\theta=0$), so we can exploit those constraints to have additional equations in order to obtain the feed forward corrections on surfaces of control $\delta_e(\alpha_r)$ and trust engine $\eta(\alpha_r)$ according to the desired attack angle α_r.

One's all the equations are established, the calculus of feedback gains and feed-forward corrections is equivalent of looking for equilibriums points for different value of the elevator control surface δ_e. Starting from an equilibrium point satisfying as well the initial equations of the plane as the additional equations and using continuation other equilibrium points are found according to the parameter δ_e. So, curves gains ensuring the set of these equilibrium points are defined as illustrated in Fig. 1-a and 1-b. for all value of α.

Note that, when deriving nonlinear equations, we consider k_α and k_q as constants of α, witch is not true. So, we have to change static gains k_α and k_q respectively to dynamic gains K_α and K_q in order that all the equation will be respected.

We have proof that replacing $k_\alpha\alpha$ by $K_\alpha\alpha$ given by equation 4, is a solution for the first omission of considering k_α as static. Fig. 3 illustrates this non linear feedback corrected.

$$K_\alpha \alpha = \int_{-\infty}^{\alpha} k_\alpha(\alpha) d\alpha + cte \tag{4}$$

We have also proof that, unfortunately, we have no solution for $K_q(\alpha)$, the only solution is to take it as series of steps defined with the value of k_q in a little segment of α.

$$\delta_e(\alpha_r) \equiv \delta_e(\alpha_r) + \int_{-\infty}^{\alpha_r} k_\alpha(\alpha) d\alpha \tag{5}$$

By changing the values of feedback gains, we have change the relation between the input and the output of the designed system. To keep this relation unchanged, we have introduced a correction to the command surface angle δ_e. This correction is given by equation 5.

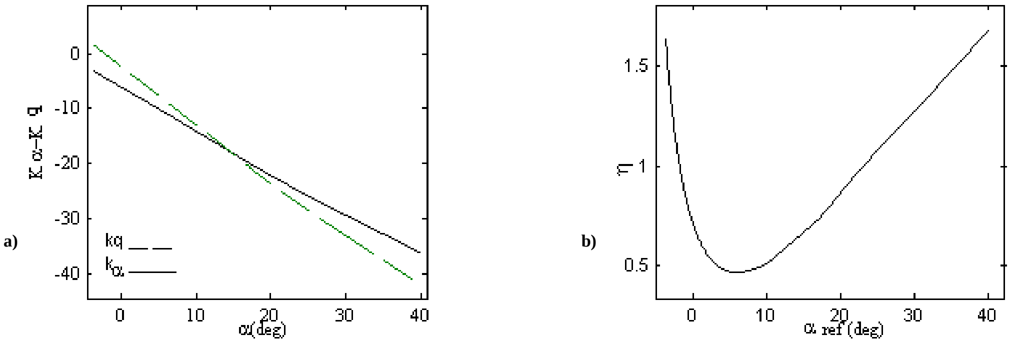

Fig. 1 **a)** Quasi-linear attack angle and pitch rate feedback gains **b)** feed-forward trust correction

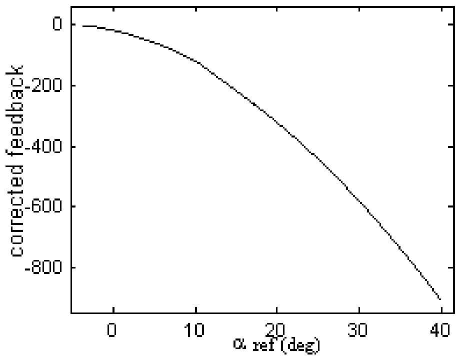

Fig. 2 : Non-linear attack angle feedback

By taking again these gains calculated in the nonlinear equations of the plane in closed loop, we can easily check the position of the new poles. The results of this stage are illustrated for the F-16 aircraft in open loop by Fig. 2 and Fig. 3 in closed one.

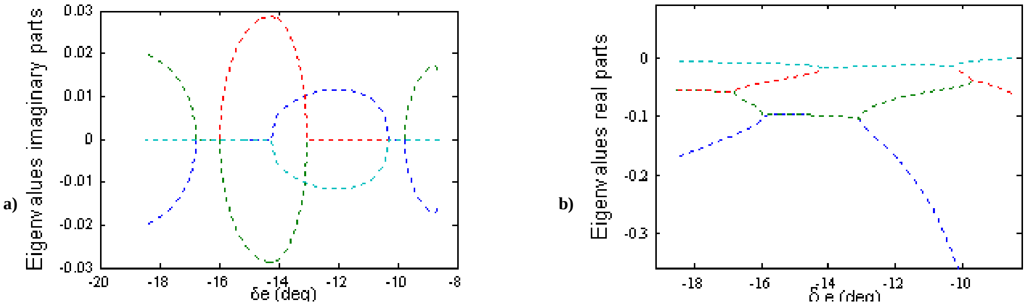

Fig. 3 **a)** Real and **b)** Imaginary eigenvalues for non linear F16 in open loop

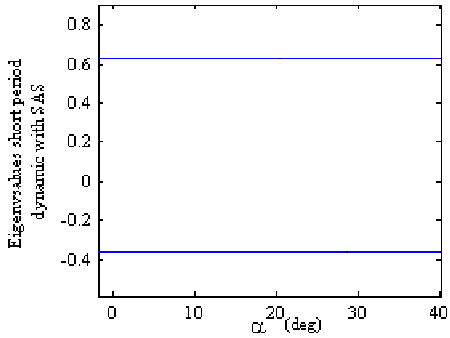

Fig. 4 : New poles position for Short Period F16 dynamic with the designed SAS

These first results obtained proved that the used technique is very encouraging and very attractive, especially that the calculation of feedback and feed forward gains done in an almost automatic way. The aircraft dynamic with the new SAS match exactly the desired system. Fig. 2-3 show the poles positions without and with the SAS. It's clear that by the SAS, we have exactly the desired system. Fig. 5-a) shows the response in open loop and how α and θ diverge. Fig. 5-b) shows that with the designed SAS the path angle γ is kept null all the time (α=θ) us considered in the design process. By Fig. 6, we show that we obtain the expected response for all the values of attack angle.

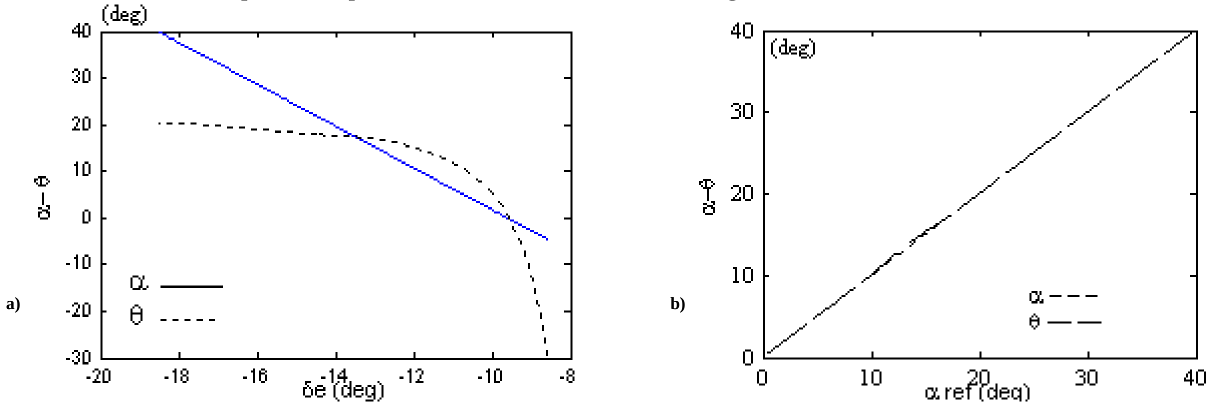

Fig. 5 **a)** Non linear f16 aircraft response in open loop **b)** Attack and pitch angle response to reference attack angle with designed SAS

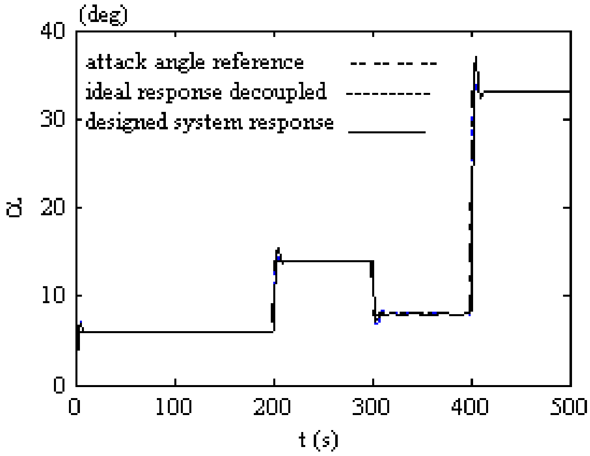

Fig. 6 Non linear f16 aircraft response with the designed SAS

V. NON LINEAR PIO ANALYSIS

Thereafter, we are interested in the effect of the interaction of the pilot represented by a linear transfer function. Thus, an analysis by the theory of bifurcation is carried out in order to guess the possibility of existence of stable and unstable limit cycles. Next paragraphs give details of used model in closed loop analysis.

A. Pilot and stick model

The model used to represent the pilot and the stick is given by Fig. 7 and is taken from [13]. The gain of the pilot is modeled by $K_p(s) = \frac{k_p}{\tau s + 1}$. There is a dead zone is limited by *(b=±5 lb)* whereas the saturation is limited by *(d_1=±25 deg)*.

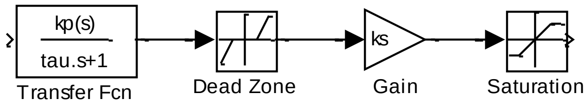

Fig. 7 The pilot and the stick model

In order, to analyze only PIO category III, we have omitted all typical non-linearities in the stick model and actuators. In fact, PIO analysis category II who is caused by typical non linearity was largely studied in our last work [14].

B. F16 aircraft longitudinal Dynamic model

The model of the plane used is extracted from [15], it exhibits a non linear model detailed in the **annex 1** that was used in SAS conception and PIO analysis.

C. Pilot-aircraft in loop mode

The model considered for the analysis of the PVS is depicted in Fig. 8.

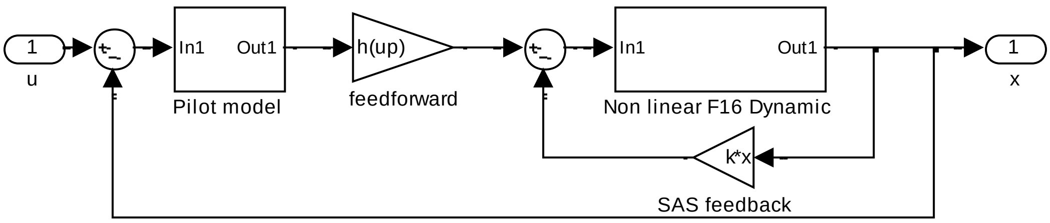

Fig. 8 Pilot-aircraft in the loop

D. PIO Analysis

A numerical pilot-induced oscillation (PIO) prediction method is developed. This method is based on modeling the PIO phenomena as limit cycle and the pilot action as feedback control. In order to define some limits for pilot maneuvers to avoid PIO, we have decided to perform the analysis by varying the pilot gain k_p for different values of the lag τ. Starting from an equilibrium point obtained by trimming the aircraft-pilot system, we have applied the continuation approach by looking for other equilibrium points as k_p varies. For τ=.3 and different values of α reference, we have established the Fig. 9 witch illustrate the existence of Hopf point who indicate the born of limit cycles.

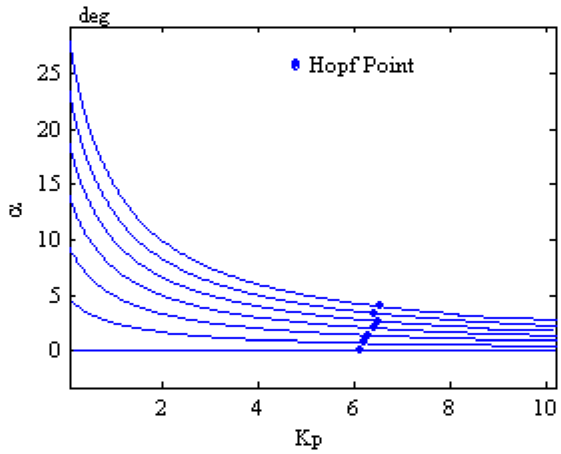

Fig. 9 Equilibrium and Hopf points as kp varies for τ.

Same thing was found different values of τ. Those limit cycles bifurcates from equilibrium point, from some of those Hopf points, we have start looking for the continuation of limit cycles. The results of this operation for α_{ref} =0 and respectively τ=.15s and τ=.2s are illustrated respectively by Fig. 10-11.

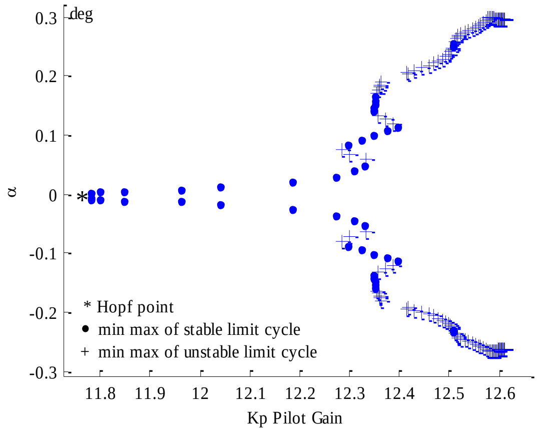

Fig. 10 Stable and unstable limit cycles for τ =.15 as Kp varies

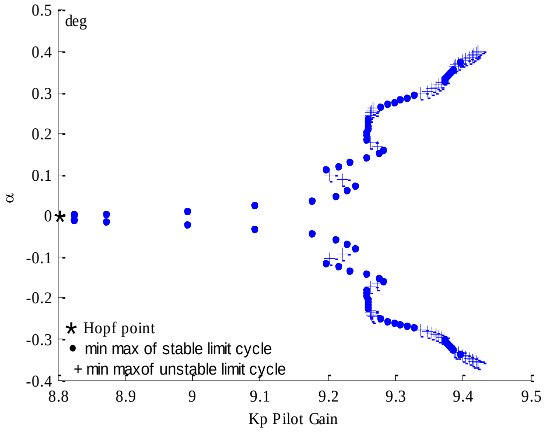

Fig. 11 Stable and unstable limit cycles for τ =.2 s as Kp varies

For both cases, the analysis stops because the system goes to chaos or diverges. This chaos is illustrated by Fig. 12.

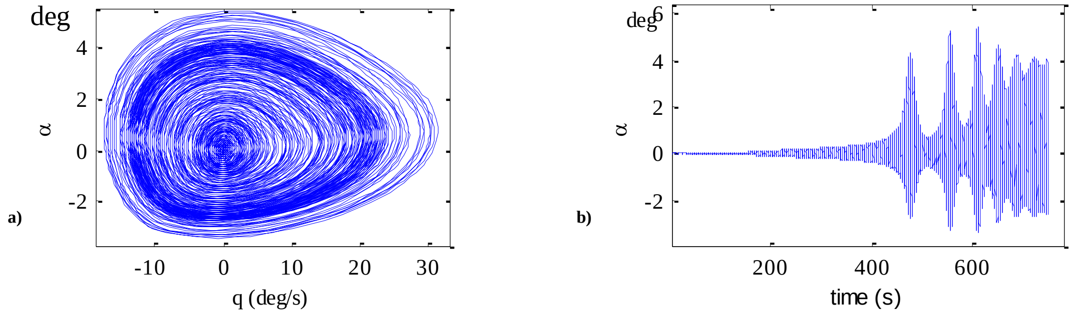

Fig. 12 Aircraft in loop in chaos situation Kp=13 τ=.15 **a)** phase diagram **b)** attack angle vs time

VI. CONCLUSION

The objective is to analyze the phenomenon of oscillations due to the interaction of the pilot. Thus, the SAS had initially been designed; this was carried out in a very fast and effective way by transforming our problem to a continuation one. A transformation of the static feed forward gains to a dynamic one was introduced to have the desired system. The results are very promising and give the exact desired system with the desired constraints. Thereafter, an analysis by bifurcation theory is done to highlight the possibility of existence of stable, unstable limit cycles and chaos phenomena at the time of the interaction of the pilot. By this study, we have discovered PIO category III presented by limit cycles with a limited amplitude but witch diverge largely or goes to chaos. We project to analyse a complete aircraft dynamic longitudinal and lateral one.

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