



Temperature and Pressure Effects on the Statistical Properties of Lipid Bilayers

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Abstract—The system that we consider the position fluctuations of a fluid membrane confined to a substrate in an external potential and at finite temperature. We took in first time a quasi-flat membrane subjected to a kind of potential van der Waals -Helfrich where it appears the temperature setting. The analytical study shows that as the temperature increases phospholipids pass a crystalline phase at very low temperature, a gel phase in an intermediate area at a high temperature fluid phase, and in second time, we interested in studying lipid bilayers where van der Waals attractions are balanced by hydration repulsion associated with a hard wall. Careful study shows that the decrease in pressure causes an increase in the depth that is to say, the membrane becomes more stable.

Keywords—Biological membranes , Confinement , Wall interact, Solvent , Temperature, Pressure.

I. INTRODUCTION

The biomembranes are omnipresent in all living beings. Indeed, they delimit cells and create compartments, and participate in almost all biological functions. These biological materials are complex systems, but they possess a natural structural organization, where each component has a specific function (e.g. [1]–[7]).

Nowadays, the scientific community recognizes that the cell membranes essentially present as a phospholipid bilayer combined with a variety of proteins and cholesterol. For example, the function of the cholesterol molecules is to ensure the bilayer fluidity. A phospholipid is commonly defined as an amphiphile molecule that is composed of a hydrophilic polar head attached to a two hydrophobic (fatty acyl) chains. We note that the phospholipids move freely on the membrane surface. On the other hand, the thickness of the bilayer is of the order of 5 nanometres. This two facts allow to consider this bilayer as a two-dimensional fluid membrane. The experiment shows that the fluid membranes, self-assembled from surfactant solutions, may have a variety of shapes and topologies (e.g. [8]). These forms have been theoretically explained in terms of bending energy (e.g. [9]–[10]). Usually, the biomembranes are not immersed in liquids of infinite extent, but they are rather confined to geometrical boundaries. Typical examples are provided by a white and red globules or liposomes, as drugs transport agents (e.g. [11]–[14], in blood vessels.

The purpose of the present paper is to develop a self-consistent theory for a fluid-free membrane floating near solid substrate (Figure 1).

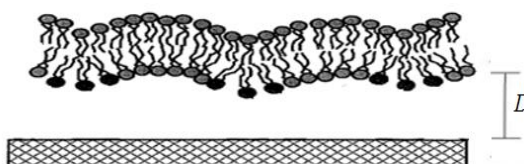


Fig1: membrane floating near solid substrate

The shape of the substrate potential determines the amplitude σ of a thermal fluctuations which it self determines the effective potential. The average membrane position and its root meansquare (rms) fluctuation amplitude are thus coupled and depend both on temperature and on bending energy of the membrane. (e.g. [15]).

The average membrane position and its root meansquare (rms) fluctuation amplitude are thus coupled and depend both on temperature and on bending energy of the membrane. This model situation is experimentally relevant as reported in a recent paper by Fragneto et al. (e.g. [16]). An important aspect of biological membranes is that they are typically not free but rather confined by other surrounding membranes, adhere to other membranes, and attach to elastic networks such as the cytoskeleton and the extracellular matrix. Several model systems with reduced compositional complexity have been designed to mimic biological membranes. These biomimetic systems include phospholipid bilayers deposited onto solid substrates (solid-supported membranes) (e.g. [17]). or on ultrathin polymer supports (polymer-supported membranes) (e.g. [18]). With the aid of biochemical tools and genetic engineering, supported membranes can be functionalized with various membrane-associated proteins (e.g. [19]). Synthetic supported membranes with reconstituted proteins are

increasingly used as controlled idealized models for studying key properties of cellular membranes (e.g. [20]). They provide a natural environment for the immobilization of proteins under non-denaturing conditions and in well-defined orientation (e.g. [21]). Another attractive application of supported membranes is the design of phantom cells exhibiting well-defined adhesive properties and receptor densities (e.g. [22]). Finally, biofunctional membranes supported by solid interfaces (semiconductors, metals, plastics) provide new classes of biosensors, diagnostic tools, and other biocompatible materials (e.g.[19],[23]). Theoretically, the thermal shape undulations of supported membranes have been addressed for various model systems. These model systems include (a) membranes that adhere to surfaces under the action of a continuum local potential(e.g.[24],[25]), (b) membranes pinned or tethered discretely to a surface (e.g.[26]-[30]), (c) membranes supported by elastic networks of springs (e.g.[30]-[42]) ,and (d) Fluctuation spectra of free and supported membrane pairs (e.g.[43]). We thus want to compute the equilibrium distance D between the bilayer and the substrate and the rms fluctuation amplitude σ for any given value of the bending rigidity κ (e.g.[44]). However, although abundant, the theoretical literature on fluctuations of a membrane submitted to an external potential does not address exactly this problem, to our knowledge. The difficulty is the following. According to Helfrich, the energy of a membrane is the sum of the external potential and the cost of bending. At finite temperature, the potential determines the spectrum of fluctuations, according to the equipartition of energy. In turn, the entropy cost of fluctuations contributes to the free energy and determines the average position . This paper is organized as follows. In Sec. II, we discuss the effect of temperature and pressure on the statistical properties of lipid bilayers III. Some concluding remarks are drawn in the last section.

II. THEORETICAL FORMULATION

A. ONE MEMBRANE SUPPORTED ON SOLID SUBSTRATE

We first consider a single membrane lying on a horizontal solid planar substrate at position $z = 0$. Membrane local position vector is, in the Monge representation, $R = x\hat{X} + y\hat{Y} + h(x,y)\hat{Z}$ where $h(x,y)$ is the height the membrane and $\eta = \langle h(x,y) \rangle$ is the average distance between the substrate and the membrane. The Hamiltonian of a supported membrane is given by the usual Helfrich Hamiltonian of a single fluctuating membrane (e.g.[9],[15]), plus a term taking into account the interaction between the membrane and the substrate:

$$H = \frac{1}{2} \int_S d^2r \left\{ \kappa (\nabla^2 h(r))^2 + \gamma (\nabla h(r))^2 + V[h(r)] \right\} \quad (1)$$

where κ is the bending modulus of the bilayer, γ its surface tension and U the membrane-substrate interaction potential. Using a quadratic approximation for $V[h(r)]$ it is possible to apply the equipartition theorem for small fluctuations:

$$\langle |U_q|^2 \rangle = \frac{1}{S} \times \frac{k_B T}{\kappa q^4 + \gamma q^2 + V''} \quad (2)$$

Note that this expression is only valid when the fluctuation amplitude is smaller than the mean distance to the substrate, which seems to be the case in our experiments. For large fluctuations one would have to take into account the reduction of entropy caused by the substrate, leading to the so-called Helfrich repulsion (e.g.[9]).

B. EFFECT OF THE PRESSURE

a. Interaction potential

The physical properties of lipid bilayers are the result of a competition between attractive and repulsive molecular interactions (Van der Waals, electrostatic, hydration). The membranes are also subject to thermal fluctuations which lead to entropic repulsion (e.g.[46],[47]). The Van der Waals interaction $U(z)$ between two membranes of thickness δ separated by a distance z can be written:

$$U_{vdW}(z) = -\frac{A}{12\pi} \left[\frac{1}{z^2} - \frac{2}{(z+\delta)^2} + \frac{1}{(z+2\delta)^2} \right] \quad (3)$$

where the Hamaker constant H is on the order of $k_B T$ and depends both on the lipids and on the solvent. At short length scales (less than 1 nm), bilayers separated by a distance d experience an exponentially decaying repulsive hydration force, the microscopic origin of which has been the matter of intense debate(e.g.[48]-[51]). We write the hydration potential

$$U_h(z) = A_h \exp(-2z/z_0), \quad (4)$$

where $z_0/2 = d_h \approx 0.2 - 0.3$ nm, (e.g.[52]) is the hydration length and $P_h \approx 10^8 - 10^9$ Pa is the hydration pressure. **Table.1** - Parameter values of effective interaction between membranes “T. Charitat, S. Lecuyer et al⁵³, And H. I. Petrache, N. Gouliarov et al⁵⁴(DSPC multilayer) .

Ref	System	$T(^{\circ}C)$	$A(\times 10^{-21} J)$	$P_h(\times 10^6 Pa)$	$d_h (nm)$
[53]	DSPC	43	0.1	27	1.3
[53]	DSPC	53	1.15	29.5	1.3
[54]	DPPC	50	2.87-9.19	13.8-19.5	1.97-2.39

Finally, as first highlighted by Helfrich (e.g.[9]), an additional entropic contribution due to the confinement of the fluctuating membranes needs to be taken into account. The total potential $V(z)$ is as follows

$$V(z) = U_{VDW}(z) + U_h(z) \tag{5}$$

the minimum position z_{min} of this potential $V(z)$ depends on the value of z_0 and the report A_h/A , the ratio A_h/A was estimated for different values of number of carbon atoms per chain in the formula Di-Cn-PC (phosphatidyl-cholines (PC) with different chain lengths), for example: di-palmitoyl (Di-C16-PC), di-heptadecanoyl (Di-C17PC), and di-stearoyl (Di-C18-PC). (e.g.[55])

Table 2. List of experimental data and literature parameters for the lipid species studied (e.g.[55]).

N	$T_p(^{\circ}C)$	$T_m(^{\circ}C)$	$SR(^{\circ}C)$	$T_s^{max}(^{\circ}C)$	$D^{chain}(nm)$	$D_w^{max}(nm)$	$D_{w,g}(nm)$	A_h/A
16	35-37	41-41.2	(35-40) ± 1	---	3.2 ± 0.2	3.2 ± 0.2	2.2 ± 0.1	80 ± 15
17	43	48-49.8	(40-45) ± 1	41.5 ± 0.5	3.4 ± 0.2	4.8 ± 0.2	1.4 ± 0.1	23 ± 3
18	50-52	54.6-55	(45-55) ± 1	51.5 ± 0.5	3.6 ± 0.2	6.0 ± 0.4	1.8 ± 0.1	41 ± 7

T_p is the temperature of transition from the gel to the ripple phase (e.g.[56]), T_m is the temperature of main transition (e.g.[56]), $SR(^{\circ}C)$ = Swelling Region are listed the regions of temperature where swelling was observed, T_s is temperature at which D_w reaches its maximum value D_w^{max} and $D_{w,g}$ is the value of D_w in Gel phase. D_{chain} is the chain region thickness, for the floating bilayer, in the gel phase (25°C).

Analytical solutions are not available for this potential, against computer simulations are easier and give the best results in the physical mechanism. In this context we consider the case $z \ll \delta$, the two terms of the development of $U_{VDW}(z)$ will be neglected in the relationship (3) can be written as, with $A' = A/12\pi$

$$V(z) = A_h \exp(-2z/z_0) - \frac{A'}{z^2}, \tag{6}$$

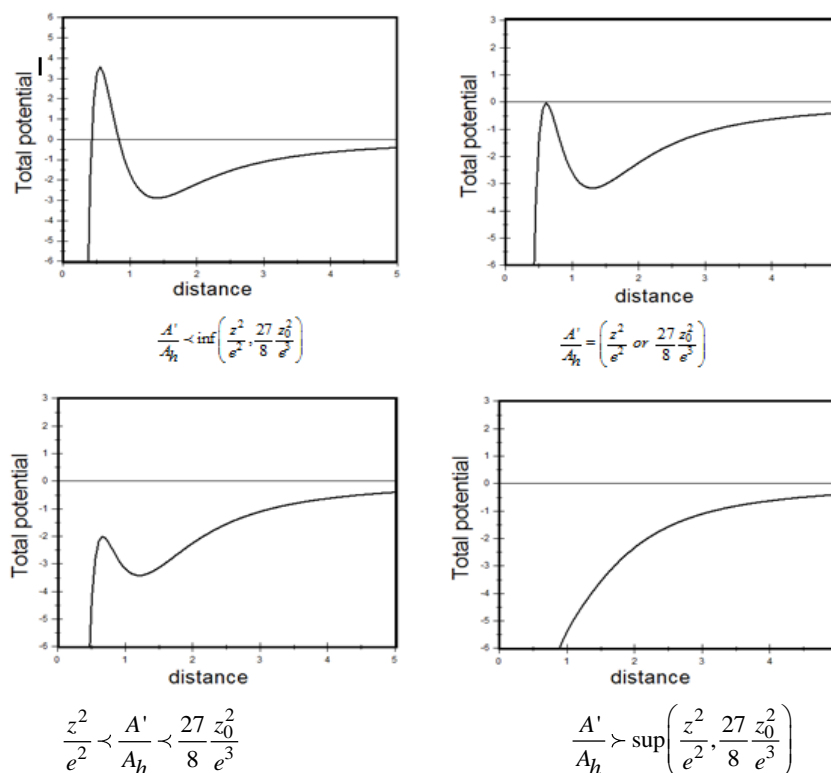


Fig 2 : The total potential as a function of distance for different cases

We first start by searching the zeros of this potential $V(z)$, that is to say the values of z for which $V(z) = 0$, a simple calculation then gives,

$$z^2 e^{-2z/z_0} = \frac{A'}{A_h} \tag{7}$$

along with $X = z/z_0$ we find

$$X^2 e^{-2X} = \frac{A'}{z_0^2 A_h} \tag{8}$$

To find the solutions of this equation, we plot the first member of the equation $X^2 e^{-2X}$ admitting point $X = 1$ maximum value, and then we cut the tangent $A'/A_h z_0^2$ to determine the conditions of zero total potential studied. When the following condition is satisfied $A_h/A' = (e/z_0)^2$ we have only one zero, and for $A_h/A' < 1/(e/z_0)^2$ we have no zero if not we have two zeros. To receive information on the concavity of the total potential, we minimize $V(z)$ with respect to the distance from the membrane to the substrate, we find explicitly

$$X^3 e^{-2X} = \frac{A'}{A_h} \frac{1}{z_0^2} \tag{9}$$

in Fig. 3 illustrates the shapes of functions $X^3 e^{-2X}$ and $A/12\pi A_h z_0^2$. According to the rule of the above-mentioned tangent, when the condition $A_h/A' = (1/z_0^2)(2e/3)^3$ is performed, one obtains a single extremum, and when $A_h/A' > (1/z_0^2)(2e/3)^3$ we have no extremum, and otherwise we have two extreme values.

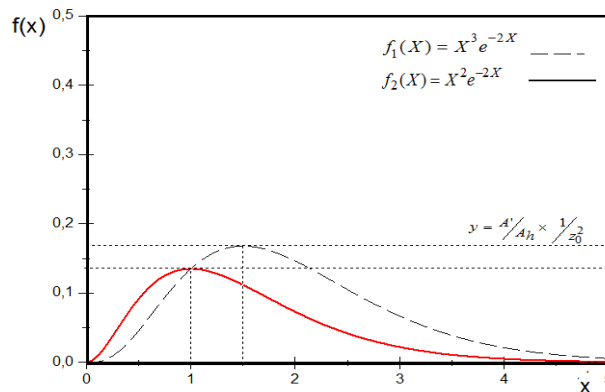


Fig 3 : The curves of the functions of the two membranes of the equation $U(z) = 0$ and $U'(z) = 0$.

In the Fig.3 we set the value of A , and z_0 is about 0.6 nm, the critical pressure, the total potential is dominated by the repulsion of hydration, because water molecules inserted between the heads hydrophilic lipid. Gradually, as the hydration P_h pressure (that is to say A_h) decreases, the gate potential decreases and the depth increases, that is to say, the membrane becomes more stable. This behavior is interpreted by a screening of repulsive interactions by the strong presence of attractive interactions.

b. The height of barrier

The height of the barrier is represented by the maximum of the curve of the total potential $V(z)$, indicates the physical barrier which prevents the stability of the membrane. To get an idea on this barrier if $z_0 > z_{max}$, we replace in the expression of the total potential 4, z by its value z_{max} corresponding to the maximum potential. We then find

$$H = U(z_{max}) = \frac{A'}{z_{max}^2} \left(\frac{z_0}{z_{max}} - 1 \right) \tag{10}$$

c. The depth of stability

The depth of the potential h is the minimum of the mean energy where the membrane is more stable. To derive the depth of the total potential in the case where $z_0 < z_{min}$, is replaced in the same way as before z_{min} in the relationship (6), and there are

$$h = -U(z_{min}) = -\frac{A'}{z_{min}^2} \left(\frac{z_0}{z_{min}} - 1 \right) \tag{11}$$

The potential shown in Fig. 2(a) has two minima: the first, a very short distance between the membrane and the substrate, is a microscopic barrier; the second, at infinite distance between the two interfaces, refers to the stability of the membrane. The relative position of these two minimum free energy changes according to the pressure of hydration: high pressure, the first minimum is an absolute minimum, while at low pressure, the second minimum is the minimum absolu.

C. EFFECT OF TEMPERATURE

When trying to investigate the effect of the temperature, one must take into account the thermal fluctuations of the membrane. The renormalization theory then shows that we must replace the actual membrane by a smoothed effective membrane whose characteristic scale dependent ζ observation (e.g.[67]).

Under the effect of thermal agitation, the membrane fluctuates around its equilibrium position. We will limit ourselves here to the case of a neutral membranes fluctuating near a single substrate (or another single membrane). Under these conditions, it is proposed to investigate the bilayer-bilayer interaction potential, which is roughly modeled by an effective potential $U_{eff}(z)$, which is formed by two contributions.

With
$$U_{eff}(z) = U_{att}(z) + U_{rep}(z), \tag{13}$$

$$U_{att}(z) = -\frac{A}{12\pi z^2}, \tag{13-a}$$

$U_{att}(z)$ is the bare attractive potential energy, approximated by its z^{-2} term, dominant at short distances. The second term is the entropic repulsion, approximated in the spirit of Helfrich, but where the hard wall is apparently shifted to the position $z = z_0$ to account for hydration repulsion. We write

$$\begin{cases} U_{rep}(z) = C_H \frac{(k_B T)^2}{\kappa} \frac{1}{(z - z_0)^2} & z > z_0, \\ U_{rep}(z) = \infty & z \leq z_0. \end{cases} \tag{13-b}$$

Where A is the Hamaker constant. For lipid membranes, κ is the bending rigidify which is typically of the order of $10-50 k_B T$, for phospholipid membranes. C_H is a constant, which is according

$$\beta = \frac{(k_B T)^2}{A\kappa}, \tag{14}$$

The precise value of coefficient C_H remains an open debate. Indeed, Helfrich gave for this coefficient value $C_H \approx 0,23$ (e.g.[9]), but computer simulations predict smaller prefactors, namely, $C_H \approx 0,16$ (e.g.[57]), $C_H \approx 0,1$ (e.g.[58]), $C_H \approx 0,07$ (e.g.[59]), $C_H \approx 0,08$ (e.g.[60]). In what follows, we will ask

$$\beta_{cri}^{-1} = 12\pi C_H, \tag{15}$$

$$C = \frac{A}{12\pi}, \tag{16}$$

With β is a dimensionless parameter that controls the movements and position of the membrane, β_{cri} is the critical value:

- If β is small, the membrane varies slowly in the minimum of microscopic potential;
- When β increases, the amplitude of the fluctuations increases and the membrane get blocked by the substrate;
- When β reaches a critical value $\beta_{cri} \approx 0.2 - 0.3$, the membrane detaches completely from the substrate. Under these conditions, the total potential can be written

$$\frac{U_{eff}(z)}{C} = -\frac{1}{z^2} + \frac{\beta}{\beta_{cri}} \frac{1}{(z - z_0)^2} \tag{17}$$

The next step is to study the mathematical structure of this potential for van der Waals-Helfrich. We first start by searching the zeros of this potential, that is, the values of z for which $U_{eff}(z^*) = 0$. A simple calculation leads to a single zero acceptable

$$z^* = \frac{z_0}{1 - (\beta/\beta_{cri})^{1/2}} > z_0. \tag{18}$$

Should $\beta < \beta_{cri}$, otherwise there is no zero. Now we are looking for the extreme points of this potential, z_m solution of the equation $U'_{eff}(z_m) = 0$. The calculation gives a single extreme

$$z_{\min} = \frac{z_0}{1 - (\beta/\beta_{\text{cri}})^{1/3}} > z_0, \tag{19}$$

with $\beta < \beta_{\text{cri}}$. However, if this condition is reversed, that is to say $\beta > \beta_{\text{cri}}$, the potential is a monotonically decreasing function (not zero). A simple calculation of the second derivative of the potential shows that z_m , if any, is minimal. Let us go now to the situation where the interaction potential has a minimum.

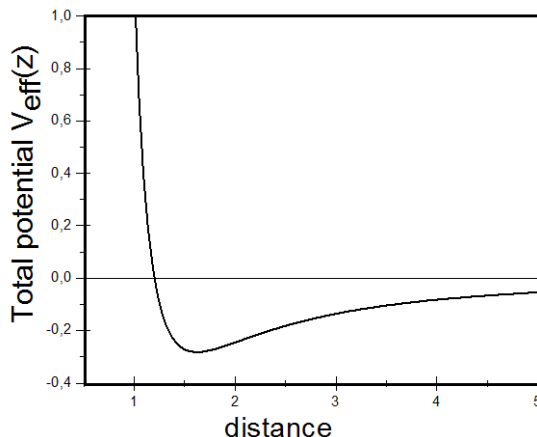


Fig.4 the total free energy of the membrane by an effective potential

D. STATISTICAL PROPERTIES

Let us go now to the situation where the interaction potential has a minimum. Well-understood, the membrane is confined in the region of distances around the minimum, or the potential can be approximated by (Taylor expansion to order two)

$$W(z) \approx W_0 + \frac{E_0}{2}(z - z_{\min})^2 \tag{20}$$

For information on the concavity of the potential, we can calculate the second derivative of the effective potential $U''_{\text{eff}}(z_m)$. We easily show that

$$E_0 = U''_{\text{eff}}(z_{\min}) = 6C(\beta_{\text{cri}}/\beta)^{1/3} z_0^4 [1 - (\beta/\beta_{\text{cri}})^{1/3}]^5. \tag{21}$$

A deep analysis of this expression, shows that $U''_{\text{eff}}(z_m)$ has the same sign with $[1 - (\beta/\beta_{\text{cri}})^{1/3}]^5$. To have a concavity

which corresponds to the potential stability of the membrane, the sign should be positive, that is to say $\beta < \beta_{\text{cri}}$. Table. Values of the second derivative of the potential interaction of the bending rigidity of the membrane, and the membrane-floating distance grafted membrane. (e.g.[55], [61])

Temperature (°C)	$E_0 = U''$ ($10^x J.m^{-4}$)	κ ($k_B T$)	d_{H_2O} (nm)
42.9 ± 0.1	12.55 ± 0.1	350 ± 20	42.9 ± 0.1
47.9 ± 0.1	12.3 ± 0.1	50 ± 5	42.9 ± 0.1
51.5 ± 0.1	12.2 ± 0.1	30 ± 5	42.9 ± 0.1
52.9 ± 0.1	12 ± 0.1	20 ± 5	42.9 ± 0.1

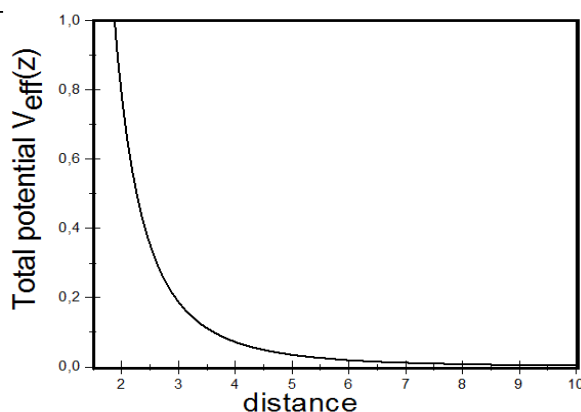


Fig.4 the total free energy of the membrane by an effective potential (repulsive case) .

Using this approximated potential, we can calculate everything, especially the mean position of the membrane, $D = \langle u \rangle$, the spreader and the amplitude of fluctuation, $\sigma^2 = \langle (u-D)^2 \rangle$. We easily show that

$$D = z_{\min} = \frac{z_0}{1 - (\beta/\beta_{crit})^{1/3}} = z_0 \frac{\beta_{crit}^{1/3}}{\beta_{crit}^{1/3} - \beta^{1/3}} > z_0 \quad , \beta < \beta_{crit} \quad , \quad (22)$$

Of which, the average position of the diaphragm coincides with the position of minimum potential. Note that $\langle u - D \rangle = 0$, because the Hamiltonian, defined below, is a quadratic function of the variable $u - D$. Expressed as the unit zero z_0 , the average position of the membrane depends only on the amplitude ratio β/β_{crit} , the repulsive and attractive parts. Now, let's calculate the height function-height correlation

$$G(\rho - \rho') = \langle [u(\rho) - D][u(\rho') - D] \rangle \quad (23)$$

with the notations: $\rho = (x, y) \in R^2$ and $\rho' = (x', y') \in R^2$. Here, the average value of a quantity, X , configuration dependent u , is defined by

$$\langle X \rangle = \frac{1}{Z_m} \int D_u X(u) \exp\left\{-\frac{H_0[u]}{k_B T}\right\}, \quad (24)$$

with the new Hamiltonian

$$H_0[u] = \iint dx dy \left[\frac{\kappa}{2} (\Delta u)^2 + \frac{\gamma}{2} (\nabla u)^2 + E_0(u-D) \right] (x, y) \quad (25)$$

and the partition function of the membrane

$$Z_m = \int D_u \exp\left\{-\frac{H_0[u]}{k_B T}\right\} \quad (26)$$

In Fourier space, this propagator is written in the following integral form. (e.g.[62], [63]).

$$G(\rho) = \int \frac{d^2 \mathbf{q}}{(2\pi)^2} \frac{e^{i\mathbf{q}\cdot\rho}}{\hat{\kappa}q^4 + \hat{\gamma}q^2 + \hat{E}_0} \quad (27)$$

with elastic constants dimensionless : $\hat{\kappa} = \kappa/k_B T$, $\hat{\gamma} = \gamma/k_B T$, $\hat{E}_0 = E_0/k_B T$. Explicitly, the previous expression becomes

$$G(\rho) = \int_0^\infty \frac{dq}{2\pi} \frac{qJ_0(q\rho)}{\hat{\kappa}q^4 + \hat{\gamma}q^2 + \hat{E}_0} = \int_0^\infty \frac{dq}{2\pi} \frac{qJ_0(q\rho)}{E(q)} \quad (28)$$

Where J_0 is the Bessel function. To calculate this integral, we adopt the following decomposition

$$\frac{q}{\hat{\kappa}q^4 + \hat{\gamma}q^2 + \hat{E}_0} = \frac{q}{\sqrt{\hat{\gamma}^2 - 4\hat{E}_0\hat{\kappa}}} \left\{ \left(q^2 + \frac{\hat{\gamma} - \sqrt{\hat{\gamma}^2 - 4\hat{E}_0\hat{\kappa}}}{2\hat{\kappa}} \right)^{-1} - \left(q^2 + \frac{\hat{\gamma} + \sqrt{\hat{\gamma}^2 - 4\hat{E}_0\hat{\kappa}}}{2\hat{\kappa}} \right)^{-1} \right\}, \quad (29)$$

Using standard mathematical formula

$$\int_0^\infty dq \frac{qJ_0(q\rho)}{q^2 + a^2} = K_0(\rho a) \quad (30)$$

We find the desired expression of the propagator

$$G(\rho) = \frac{K_0(\rho\xi_-) - K_0(\rho\xi_+)}{2\pi\sqrt{\hat{\gamma}^2 - 4\hat{E}_0\hat{\kappa}}}, \quad (31)$$

$K_0(z)$ is the modified Bessel function. We ratings

$$\xi_-^2 = \frac{\hat{\gamma} - \sqrt{\hat{\gamma}^2 - 4\hat{E}_0\hat{\kappa}}}{2\hat{\kappa}}, \quad \xi_+^2 = \frac{\hat{\gamma} + \sqrt{\hat{\gamma}^2 - 4\hat{E}_0\hat{\kappa}}}{2\hat{\kappa}} \quad (32)$$

For membranes without voltage, ie $\gamma = 0$, becomes the previous propagator

$$G(\rho) = \frac{K_0(-\sqrt{i}(\hat{E}_0/\hat{\kappa})^{1/4}\rho) - K_0(\sqrt{i}(\hat{E}_0/\hat{\kappa})^{1/4}\rho)}{4\pi i\sqrt{\hat{E}_0\hat{\kappa}}} \quad (33)$$

It is possible to rewrite this expression in terms of the Kelvin function (Thomson function), $kei(z)$, defined by

$$K_0(z\sqrt{i}) - K_0(-z\sqrt{-i}) = 2i \times Kei(z). \quad (34)$$

Using these considerations, the propagator becomes

$$(35)$$

$$G(\rho) = -\frac{1}{2\pi\kappa} (\hat{E}_0/\kappa)^{1/2} Kei(\hat{E}_0/\kappa)^{1/4} \rho).$$

With the in-

plane correlation length is $\xi_{\square} = (\hat{E}_0/4\kappa)^{-1/4}$ and $\beta = (k_B T)^{-1}$. The mean squared width of the fluctuations is

$$\xi_{\perp}^2 = G(\rho = 0) = \frac{1}{4\xi_{\square}^2} \frac{1}{\beta E_0} = \frac{1}{16\beta\kappa} \xi_{\square}^2 \quad (36)$$

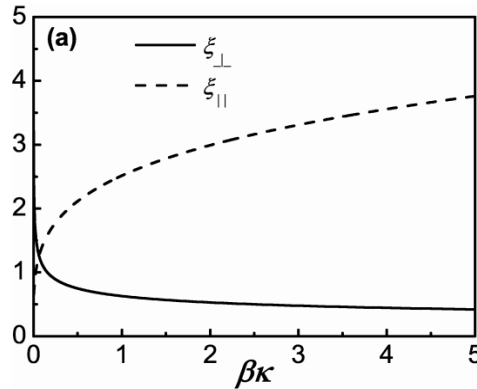


Fig 5 a: Correlation length as a function of the rigidity of the membrane κ for given value of E_0 . (e.g.[64])

Figure (5 a) shows the behavior of the correlation lengths, ξ_{\perp} and ξ_{\square} , when the bending κ changes keeping the value of μ constant. In the limiting case where $\kappa \rightarrow 0$ (where thermal fluctuations easily modify the shape of the membrane), also $\xi_{\square} \rightarrow 0$, but if $\kappa \rightarrow \infty$ (flat wall) then $\xi_{\square} \rightarrow \infty$; (e.g.[64]) and ξ_{\perp} behaves inversely at both limits. We then define the mean roughness of the membrane (Figure b)

$$\xi_{\perp}^2 = G(\rho = 0) = \frac{1}{8} \frac{1}{\beta(\kappa E_0)^{1/2}} \quad (37-a)$$

or

$$\xi_{\perp} = G^{1/2}(0) = 2^{-3/2} (\kappa / \hat{E}_0)^{1/4} \quad (37-b)$$

that the result (35) was recently derived in Ref. . (e.g.[62], [65]) . Bending undulations of wavelength ξ_{\square} are characterized by the roughness

$$\xi_{\perp} \square (T/\kappa)^{1/2} \xi_{\square}, \quad (38)$$

with temperature T and bending rigidity κ (e.g.[66]) .The bending rigidity has the dimensions of an energy. For phospholipid bilayers in their fluid state, it typically varies between $0.2 \times 10^{-19} J$ and $4 \times 10^{-19} J$ which implies $0.01 \leq T/\kappa \leq 0.2$ at room temperature $T \square 0.04 \times 10^{-19} J$ (e.g.[9], [10]) .

Thus ,each segment forms a hump of longitudinal and perpendicular extension ξ_{\perp} and ξ_{\square} , respectively, and thus of volume $v = \xi_{\square} \xi_{\perp}$. Using the ideal gas law $Pv = T$ for single degree of the freedom and the relation (36), one then arrives at the pressure (e.g.[57], [71])

$$P \square T/\kappa \xi_{\perp}^3 \quad (39)$$

Thus, the bending undulations lead to disjoining pressure P which decays slowly

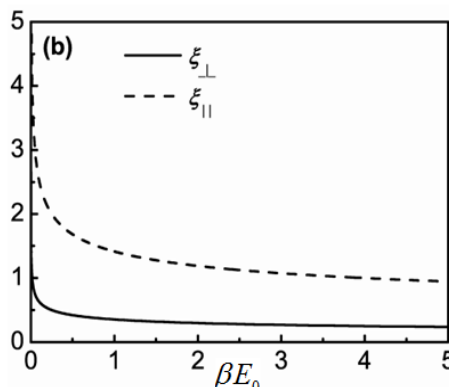


Figure 5 b : Correlation length as a function of the mass of the membrane E_0 for given value of κ (e.g.[64]) .

Figure (5b) shows the variation in both correlation lengths for the constant κ by changing the membrane mass. On the other hand, ξ_{\perp} and ξ_{\parallel} make it possible to scale all the observable quantities.

In case $\gamma \neq 0$, using the definition (24) of the correlation function with the parallel correlation length ξ_{\parallel} defined by $G(\rho \rightarrow \infty) = \exp(-\rho/\xi_{\parallel})$ and the mean squared width $\sigma^2 = G(\rho = 0)$, one obtains the exact relation is then obtained the exact relation (e.g.[64])

$$\sigma^2 = G(\rho = 0) = \frac{1}{2\pi\sqrt{\hat{\gamma}^2 - 4\hat{E}_0\hat{\kappa}}} \begin{cases} \text{Arc tan} \frac{1}{\hat{\gamma}} \sqrt{4\hat{E}_0\hat{\kappa} - \hat{\gamma}^2} & \hat{\gamma}^2 < 4\hat{E}_0\hat{\kappa} \\ \frac{1}{2} \log \frac{\hat{\gamma} + \sqrt{\hat{\gamma}^2 - 4\hat{E}_0\hat{\kappa}}}{\hat{\gamma} - \sqrt{\hat{\gamma}^2 - 4\hat{E}_0\hat{\kappa}}} & \hat{\gamma}^2 > 4\hat{E}_0\hat{\kappa} \end{cases} \quad (40)$$

with the limiting expressions

$$\sigma^2 \simeq \begin{cases} \frac{K_B T}{8\sqrt{\kappa E_0}} & \gamma^2 \ll 4\kappa E_0 \\ \frac{K_B T}{2\pi \gamma \sqrt{1 - \frac{4\kappa E_0}{\gamma^2}}} \log \frac{\gamma}{\sqrt{\kappa E_0}} & \gamma^2 \gg 4\kappa E_0 \end{cases} \quad (41)$$

The relation between the perpendicular and the parallel correlations length may be approximated as follows [72]

$$\xi_{\perp} = \frac{\kappa}{\gamma} \left(\exp\left(\frac{4\pi\gamma}{k_B T} \sigma^2\right) - 1 \right) \rightarrow \begin{cases} \frac{4\pi\kappa}{k_B T} \sigma^2 = \frac{\pi}{2} \sqrt{\frac{\kappa}{E_0}} & \gamma^2 \ll 4\kappa E_0 \\ \frac{\kappa}{\gamma} \exp\left(\frac{4\pi\gamma}{k_B T} \sigma^2\right) = \frac{\gamma}{E_0} & \gamma^2 \gg 4\kappa E_0 \end{cases} \quad (42)$$

yielding the two exact limiting cases for vanishing surface tension γ and bending rigidity κ , respectively. Note that in the former the factor 4π is often replaced by 16, a difference which is not relevant.

III. CONCLUSIONS

We recall that the aim of this work was to study the statistical properties of a fluid membrane in contact with a plane wall (quasi-planar membrane). The latter develops an external potential to a body, which is felt by the membrane. To conduct the calculations of these properties, we started an extensive Cahn-Helfrich Hamiltonian which takes into account the presence of the external potential. This same potential is the sum of two contributions, a repulsive potential and an attractive potential type of van der Waals forces. We first studied all the analytical properties of the considered potential (zeros extrema variations). We found that there are two situations, depending on the relative values of the amplitudes of the repulsive and attractive parts: A situation where the full potential is a decreasing function of the distance (dominant repulsive forces), and another where the potential has a minimum (dominant attractive forces). Then, this potential is developed around its minimum (when available). This who gave birth to a new Hamiltonian which is quadratic in the deviation of the height according to the average position of the membrane. Using this new Hamiltonian, we were able to calculate First the propagator, hence, we extracted the amplitude of fluctuations.

Finally, note that its same computational techniques could be extended to even or other interaction potential, and the case of two adjacent membranes floating a third fixed.

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