Volume 4, Issue 11, November 2014 ISSN: 2277 128X
International Journal of Advanced Research in Computer Science and Software Engineering
Research Paper
Available online at: www.ijarcsse.com

Humanoid Push Recovery: An Analytical Solution

Leonardo Lanari
DIAG Universitàdi Roma La Sapienza,
Italy

Abstract— ***Weextend the concept of capture point for humanoidsto the presence of persistent known disturbances, such as constant or sinusoidal forces. An analytical general solution is provided for the Linear Inverted Pendulum given any known input function, allowing not only the removal of some previous difficulties in the use of the capture point, but also its extension to more general situations.***

Keywords— ***Robot, humanoids, control, gait generation, push recovery***

I. INTRODUCTION

Some of the recent research on humanoids investigates the capabilities of recovering from external disturbances such as a sudden push; this problem is known as "Push Recovery" and different strategies have been proposed. The seminal paper of Pratt [1] introduced the capture point as the point on the ground where the humanoid had to step in order to come to a complete stop after the push has vanished. The analysis carried on the Linear Inverted Pendulum (LIPM) was enriched with the use of an ankle torque or a flywheel to maintain balance before a step was needed. A complementary result defining decision surfaces was presented in [2]. An interesting extension was presented in [3] with the introduction of the Foot Point Estimation where a nonlinear model of a rimless wheel with only two spokes was used instead of the LIPM allowing the inclusion of the energy loss due to the ground impact. We refer to the bibliography of [4] for a complete overview.

One of the difficulties, even with the LIPM, has been to single out the non-diverging solution of the pendulum dynamics given a generic input. In most cases the input has either been considered piecewise constant or periodical. In this paper we give the analytical solution of this particular bounded trajectory for a given input and therefore allowto extend the concept of capture point in the presence of known disturbances acting on the system.

After a quick review of the LIPM model the concept of Capture Point is recalled and explained in detail. The general result is presented in Section IV and a possible interesting applicationis discussed in Section V for illustration purposes: recovery under the effect of a persistent sinusoidal push.

II. THE LINEAR INVERTED PENDULUM

Let us consider the basic dynamic equations of the Linear Inverted Pendulum LIPM [5], [4] in the sagittal plane

$$\ddot{x}_c = \omega_o^2 x_c - \omega_o^2 u + \frac{F}{m} \qquad (1)$$

with x_c the centre of mass (CoM) x-position of the concentrated m, $\omega_o = \sqrt{g/z_o}$ the pendulum angular frequency, z_o the constant height of the centre of mass, u a control input and F a disturbance force acting in the x-direction as illustrated in Fig. 1. How this LIPM model applies to a real humanoid is shown in Fig. 2.

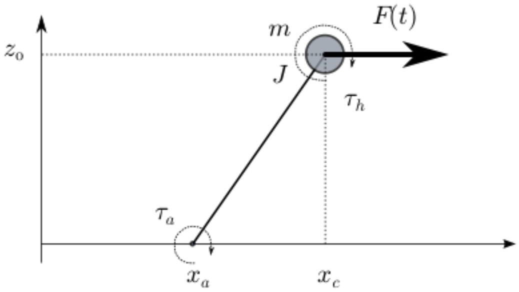

Fig. 1 - The Linear Inverted Pendulum (LIPM)

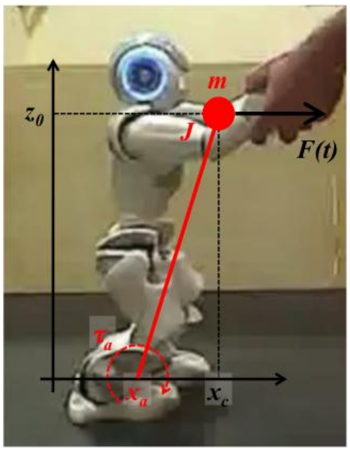

Fig. 2 - An example of the LIPM concept

Following the clear formulation of [4], the linear dynamic equation (1) can represent three simple basic gait models as summarized in TABLE I.

Table I. Three Simple Gait Models

Type	**Input** u
Point-foot	point-foot location x_a
Finite-sized foot	$x_{cop} = x_a - \frac{\tau_a}{mg}$
Reaction mass	$x_{cmp} = x_a - \frac{\tau_a - \tau_h}{mg}$

with x_{cop} the centre of pressure, x_{cmp} the centroidal moment pivot of pressure, τ_a an ankle torque and τ_h a hip torque acting on a reaction mass. For flat foot contact on a horizontal surface the centre of pressure coincides with the Zero Moment Point (ZMP) [6] i.e. $x_{cop} = x_{zmp}$. In this context, the CoP is used as a control input.

III. THE CAPTURE POINT

Several points on the ground such as the ZMP, CMP or FRI (Foot Rotation Indicator) have been identified as of particular interest in the control of legged robots [7]. Later the Capture Point (CP) was presented in [1] as a basic tool for analyzing and controlling push recovery.

The Capture Point concept was introduced in a pure free response (i.e. zero input) context: the basic definition was "*the point on the ground where the humanoid had to step in order to come to a complete stop*" after an impulsive push.

The scenario considers the humanoid, right after an impulsive push, in the state $(x_c(0), \dot{x}_c(0))$. To have a converging unforced response, this initial state should belong to the stable manifold Π_s generated by $-\omega_o$. We have then the following Capture Point condition

$$\begin{pmatrix} x_c(0) \\ \dot{x}_c(0) \end{pmatrix} \in \Pi_s = gen\left\{ \begin{pmatrix} 1 \\ -\omega_o \end{pmatrix} \right\} \quad (2)$$

Assuming instantaneous displacement of the swinging leg and no change in the initial velocity (no loss of energy due to impact), the new configuration should then be $(-\dot{x}_c(0)/\omega_o, \dot{x}_c(0)) \in \Pi_s$ which is equivalent, as shown in Fig. 3 for a positive initial velocity, to stepping in the new location

$$x_c(0) + \frac{\dot{x}_c(0)}{\omega_o} = x_{cp} \quad (3)$$

which is usually defined as the Capture Point x_{cp}. After this particular instantaneous step, the pendulum would naturally in an unforced and stable way regain its upright position in the new foot location x_{cp}.

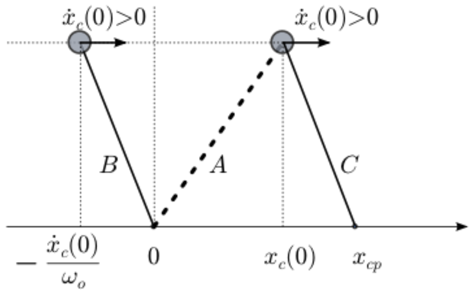

Fig. 3 – A graphical explanation of the capture point

We have the following interpretations of the Capture Point condition (2).

- Fixed initial velocity - Make a step in x_{cp} (ideal LIPM case) maintaining the original velocity $\dot{x}_c(0)$.
- Fixed initial position - Change instantaneously the velocity to $-\omega_o x_c(0)$, this would let the pendulum go back to the vertical position in x_c.
- Find the appropriate control input so that, at a desired instant t^* the CoM will find itself satisfying the Capture Point condition (2). A similar controllability approach has been used in [8] for the ZMP.

When the input, as an ankle or flywheel torque, is taken into account explicitly, it is usually constrained to be *piecewise-constant* as in [1] or in [4] with the*Equivalent Constant CoP* idea. Once the input structure has been decided, it is possible to compute where the final state will be at the end of the input duration $t = t_f$ i.e. when the free response will start. This solution is parametrizedw.r.t. the initial conditions at $t = 0$, therefore forcing this final state to be on the stable manifold allows to back propagate this condition to the initial state in $t = 0$.

Itis still a zero-input concept, as it should be, since the disturbance or the input is considered to be time-limited.

A slightly different point of view is adopted in [2] where the balancing conditions under ankle and/or flywheel torque restrictions are studied. For example, a saturated torque can be translated in a limitation on the support region through the CoP or equivalently into a finite-sized foot. Therefore we have balancing if the CP stays within this region. In other terms the support region for a finite-sized foot can be seen as a capture region for an equivalent LIPM with point foot. The same applies when a flywheel is added.

IV. MAIN RESULT

We first illustrate different state-space representations of the LIPM system and convert the capture point condition. This will allow us to state and solve our problem.

A. LIPM equivalent state-space representations

The state-space representation in the CoM position and velocity $(x_c, \dot{x}_c)$ coordinates is

$$S_c : \begin{pmatrix} x_c \\ \dot{x}_c \end{pmatrix} \rightarrow A_c = \begin{pmatrix} 0 & 1 \\ \omega_o^2 & 0 \end{pmatrix}, B_c = \begin{pmatrix} 0 \\ -\omega_o^2 \end{pmatrix} \tag{4}$$

The same system can however be looked at choosing the capture point as one of the coordinates

$$\begin{pmatrix} x_c \\ x_{cp} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1/\omega_o \end{pmatrix} \begin{pmatrix} x_c \\ \dot{x}_c \end{pmatrix} \tag{5}$$

leading to

$$S_{cp} : \begin{pmatrix} x_c \\ x_{cp} \end{pmatrix} \rightarrow A_{cp} = \begin{pmatrix} -\omega_o & \omega_o \\ 0 & \omega_o \end{pmatrix}, B_{cp} = \begin{pmatrix} 0 \\ -\omega_o \end{pmatrix} \tag{6}$$

The same change of coordinates has been used in [9] in the finite-sized foot case i.e. with input $u = x_{zmp}$ .Another useful representation is along the stable and unstable eigenvectors also called in [10] the "convergent and divergent components of motion"

$$\begin{pmatrix} x_s \\ x_u \end{pmatrix} = \begin{pmatrix} 1 & -1/\omega_o \\ 1 & 1/\omega_o \end{pmatrix} \begin{pmatrix} x_c \\ \dot{x}_c \end{pmatrix} \tag{7}$$

which results in

$$S_{su} : \begin{pmatrix} x_s \\ x_u \end{pmatrix} \rightarrow A_{su} = \begin{pmatrix} -\omega_o & 0 \\ 0 & \omega_o \end{pmatrix}, B_{su} = \begin{pmatrix} \omega_o \\ -\omega_o \end{pmatrix} \tag{8}$$

It has been noted in [4]that x_s corresponds to the "point reflection of the instantaneous capture point across the projection of the point mass onto the ground". Note that we have $x_s = 2x_c - x_{cp}$. In both S_{cp} and S_{su} the stable and unstable dynamics are clearly isolated, while the connection is different, series in S_{cp} and parallel in S_{su}.

B. Problem formulation

The key point in the introduction of the capture point concept lies in condition (2) which can be rewritten for the S_{su} coordinates as

$$\begin{pmatrix} x_c(0) \\ x_{cp}(0) \end{pmatrix} \in \Pi_s = gen \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\} \tag{9}$$

in other words the condition simply states that the unstable dynamics -- x_{cp} or equivalently x_u -- should not be excited by the initial conditions while the stable part is totally irrelevant. Let us consider the two sub-systems of S_{cp} i.e. the unstable dynamics of x_{cp}

$$S_{div} : \begin{cases} \dot{x}_{cp} = \omega_o x_{cp} - \omega_o u \\ y_1 = x_{cp} \end{cases} \tag{10}$$

and the stable one

$$S_{conv} : \begin{cases} \dot{x}_c = -\omega_o x_c + \omega_o x_{cp} \\ y_2 = x_c \end{cases} \tag{11}$$

We need to concentrate only on the unstable system S_{div}. The capture point problem reduces to that of finding the proper initial condition of S_{div} so that the unforced evolution remains bounded. In this trivial formulation the answer is obviously 0.

The extension we would like to introduce considers the same problem but for the total response, unforced plus forced terms, thus allowing also taking into account disturbances or the effect of control inputs in the generation of stable gaits for a humanoid.

Problem

For the unstable system S_{div}, find the proper initial conditions so that, given a known input $u(t)$, the total response remains bounded.

The term "bounded" here is referred to the given input, i.e. we want to avoid the divergence due to the unstable eigenvalue ω_o and obtain a behaviour similar to the concept of steady-state. If the input is for example a ramp we can admit a similar behaviour for the sought solution even if it is diverging.

We will use the early result of [11] - p. 1406 - or equivalently of [12].Although at first this may seem quite counterintuitive, just think about letting a coin roll on the cutting edge of an inclined knife: this is possible only for some specific initial conditions.

C. Analytical solution

We will state directly the result on the unstable dynamics (10) of S_{div} but obviously the result is general. The complete solution

$$x_{cp}(t) = e^{\omega_o t} x_{cp}(0) - \int_0^t e^{\omega_o (t-\tau)} u(\tau) d\tau \tag{12}$$

in general diverges, but if among all possible initial conditions we choose

$$x_{cp}^*(0) = \omega_o \int_0^\infty e^{-\omega_o \tau} u(\tau) d\tau \tag{13}$$

we obtain the particular solution

$$x_{cp}^*(t) = \omega_o \int_0^\infty e^{-\omega_o \tau} u(\tau + t) d\tau \tag{14}$$

which is bounded under mild boundedness conditions on the input $u(t)$. In a very broad sense $x_{cp}^*(t)$ is a form of "steady-state" solution associated to a given input $u(t)$.Obviously if $u(t) \equiv 0$ the resulting initial condition $x_{cp}^*(0)$ is also equal to 0 consistently with the Capture Point condition.

It is important to notice that the particular solution (14) is non-causal since the actual trajectory requires knowledge of future values of the input. Although it may sound odd it has already been noticed in [5] where an identical problem has been tackled for the stable generation of a CoM trajectory associated to a given desired ZMP one.

In this framework we can now turn to the stable dynamics S_{conv}, for which a true steady-state solution exists, driven by the input $x_{cp}^*(t)$. The particularity of this equation is that the input starts theoretically at $-\infty$. Therefore we can split the input into a past forcing term that brings the CoM in an initial state $x_{cp}^*(0)$ and then add the forced response to the present and future input. This can be written as

$$\begin{aligned} x_c^*(t) &= \lim_{t_0 \to -\infty} x_c(t) \\ &= e^{-\omega_o t} x_{cp}^*(0) + \omega_o \int_0^t e^{-\omega_o (t-\tau)} x_{cp}^*(\tau) d\tau \\ &= \omega_o \int_{-\infty}^0 e^{\omega_o \nu} x_{cp}^*(t+\nu) d\nu \\ &= \omega_o \int_0^\infty e^{-\omega_o \tau} x_{cp}^*(t-\tau) d\tau \end{aligned} \tag{15}$$

i.e. we choose the particular initial condition

$$x_c^*(0) = \omega_o \int_{-\infty}^0 e^{\omega_o \tau} x_{cp}^*(\tau) d\tau \tag{16}$$

to select the steady-state behaviour associated to the given input x_{cp}^*. Note that, with these particular choices, we have associated to a given input a unique state bounded trajectory. This is achieved choosing an input-dependent initial condition, both in the stable and unstable dynamics, so there is a one-to-one correspondence between the input and this particular trajectory. It is true, as clearly shown in both the unstable and stable special chosen trajectories, that we need to know the input for all future time. Usually this for a stable system is not considered a problem while for the unstable case it is sometimes pointed out, with no reason, as a drawback.

We can directly obtain the CoM corresponding evolution by summing both contributions and obtain

$$x_c^*(t) = \frac{\omega_o}{2} \int_0^\infty e^{-\omega_o \tau} [u(t+\tau) + u(t-\tau)] d\tau \tag{17}$$

V. APPLICATION EXAMPLE

Imagine the humanoid being pushed by a sinusoidal force

$$F(t) = \sin(\overline{\omega} t) \delta_{-1}(t-T) \tag{18}$$

For example, the effect of a sinusoidal boat rolling could be seen as a disturbance in the lateral (or even sagittal) plane. Again we may have two different approaches, one trying to contrast the induced movement (pure disturbance cancellation) or adapt to the rolling. The energy consumption differences resulting from these two alternatives are quite evident.

The ideal CP trajectory, using the ideal solution, is

$$x_{cp}^{*}(t)=\begin{cases}-\dfrac{\omega_o \sin\varpi T+\varpi\cos\varpi T}{m\omega_o(\omega_o^2+\varpi^2)}e^{\omega_o(t-T)} & if \quad t\le T\\ -\dfrac{\omega_o \sin\varpi t+\varpi\cos\varpi t}{m\omega_o(\omega_o^2+\varpi^2)} & if \quad t\ge T\end{cases} \tag{19}$$

The resulting CoM trajectory can be computed also as the total response from the initial condition

$$x_c^*(T)=-\frac{\omega_o \sin\varpi T+\varpi\cos\varpi T}{2m\omega_o(\omega_o^2+\varpi^2)} \tag{20}$$

given input x_{cp}^* for $t\ge T$. An example of the final time evolution is reported in Fig. 4. First note that, after T, the capture point is not constant but input-like as the steady-state behaviour of a linear system should be although the system is unstable. The oscillation amplitude of the centre of mass and the capture point depend upon both the input frequency ϖ and the pendulum angular frequency ω_o. Finally note that x_{cp}^* is not differentiable in T. This behaviour is however consistent since the discontinuity of the input F implies that of the velocity. Note however that both x_c^* and $\dot{x}_c^*$ are continuous.

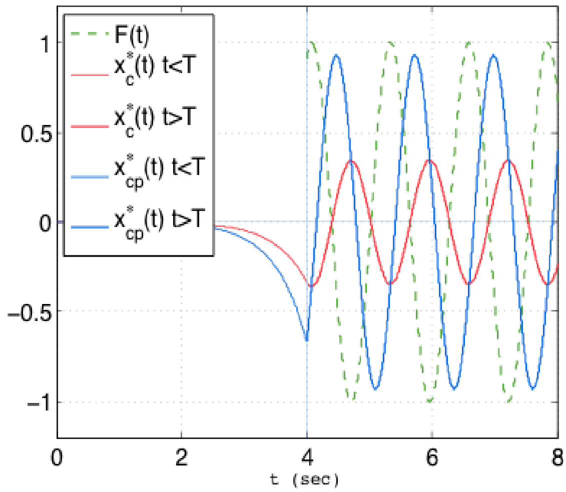

Fig. 4 – Sinusoidal disturbance: center of mass and capture point ideal trajectories

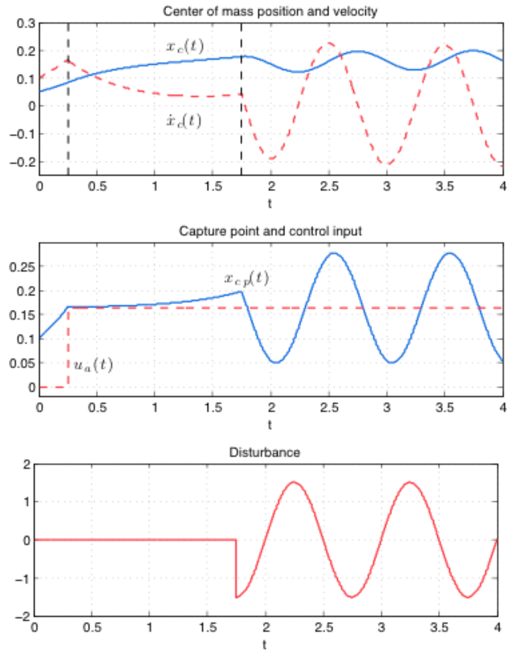

Fig. 5 – Sinusoidal disturbance: an anticipative step $u_a(t)$ counterbalances the future sinusoidal disturbance

An interesting situation is also reported in Fig. 5 where a proper amplitude anticipative step is applied in order to cope with the known future sinusoidal input that will be applied on the humanoid.

VI. Conclusions

In this paper we have explored the effect of disturbances on the simple but representative LIPM model of a humanoid. In particular we have shown that, if the disturbance is known a priori, we can select particular initial conditions in order to make the final state trajectories, when the disturbance is applied, non diverging. These closed-form solutions give an insight on how to use as much as possible the natural dynamics of the humanoid.

References

[1] J. Pratt, J. Carff, S. Drakunov, and A. Goswami, "Capture point: A step towardhumanoid push recovery," in *Humanoid Robots, 2006 6th IEEE-RAS International Conference on*, 2006, pp. 200-207.

[2] B. Stephens, "Humanoid push recovery," in *Humanoid Robots, 2007 7th IEEE-RAS International Conference on*, 2007, pp. 589-595.

[3] D. Wight, E. Kubica, and D. Wand, "Introduction of the foot placement estimator: A dynamic measure of balance for bipedal robotics,"*Journal of Computational and Nonlinear Dynamics*, vol. 3, pp. 1-9,2007.

[4] T. Koolen, T. de Boer, J. Rebula, A. Goswami, and J. Pratt,"Capturability-based analysis and control of legged locomotion, part 1:Theory and application to three simple gait models,"*The International Journal of Robotics Research*, vol. 31, no. 9, pp. 1094-1113, 2012.

[5] S. Kajita, F. Kanehiro, K. Kaneko, K. Fujiwara, K. Harada, K. Yokoi, andH. Hirukawa, "Biped walking pattern generation by using preview control ofzero-moment point," in *Robotics and Automation, 2003. Proceedings ICRA '03. IEEE International Conference on*, vol. 2, 2003, pp. 1620-1626.

[6] M. Vukobratovic and D. Juricic, "Contribution to the synthesis of biped gait," *Biomedical Engineering, IEEE Transactions on*, vol. BME-16, no.1, pp. 1-6, 1969.

[7] M. B. Popovic, A. Goswami, and H. Herr, "Ground reference points in legged locomotion: Definitions, biological trajectories and control implications," *The International Journal of Robotics Research*, vol.~24, no.~12, pp. 1013-1032, 2005.

[8] C. Santacruz and Y. Nakamura, "Walking motion generation of humanoid robots: Connection of orbital energy trajectories via minimal energy control," in*Humanoid Robots (Humanoids), 2011 11th IEEE-RAS International Conference on*, 2011, pp. 695--700.

[9] J. Englsberger, C. Ott, M. Roa, A. Albu-Schaffer, and G. Hirzinger, "Bipedal walking control based on capture point dynamics," in *Intelligent Robots and Systems (IROS), 2011 IEEE/RSJ International Conference on*, 2011, pp. 4420-4427.

[10] T. Takenaka, T. Matsumoto, and T. Yoshiike, "Real time motion generation and control for biped robot -1st report: Walking gait pattern generation," in *Intelligent Robots and Systems, 2009. IROS 2009. IEEE/RSJ International Conference on*, 2009, pp. 1084-1091.

[11] L. Lanari and J. Wen, "Feedforward calculation in tracking control of flexible robots," in *Decision and Control, 1991, Proceedings of the 30th IEEE Conference on,* 1991, pp. 1403-1408 vol.2.

[12] S. Devasia and B. Paden, "Exact output tracking for nonlinear time-varying systems," in *Decision and Control, 1994, Proceedings of the 33rd IEEE Conference on*, vol.3, 1994, pp. 2346-2355 vol.3.

