

**A Study on Similarity Measure for Fuzzy Sets**

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Abstract— Similarity measure for Fuzzy sets is one of the researched topics of Fuzzy set theory. The literature is vast and growing. The main aim of this article is to revisit the definitions of Similarity measure which are predominant in the literature of Fuzzy set theory. We wish to analyze these existing definitions with the extended definition of complementation of Fuzzy sets on the basis of reference function. This paper summarizes some of the developments in this research area of Fuzzy set theory. Here efforts have been made to show that this kind of representation is not logical from the standpoints of new definition of complementation of Fuzzy sets. Our purpose is to provide a synthesis of some published researches in this area, analyze them on the basis of new definition of complementation and stimulate further research interests and efforts in this identified topic.

Keywords— Complement of a Fuzzy set, Fuzzy membership function, Fuzzy membership value, Fuzzy reference function, Fuzzy set, Similarity measure.

I. INTRODUCTION

Similarity measure for Fuzzy sets is one of the active research and application areas of Fuzzy set theory. The literature is vast and growing. This paper focuses on some of the important developments in this area of Fuzzy set theory. Since Zadeh [1] introduced Fuzzy sets in 1965, many approaches and theories treating imprecision and uncertainty have been proposed. Some of these theories, such as intuitionistic Fuzzy sets (IFS), interval-valued Fuzzy sets (IVFS), and interval-valued intuitionistic Fuzzy sets (IVIFS), are extensions of Fuzzy set theory introduced by Zadeh . Many contributions on the measure of Similarity between two Fuzzy sets have already been made. Szmidt and Kacprzyk[5] proposed a similarity measure for intuitionistic Fuzzy sets and showed its usefulness in medical diagnostic reasoning. Ju and Wang [6] proposed a similarity measure for interval-valued Fuzzy sets and showed its usefulness in medical diagnostic reasoning. These already proposed Similarity measures are based on Zadehian definition of Fuzzy set. Zadeh defined Fuzzy set in a way where it has been believed that the classical set theoretic axioms of exclusion and contradiction are not satisfied for Fuzzy sets. Regarding this, Baruah [2,3] proposed that two functions, namely Fuzzy membership function and Fuzzy reference function are necessary to represent a Fuzzy set. Therefore , Baruah [2, 3] reintroduced the notion of complement of a Fuzzy set in a way that the set theoretic axioms of exclusion and contradiction can be seen valid for Fuzzy sets also. In recent years researchers have contributed a lot towards Fuzzy set theory. Neog and Sut [4] have generalized the concept of complement of a Fuzzy set introduced by Baruah[2,3] when the Fuzzy reference function is not zero and defined arbitrary Fuzzy union and intersection extending the definitions of Fuzzy sets given by Baruah [2, 3]. As a consequence of which, if we take the definition of complementation initiated by Baruah[2,3,4] into consideration then the existing Similarity measures are not at all logical in our standpoint and hence becomes unacceptable. In this paper, we discuss the errors in the existing Similarity measures and put forward an alternative way in which we can redefine it to make it error free.

The overall organization of this paper is as follows. In section 2 we visit some published papers related with Similarity measure for Fuzzy sets. In section 3 we discuss the new and extended definition of complementation of Fuzzy set. In section 4 we analyze the errors in the existing Similarity measures on the basis of the extended definition of complementation and suggest a different way to overcome it. Finally, some conclusions are given in section 5.

II. SOME PAPERS DEALING WITH SIMILARITY MEASURE**A. Szmidt and Kacprzyk's Similarity Measure**

As opposed to a Fuzzy set in X (Zadeh [1]), given by

$$A' = \{ \langle x, \mu_A(x) \rangle \mid x \in X \}$$

where $\mu_A(x) \in [0, 1]$ is the membership function of the Fuzzy set A' , an intuitionistic Fuzzy set (IFS) A is given by

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$$

where $\mu_A: X \rightarrow [0, 1]$ and $\nu_A: X \rightarrow [0, 1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1$$

and $\mu_A(x), \nu_A(x) \in [0, 1]$ denote a degree of membership and a degree of non membership of $x \in A$ respectively.

Obviously each Fuzzy set may be represented by the following intuitionistic Fuzzy set

$$A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle \mid x \in X \}$$

For each intuitionistic Fuzzy set in X , $\pi_A = 1 - \mu_A(x) - \nu_A(x)$ is called an intuitionistic Fuzzy index (or a hesitation margin) of $x \in A$ and, it expresses a lack of knowledge of whether x belongs to A or not.

It is obvious that $0 \leq \pi_A(x) \leq 1$ for each $x \in X$.

The complement of IFS, A is given by

$$A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in X \}$$

With this definition IFS, Szmidt and Kacprzyk[5] proposed a similarity measure for two IFSs X and F in the following manner-

$$\begin{aligned} \text{Sim}(X, F) &= \frac{I_{\text{IFS}}(X, F)}{I_{\text{IFS}}(X, F^c)} = \frac{I_{\text{IFS}}(X^c, F^c)}{I_{\text{IFS}}(X, F^c)} = \\ &= \frac{I_{\text{IFS}}(X, F)}{I_{\text{IFS}}(X^c, F)} = \frac{I_{\text{IFS}}(X^c, F^c)}{I_{\text{IFS}}(X^c, F)} \end{aligned}$$

1. where $\text{Sim}(X, F) = 0$ means the identity of X and F .
2. when $\text{Sim}(X, F) = 1$ means that X is to the same extent similar to F and F^c (i.e. values bigger than 1 means in fact a closer similarity of X and F^c to X and F).
3. when $X = F^c$ (or $X^c = F$) i.e. $I_{\text{IFS}}(X, F^c) = I_{\text{IFS}}(X^c, F) = 0$ means the complete dissimilarity of X and F (or in other words the identity of X and F^c) and then $\text{Sim}(X, F) \rightarrow \infty$.
4. when $X = F = F^c$ means the highest possible entropy for both elements F and X i.e. the highest fuzziness-not too constructive a case when looking for compatibility(both similarity and dissimilarity).

In other words, when applying this similarity measure to analyse the similarity of two objects, one should be interested in the values $0 \leq \text{Sim}(X, F) < 1$.

B. Ju and Wang's Similarity Measure

As opposed to a Fuzzy set in X (Zadeh [1]), given by

$$A = \{ \langle x, A(x) \rangle \mid x \in X \}$$

where $A(x) \in [0, 1]$ is the membership function of the Fuzzy set A , an interval-valued Fuzzy set (IVFS) A is given by

$$A = \{ \langle x, [A^-(x), A^+(x)] \rangle \mid x \in X \}$$

where $A^-, A^+ : X \rightarrow [0, 1]$ such that

$$0 \leq A^-(x) \leq A^+(x) \leq 1, x \in X$$

and $[A^-(x), A^+(x)]$ is the interval degree of membership function of an element x to the set A . Obviously, each Fuzzy set may be represented by the following interval-valued Fuzzy set,

$$A = \{ \langle x, [A^-(x), A^+(x)] \rangle \mid x \in X \}$$

For each interval valued Fuzzy set in X , $\pi_A(x) = A^+(x) - A^-(x)$ is called an interval valued Fuzzy index (or a hesitation margin) of $x \in A$ and, it expresses a lack of knowledge of the degree x belongs to A .

It is obvious that $0 \leq \pi_A(x) \leq 1$ for each $x \in X$.

The complement of IVFS, A is given by

$$A^c = \{ \langle x, [1 - A^+(x), 1 - A^-(x)] \rangle \mid x \in X \}$$

With this definition IVFS, Ju and Wang [6] proposed a similarity measure for two IVFSs E and F in the following manner-

$$\text{Sim}(E, F) = \frac{I_{\text{IVFS}}(E, F)}{I_{\text{IVFS}}(E, F^c)} = \frac{a}{b}$$

where a is distance (E, F) from $E[E^-, E^+, \pi_E]$ to $F[F^-, F^+, \pi_F]$, b is the distance (E, F^c) from $E[E^-, E^+, \pi_E]$ to $F^c[1 - F^+, 1 - F^-, \pi_F]$.

For this similarity measure, we have, $0 < \text{Sim}(E, F) \leq \infty$ and $\text{Sim}(E, F) = \text{Sim}(F, E)$.

Note that:

1. $\text{Sim}(E, F) = 0$ means the identity of E and F .
2. $\text{Sim}(E, F) = 1$ means that E is to the same extent similar to F and F^c (i.e. value bigger than 1 mean in fact a closer similarity of E and F^c to E and F).
3. When $E = F^c$ (or $E^c = F$) i.e. $I_{\text{IVFS}}(E, F^c) = I_{\text{IVFS}}(E^c, F) = 0$ means the complete dissimilarity of E and F (or the identity of E and F^c) and then $\text{Sim}(E, F) \rightarrow \infty$.
4. When $E = F = F^c$ means the highest entropy for both elements E and F i.e. the highest fuzziness-not too constructive a case when looking for compatibility(both similarity and dissimilarity).

In other words, when applying this similarity measure to analyse the similarity of two objects, one should be interested in the values $0 \leq \text{Sim}(E, F) < 1$.

III. BARUAH'S DEFINITION OF COMPLEMENTATION OF FUZZY SETS

Baruah put forward an extended definition of Fuzzy sets in the following manner –

Let $\mu_1(x)$ and $\mu_2(x)$ be two functions, such that $0 \leq \mu_2(x) \leq \mu_1(x) \leq 1$. For a Fuzzy number denoted by $\{x, \mu_1(x), \mu_2(x); x \in U\}$ we would call $\mu_1(x)$, the Fuzzy membership function and $\mu_2(x)$, a reference function such that $\{\mu_1(x)-\mu_2(x)\}$ is the Fuzzy membership value for any x .

In the definition of complement of a Fuzzy set, the Fuzzy membership value and the Fuzzy membership function have to be different in the sense that for a usual Fuzzy set the membership value and the membership function are of course equivalent. This can be visualized with the help of following diagram:

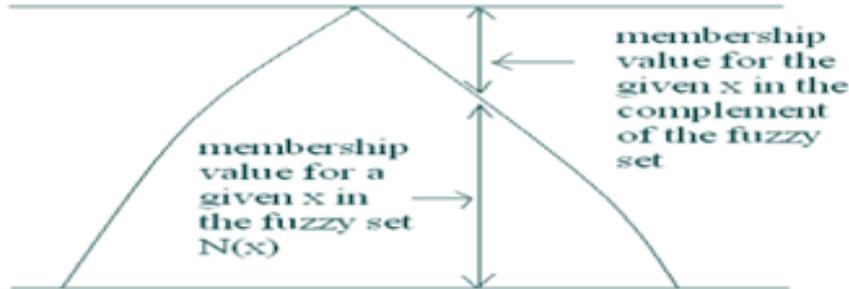


Fig. 1 Baruah's extended definition of Fuzzy set $N(x)$.

A. Extended Definition of Union and Intersection of Fuzzy Sets

With the help of the extended definition, Baruah[2,3] put forward the notion of union and intersection of two Fuzzy sets in the following manner –

Let $A(\mu_1, \mu_2) = \{x, \mu_1(x), \mu_2(x); x \in U\}$ and $B(\mu_3, \mu_4) = \{x, \mu_3(x), \mu_4(x); x \in U\}$ be two Fuzzy sets defined over the same universe U , where μ_1, μ_2 and μ_3, μ_4 are membership and reference functions of A and B respectively.

Then the operations intersection and union are defined as

$$A(\mu_1, \mu_2) \cap B(\mu_3, \mu_4) = \{x, \min(\mu_1(x), \mu_3(x)), \max(\mu_2(x), \mu_4(x)); x \in U\} \text{ and}$$

$$A(\mu_1, \mu_2) \cup B(\mu_3, \mu_4) = \{x, \max(\mu_1(x), \mu_3(x)), \min(\mu_2(x), \mu_4(x)); x \in U\}$$

Two Fuzzy sets $C = \{x, \mu_C(x); x \in U\}$ and $D = \{x, \mu_D(x); x \in U\}$ in the usual definition would be expressed as

$$C(\mu_C, 0) = \{x, \mu_C(x), 0; x \in U\} \text{ and } D(\mu_D, 0) = \{x, \mu_D(x), 0; x \in U\}$$

Accordingly, we have,

$$\begin{aligned} C(\mu_C, 0) \cap D(\mu_D, 0) &= \{x, \min(\mu_C(x), \mu_D(x)), \max(0, 0); x \in U\} \\ &= \{x, \min(\mu_C(x), \mu_D(x)), 0; x \in U\} \\ &= \{x, \mu_C(x) \wedge \mu_D(x); x \in U\} \end{aligned}$$

which in the usual definition is nothing but $C \cap D$.

Similarly we have,

$$\begin{aligned} C(\mu_C, 0) \cup D(\mu_D, 0) &= \{x, \max(\mu_C(x), \mu_D(x)), \min(0, 0); x \in U\} \\ &= \{x, \max(\mu_C(x), \mu_D(x)), 0; x \in U\} \\ &= \{x, \mu_C(x) \vee \mu_D(x); x \in U\} \end{aligned}$$

which in the usual definition is nothing but $C \cup D$.

Neog and Sut [4] showed by an example that this definition sometimes gives degenerate cases and revised the above definition as follows -

Let $A(\mu_1, \mu_2) = \{x, \mu_1(x), \mu_2(x); x \in U\}$ and $B(\mu_3, \mu_4) = \{x, \mu_3(x), \mu_4(x); x \in U\}$ be two Fuzzy sets defined over the same universe U . The operation intersection is defined as

$$A(\mu_1, \mu_2) \cap B(\mu_3, \mu_4) = \{x, \min(\mu_1(x), \mu_3(x)), \max(\mu_2(x), \mu_4(x)); x \in U\}$$

with the condition that $\min(\mu_1(x), \mu_3(x)) > \max(\mu_2(x), \mu_4(x)) \forall x \in U$.

Now if for some $x \in U$,

$$\min(\mu_1(x), \mu_3(x)) < \max(\mu_2(x), \mu_4(x))$$

Then our conclusion is that $A \cap B = \emptyset$.

and if for some $x \in U$,

$$\min(\mu_1(x), \mu_3(x)) = \max(\mu_2(x), \mu_4(x)) \text{ then also } A \cap B = \emptyset.$$

Further the operation Union is defined as

$$A(\mu_1, \mu_2) \cup B(\mu_3, \mu_4) = \{x, \max(\mu_1(x), \mu_3(x)), \min(\mu_2(x), \mu_4(x)); x \in U\}$$

with the condition that $\min(\mu_1(x), \mu_3(x)) \geq \max(\mu_2(x), \mu_4(x)) \forall x \in U$.

if for some $x \in U$,

$$\min(\mu_1(x), \mu_3(x)) < \max(\mu_2(x), \mu_4(x))$$

then the union of Fuzzy sets A and B cannot be expressed as one single Fuzzy set.

The union, however, can be expressed in one single Fuzzy set if

for some $x \in U$,

$$\min(\mu_1(x), \mu_3(x)) = \max(\mu_2(x), \mu_4(x)).$$

B. Complement of a Fuzzy Set Using Extended Definition

Baruah put forward the notion of complement of usual Fuzzy sets with Fuzzy reference function 0 in the following way –

Let $A(\mu, 0) = \{x, \mu(x), 0; x \in U\}$ and $B(1, \mu) = \{x, 1, \mu(x); x \in U\}$ be two Fuzzy sets defined over the same universe U. Now we have

$$\begin{aligned} A(\mu, 0) \cap B(1, \mu) &= \{x, \min(\mu(x), 1), \max(0, \mu(x)); x \in U\} \\ &= \{x, \mu(x), \mu(x); x \in U\} \end{aligned}$$

which is nothing but the null/empty set \emptyset [since $\mu(x) - \mu(x) = 0$] and

$$\begin{aligned} A(\mu, 0) \cup B(1, \mu) &= \{x, \max(\mu(x), 1), \min(0, \mu(x)); x \in U\} \\ &= \{x, 1, 0; x \in U\} \end{aligned}$$

which is nothing but the universal set U.

This means if we define a Fuzzy set

$$(A(\mu, 0))^c = \{x, 1, \mu(x); x \in U\}$$

it is nothing but the complement of $A(\mu, 0) = \{x, \mu(x), 0; x \in U\}$.

Neog and Sut [4] have generalized the concept of complement of a Fuzzy set when the Fuzzy reference function is not zero extending definition of complement of Fuzzy sets introduced by Baruah [2, 3] in the following manner-

Let $A(\mu_1, \mu_2) = \{x, \mu_1(x), \mu_2(x); x \in U\}$ be a Fuzzy set defined over the universe U. The complement of the Fuzzy set $A(\mu_1, \mu_2)$ is defined as

$$\begin{aligned} (A(\mu_1, \mu_2))^c &= \{x, \mu_1(x), \mu_2(x); x \in U\}^c \\ &= \{x, \mu_2(x), 0; x \in U\} \cup \{x, 1, \mu_1(x); x \in U\} \end{aligned}$$

Membership value of x in $(A(\mu_1, \mu_2))^c$ is given by

$$\mu_2(x) + (1 - \mu_1(x)) = 1 + \mu_2(x) - \mu_1(x).$$

If $\mu_2(x) = 0$, then membership value of x is $1 + 0 - \mu_1(x) = 1 - \mu_1(x)$.

Since, for $x \in U$, $\min(\mu_2(x), 1) < \max(0, \mu_1(x))$, so the union of these two Fuzzy sets cannot be expressed as one single Fuzzy set.

The all complement properties hold good also when we take Fuzzy reference function = 0 $\forall x \in U$.

Thus, we have understood that for the complement of a Fuzzy set the Fuzzy membership value and the Fuzzy membership function are two different things although for a usual Fuzzy set they are not different because the value of the function is counted from 0 in the usual case.

These extended definitions of Fuzzy set has satisfied the set theoretic axioms of contradiction and exclusion in the following manner-

C. Law of Contradiction

Let $A(\mu_1, \mu_2)$ be a Fuzzy set defined on the set of universe U. Now we have,

$$\begin{aligned} A(\mu_1, \mu_2) \cap (A(\mu_1, \mu_2))^c &= \{x, \mu_1(x), \mu_2(x); x \in U\} \cap [\{x, \mu_2(x), 0; x \in U\} \cup \{x, 1, \mu_1(x); x \in U\}] \\ &= [\{x, \mu_1(x), \mu_2(x); x \in U\} \cap \{x, \mu_2(x), 0; x \in U\}] \\ &\quad \cup [\{x, \mu_1(x), \mu_2(x); x \in U\} \cap \{x, 1, \mu_1(x); x \in U\}] \\ &= [\{x, \mu_2(x), \mu_2(x); x \in U\}] \cup [\{x, \mu_1(x), \mu_1(x); x \in U\}] \\ &= (\text{Empty Set}) \cup (\text{Empty Set}) \\ &= \emptyset \cup \emptyset = \emptyset \end{aligned}$$

D. Law of Exclusion

Let $A(\mu_1, \mu_2)$ be a Fuzzy set defined on the set of universe U . Now we have,

$$\begin{aligned} A(\mu_1, \mu_2) \cup (A(\mu_1, \mu_2))^c &= \{x, \mu_1(x), \mu_2(x); x \in U\} \cup [\{x, \mu_2(x), 0; x \in U\} \cup \{x, 1, \mu_1(x); x \in U\}] \\ &= [\{x, \mu_1(x), \mu_2(x); x \in U\} \cup \{x, \mu_2(x), 0; x \in U\}] \\ &\quad \cup [\{x, \mu_1(x), \mu_2(x); x \in U\} \cup \{x, 1, \mu_1(x); x \in U\}] \\ &= [x, \mu_1(x), 0; x \in U] \cup [x, 1, \mu_2(x); x \in U] \\ &= [x, 1, 0; x \in U] \\ &= U \end{aligned}$$

Neog and Sut [4] put forward the notion of Fuzzy subset using the extended notion of Fuzzy sets in the following manner -

E. Fuzzy Subset and Properties

Let $A(\mu_1, \mu_2) = \{x, \mu_1(x), \mu_2(x); x \in U\}$ and $B(\mu_3, \mu_4) = \{x, \mu_3(x), \mu_4(x); x \in U\}$ be two Fuzzy sets defined over the same universe U .

Now the Fuzzy set $A(\mu_1, \mu_2)$ is a subset of the Fuzzy set $B(\mu_3, \mu_4)$ if $\forall x \in U, \mu_1(x) \leq \mu_3(x)$ and $\mu_2(x) \leq \mu_4(x)$.

Two Fuzzy sets $C = \{x, \mu_C(x); x \in U\}$ and $D = \{x, \mu_D(x); x \in U\}$ in the usual definition would be expressed as

$$C(\mu_C, 0) = \{x, \mu_C(x), 0; x \in U\} \text{ and } D(\mu_D, 0) = \{x, \mu_D(x), 0; x \in U\}$$

Accordingly we have,

$$C(\mu_C, 0) \subseteq D(\mu_D, 0)$$

If $\forall x \in U, \mu_C(x) \leq \mu_D(x)$, which can be obtained by putting $\mu_2(x) = \mu_4(x) = 0$ in our definition of Fuzzy set.

IV. ANALYSIS OF THE EXISTING SIMILARITY MEASURES

If we analyze the existing definitions of Similarity measure visited in section 2 on the basis of Baruah's definition of complementation of Fuzzy sets based on reference function [2,3] we can show that some parts of this kind of representation is not logical.

In the definition of Similarity measure presented by Szmidi and Kacprzyk [5] for two IFSs X and F , the possible case 4 states,

$$X = F = F^c \text{ means the highest possible entropy [7] for both elements } F \text{ and } X$$

Also, in the definition of Similarity measure presented by Ju and Wang [6] for two IVFSs E and F , for the possible case 4,

$$E = F = F^c \text{ means the highest entropy [8] for both elements } E \text{ and } F$$

Now according to the definition of Entropy [7, 8],

For a Fuzzy set A , Entropy is given by

$$E(A) = \frac{|A \cap A^c|}{|A \cup A^c|} \quad (1)$$

where $|A \cap A^c|$ and $|A \cup A^c|$ denote the cardinalities of the sets $A \cap A^c$ and $A \cup A^c$ respectively, where A^c stands for the complement of the set A , which is defined with the help of the membership function only as below,

$$\mu_{A^c}(x) = 1 - \mu_A(x) \quad \forall x \in \Omega$$

while defining entropy in this manner, it was assumed that the two laws –the Law of Contradiction and the Law of Excluded Middle are violated by Fuzzy sets.

Hence it has been seen that entropy definition [7,8] is based on Zadehian definition of complementation where it is believed that the classical set theoretic axioms of exclusion and contradiction are not satisfied. In this regard, Baruah [2,3] proposed that two functions, namely Fuzzy membership function and Fuzzy reference function are necessary to represent a Fuzzy set (as given in section 3). Baruah [2, 3] reintroduced the notion of complement of a Fuzzy set in a way that the set theoretic axioms of exclusion and contradiction can be seen valid for Fuzzy sets also.

Now if we apply the extended definition of complementation [2,3,4] we can obtain that $A \cap A^c =$ the null set ϕ and hence the cardinality of this set is always zero according to the definition of cardinality. Now if the cardinality of this Fuzzy set is zero then the entropy defined in the manner (1) will reduce to $E(A) = 0$ always. Thus we will never be able to find any measure of fuzziness from this existing definition. Also for the same reason, there will be no difference between the possible case 4 and the possible case 1 of both the existing Similarity measures discussed in section 2. Therefore if we take Baruah's definition of complementation into consideration then we find the existing measures are not at all logical in our standpoint and hence becomes unacceptable.

In this article, we intended to revisit the existing definition of Similarity measure of Fuzzy sets and in the process it is found that the Similarity measure of a Fuzzy set especially when dealing with complementation is not defined logically. The reason behind such a claim is contributed to the fact that the definition of complementation used in the existing Similarity measures is not logically defined. Hence it is obvious that any result which is obtained with the help of something which itself is controversial cannot yield a suitable result. It is observed that the complementation defined

with the help of reference function [2, 3,4] seems more logical than the existing one. It is due to this reason that we would like to propose a new definition of Similarity measure for Fuzzy sets on the basis of complementation based on reference function.

So using the definition of complementation based on reference function [2, 3, 4] we would like to state the Similarity measure with the four possible cases in a different manner as follows:

Let A and B be two elements belonging to a Fuzzy set (or sets) .Now we can measure the similarity between A and B as below:

$$\text{Sim}(A, B) = \frac{I_{FS}(A,B)}{I_{FS}(A,B^C)} = \frac{a}{b} \quad (2)$$

where a is distance from $A(\mu_m, \mu_r, \mu_v)$ to $B(\mu_m, \mu_r, \mu_v)$ and b is a distance from $A(\mu_m, \mu_r, \mu_v)$ to $B^C(\mu_m, \mu_r, \mu_v)$ where μ_m, μ_r, μ_v are membership function, reference function and membership value respectively.

For this similarity measure, we have,

$$0 \leq \text{Sim}(A, B) \leq \alpha$$

Similarly we can calculate the Similarity between two Fuzzy sets:

Let A and B be two Fuzzy sets defined on the same set of universe of discourse. Now we can measure the similarity between A and B by assessing similarity of the corresponding elements belonging to A and B, as defined in the eqn (2).

Now using Baruah's definition of Fuzzy set, for the Similarity measure of A and B, we can obtain the following 4 possibilities,

A and B may be two exactly similar sets.

or A and B^C may be two exactly similar sets.

or A may be more similar to B than to B^C .

or A may be more similar to B^C than to B.

But A can never be similar to B and B^C together i.e. $A=B=B^C$ is never possible according to the new definition of complementation of Fuzzy set [2, 3].

Therefore from the above analysis, for the Similarity measure of A and B, we can conclude four possible cases as follows:

Case 1: $\text{Sim}(A,B)=0$ when $A=B$ i.e. $AB=0$.

Case 2: $\text{Sim}(A,B)=\infty$ when $A=B^C$ i.e. $AB^C=0$.

Case 3: $\text{Sim}(A,B) > 1$ when $AB > AB^C$.

Case 4: $\text{Sim}(A,B) < 1$ when $AB < AB^C$.

Hence to measure the similarity between the two Fuzzy sets A and B, one should be interested in the values $0 \leq \text{Sim}(A,B) < 1$.

V. CONCLUSIONS

In this article, some existing measures of Similarity for Fuzzy sets, as can be found in the literature of Fuzzy set theory are revisited. Further the relationship between entropy and Similarity measure has been discussed and commented on. All these are analyzed with the help of the definition of complementation of Fuzzy sets on the basis of reference function. Here efforts have been made to show that those existing definitions of Similarity measure are not acceptable since Fuzzy entropy always becomes zero when the new definition of complementation is used. It has been observed that the existing definitions of Similarity measure are not sufficient if it is tried to be defined on the basis of reference function and so the need for a new definition of Similarity measure for Fuzzy sets is highlighted in this article. Finally, we suggest an alternative way to redefine Similarity measure for Fuzzy sets using the extended definition of complementation based on reference function which can be taken for further research so that it becomes free from any further controversy.

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