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Characteristics of Tunable Digital Filters

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Abstract- Tunable filters change their passband and stopband edge frequencies according to the requirement of the application. There are numerous applications of Tunable filters. In this paper we have discussed basic theory of tunable filters and then analyzed the characteristics of tunable filters.

Keywords- spectral transformation, IIR, FIR,

I. INTRODUCTION

Filters are the manipulation of the amplitude and/or phase response of a signal according to their frequency. These are the basic components of all signal processing and telecommunication systems. There are two kinds of filters- fixed and tunable. Fixed filters are those in which passband frequencies and stopband frequencies are fixed whereas in case of tunable filters, passband and stopband frequencies are variable. These frequencies can be changed according to the requirement of the applications. Tunable digital filters are widely employed in telecommunications, medical electronics, digital audio equipment and control systems. These filters are also known as variable digital filters [1]. Tunable digital filters are used in telecommunication system in the front end of a receiver to select a particular band of frequencies. In medical electronics, tunable notch filters are used to suppress the power line interference [2]. The bases for the design of the tunable digital filters is the spectral transformation [3] [4]. It is basically used to modify the characteristics of a filter to meet new specifications without repeating the filter design procedure. This modification is done by changing a Low pass(LP) digital filters to Low pass(LP) filters with different cutoff frequencies or to a High pass(HP), Band pass(BP) or Band stop(BS) filters. The variable Band pass (BP) and Band stop (BS) filters are used to eliminate and retrieve some narrow band signals. In [5] variable band pass and band stop filters are shown with high accuracy and independent tuning characteristics.

II. TUNABLE IIR FILTERS USING SPECTRAL TRANSFORMATION

Sometimes it is necessary to modify the characteristics of a filter to meet the new specifications without repeating the filter design procedure. For example, after a lowpass filter with a passband edge at 2.5 KHz has been designed, it may be required to move the passband edge to 2.6 KHz. It is possible to design a digital filter with highpass or bandpass or bandstop characteristics by transforming a given digital lowpass filter. The spectral transformation [6] is described here to transform a given lowpass digital IIR transfer function $G_L(z)$ to another digital transfer function $G_D(z)$ that could be a lowpass, highpass, bandpass, or bandstop filter. To eliminate the confusion between the complex variable z of the lowpass transfer function $G_L(z)$ and that of the desired transfer function $G_D(z)$, we shall use different symbols. Thus, we shall use z^{-1} to denote the unit delay in the prototype lowpass digital filter $G_L(z)$ and $\hat{z}^{-1}$ to denote unit delay in the transformed filter $G_D(\hat{z})$. The unit circle in the z- and $\hat{z}$-planes are defined by

$$z = e^{j\omega}, \hat{z} = e^{j\hat{\omega}} \qquad (1)$$

The transformation from the z-domain to the $\hat{z}$-domain is denoted as

$$z = F(\hat{z}) \qquad (2)$$

Then, $G_L(z)$ is transformed to $G_D(\hat{z})$ through

$$G_D(\hat{z}) = G_L\{F(\hat{z})\} \qquad (3)$$

To transform a rational $G_L(z)$ into a rational $G_D(\hat{z})$, $F(\hat{z})$ must be a rational function of $\hat{z}$. And to guarantee the stability of $G_D(\hat{z})$, the transformation should be such that the inside of the unit circle of the z-plane is mapped into the inside of the unit circle of $\hat{z}$-plane. Finally, to ensure that a lowpass magnitude response is mapped into one of the four basic types of magnitude responses, points on the unit circle of the z-plane should be mapped to points on the unit circle of the $\hat{z}$-plane. In the z-plane, a point on the unit circle is characterized by |z| = 1, a point inside the unit circle is given by |z| < 1, and a point outside the unit circle is defined by |z| > 1. Therefore,

$$|F(\hat{z})| \begin{cases} > 1, & if\ |z| > 1, \\ = 1, & if\ |z| = 1, \\ < 1, & if\ |z| < 1. \end{cases} \qquad (4)$$

The most general form of $F(\hat{z})$ with real coefficients is given by

$$F(\hat{z}) = \pm \prod_{l=1}^{L} \left(\frac{\hat{z} - \lambda_l}{1 - \lambda_l^* \hat{z}} \right) \qquad (5)$$

Where λ_l is either real or occurs in complex conjugate pairs $|\lambda_l| < 1$ for stability.

III. Lowpass-to-Lowpass Transformation

To transform a prototype lowpass filter $G_L(z)$ with a cutoff frequency ω_c to another lowpass filter $G_D(\hat{z})$ with a cutoff frequency$\widehat{\omega}_c$, we use the transformation

$$z^{-1} = F^{-1}(\hat{z}) = \frac{1 - \lambda\hat{z}}{\hat{z} - \lambda} \qquad (6)$$

with λ real. On the unit circle, above transformation reduces to

$$e^{-j\omega} = \frac{e^{-j\widehat{\omega}} - \lambda}{1 - \lambda e^{-j\widehat{\omega}}}, \qquad (7)$$

From which we arrive at

$$\tan\left(\frac{\omega}{2}\right) = \left(\frac{1+\lambda}{1-\lambda}\right) \tan\left(\frac{\widehat{\omega}}{2}\right). \qquad (8)$$

Note that the mapping is nonlinear except for $\lambda = 0$, resulting in a warping of the frequency scale for nonzero values of λ. However, if $G_L(z)$ is a piecewise constant lowpass magnitude response, then the transformed filter $G_D(\hat{z})$ will likewise have a similar piecewise constant lowpass magnitude response due to the monotonicity of the transformationof Eq. (8). The relation between the cutoff frequency ω_c of $G_L(z)$ with the cutoff frequency $\widehat{\omega}_c$ of $G_D(\hat{z})$ follows from Eq. (8):

$$\tan\left(\frac{\omega_c}{2}\right) = \left(\frac{1+\lambda}{1-\lambda}\right) \tan\left(\frac{\widehat{\omega}_c}{2}\right)$$

Which can be solved for λ yielding:

$$\lambda = \frac{\tan\left(\omega_c/2\right) - \tan\left(\widehat{\omega}_c/2\right)}{\tan\left(\omega_c/2\right) + \tan\left(\widehat{\omega}_c/2\right)} = \frac{\sin\left(\frac{\omega_c - \widehat{\omega}_c}{2}\right)}{\sin\left(\frac{\omega_c + \widehat{\omega}_c}{2}\right)}. \qquad (9)$$

It should be noted that the lowpass-to-lowpass transformation can also be used to transform a highpass filter with a cutoff at ω_c to another highpass filter with a cutoff at $\widehat{\omega}_c$, a bandpass filter with a center frequency at ω_0 to another bandpass filter with center frequency $\widehat{\omega}_0$, and a bandstop filter with a center frequency at ω_0 to another bandstop filter with a center frequency at $\widehat{\omega}_0$.

IV. OTHER TRANSFORMATIONS

Other transformations such as the lowpass-to-highpass, lowpass-to-bandpass, and lowpass-to-bandstop transformations are shown in Table. 3.1 [6]. Lowpass-to-lowpass transformation was discussed in article III. It should be noted that these spectral transformations can be used only to map one frequency point ω_c in the magnitude response of the lowpass prototype filter into a new position $\widehat{\omega}_c$; with the same magnitude response value for the transformed lowpass and highpass filters; or into two new positions, $\widehat{\omega}_{c1}$ and $\widehat{\omega}_{c2}$, with the same magnitude response values for the transformed bandpass and bandstop filters. Hence it is possible only to map either the passband edge or the stopband edge of the lowpass prototype filter onto the desired position(s), but not both.

Table 1 Spectral Transformation of a Lowpass Filter with a Cutoff Frequency ω_c

Filter type	**Spectral transformation**	**Design parameters**
Lowpass	$z^{-1} = \frac{\hat{z}^{-1} - \lambda}{1 - \lambda\hat{z}^{-1}}$	$\lambda = \frac{\sin\left(\frac{\omega_c - \widehat{\omega}_c}{2}\right)}{\sin\left(\frac{\omega_c + \widehat{\omega}_c}{2}\right)}$ $\widehat{\omega}_c$ =desired cutoff frequency
Highpass	$z^{-1} = -\frac{\hat{z}^{-1} + \lambda}{1 + \lambda\hat{z}^{-1}}$	$\lambda = -\frac{\cos\left(\frac{\omega_c + \widehat{\omega}_c}{2}\right)}{\cos\left(\frac{\omega_c - \widehat{\omega}_c}{2}\right)}$ $\widehat{\omega}_c$ =desired cutoff frequency

Bandpass	$z^{-1} = -\dfrac{\hat{z}^{-2} - \frac{2\lambda\rho}{\rho+1}\hat{z}^{-1} + \frac{\rho-1}{\rho+1}}{\frac{\rho-1}{\rho+1}\hat{z}^{-2} - \frac{2\lambda\rho}{\rho+1}\hat{z}^{-1} + 1}$	$\lambda = \dfrac{\cos\left(\frac{\hat{\omega}_{c2}+\hat{\omega}_{c1}}{2}\right)}{\cos\left(\frac{\widehat{\omega_{c2}-\omega_{c1}}}{2}\right)}$ $\rho = \cot\left(\dfrac{\widehat{\omega}_{c2} - \widehat{\omega_{c1}}}{2}\right)\tan\left(\dfrac{\omega_c}{2}\right)$ $\widehat{\omega}_{c2}, \widehat{\omega}_{c1}$ =desired upper and lower cutoff frequencies
Bandstop	$z^{-1} = -\dfrac{\hat{z}^{-2} - \frac{2\lambda}{\rho+1}\hat{z}^{-1} + \frac{1-\rho}{\rho+1}}{\frac{1-\rho}{\rho+1}\hat{z}^{-2} - \frac{2\lambda}{\rho+1}\hat{z}^{-1} + 1}$	$\lambda = \dfrac{\cos\left(\frac{\hat{\omega}_{c2}+\hat{\omega}_{c1}}{2}\right)}{\cos\left(\frac{\hat{\omega}_{c2}-\widehat{\omega_{c1}}}{2}\right)}$ $\rho = \tan\left(\dfrac{\widehat{\omega}_{c2} - \widehat{\omega}_{c1}}{2}\right)\tan\left(\dfrac{\omega_c}{2}\right)$ $\widehat{\omega}_{c2}, \widehat{\omega}_{c1}$ =desired upper and lower cutoff frequencies

V. SIMULATION OF TUNABLE FILTERS IN MATLAB

As it was said in theory part that the cut off frequency of a tunable filter can be changed without changing the design procedure. By using Spectral Transformation method, a low pass filter can be transformed to a low pass filter with different cut off frequency. Low pass filter can also be transformed to high pass, band pass and band stop filter. A low pass to band pass Butterworth filter using spectral transformation has been implemented in MATLAB as shown below.

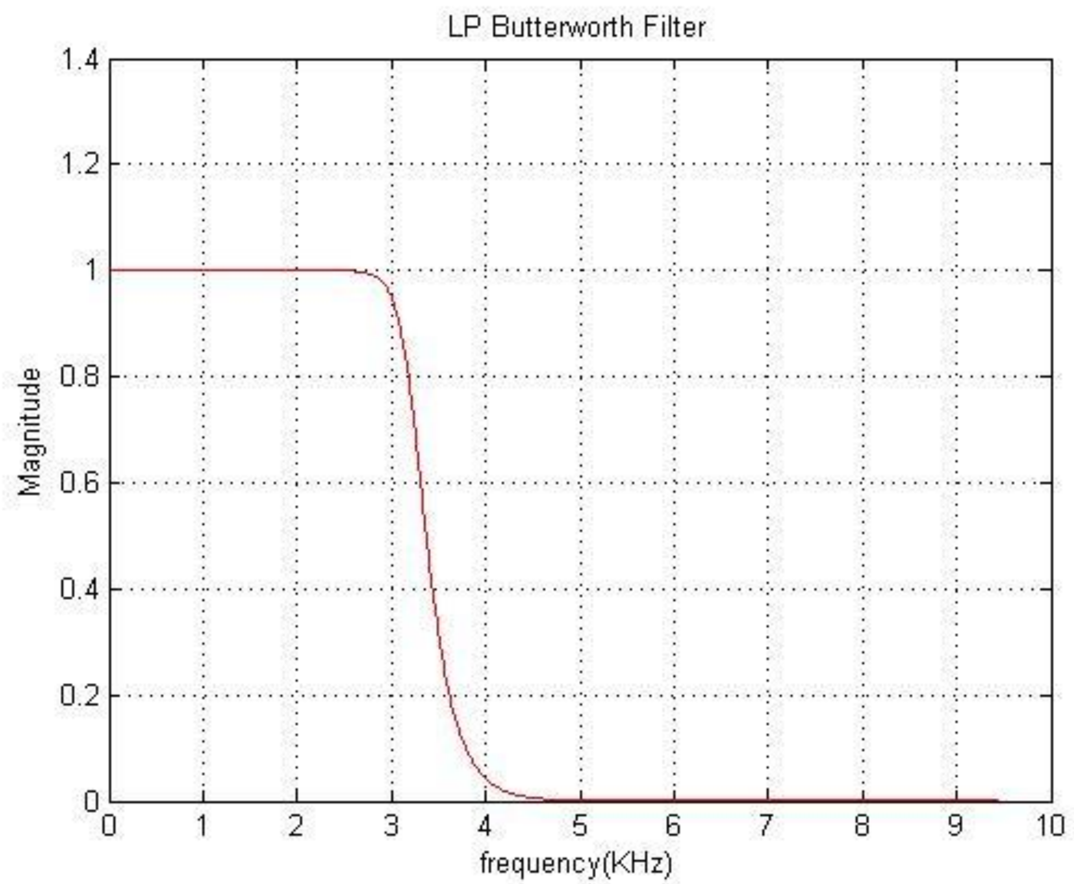

Fig. 1 Low pass Butterworth filter with cut off frequency 3 KHz

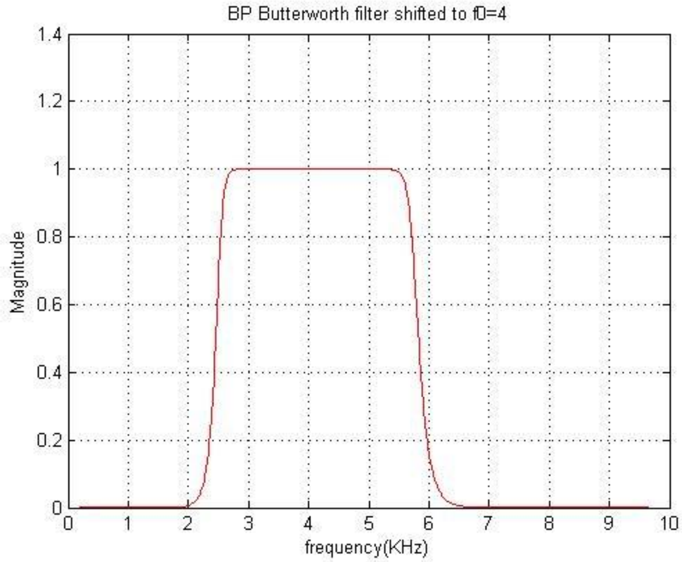

Fig. 2 Band pass Butterworth filter with center frequency f_0 = 4 KHz

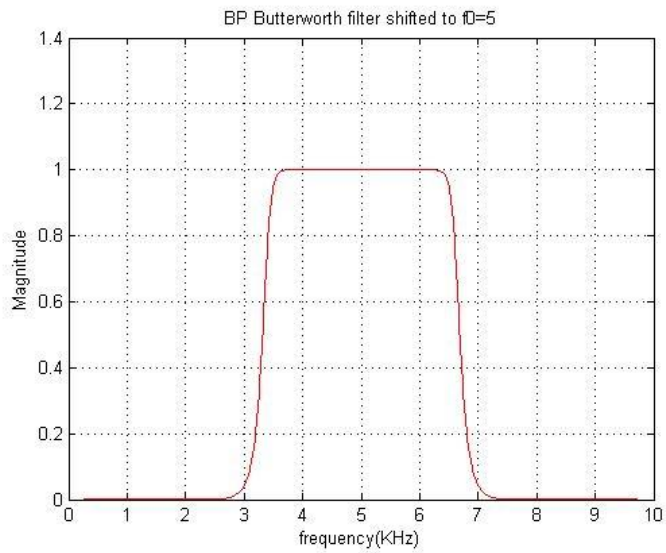

Fig. 3 Band pass Butterworth filter with center frequency f5 KHz

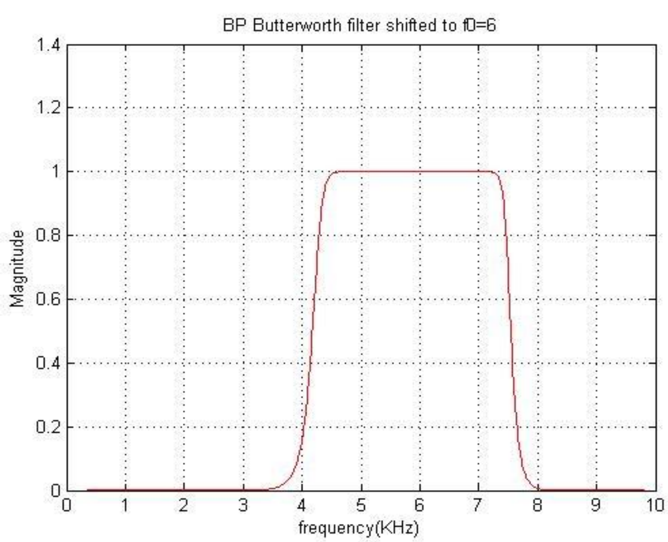

Fig. 4 Band pass Butterworth filter with center frequency f_0 = 6 KHz

Low pass Butterworth filter has cut off frequency of 3 KHz which is transformed to band pass filters with center frequencies 4 KHz, 5 KHz and 6 KHz. As the center frequencies are changed, the transformed filters is changing its position according to the center frequency. So the center frequency is the tuning element of these filters. As center frequencies are changing, they are getting tuned to different frequencies.

We can also apply this low pass to band pass transformation using Chebyshev or Elliptic filters. Let us take an example with Elliptic filters. As shown in figures 5 – 8, a low pass Elliptic filters are transformed to band pass elliptic filters using spectral transformation. Low pass filter is having cut off frequency of 3 KHz which is transformed to band pass filters with 4, 5 and 6 KHz as center frequencies. Here also the tuning element is the center frequency.

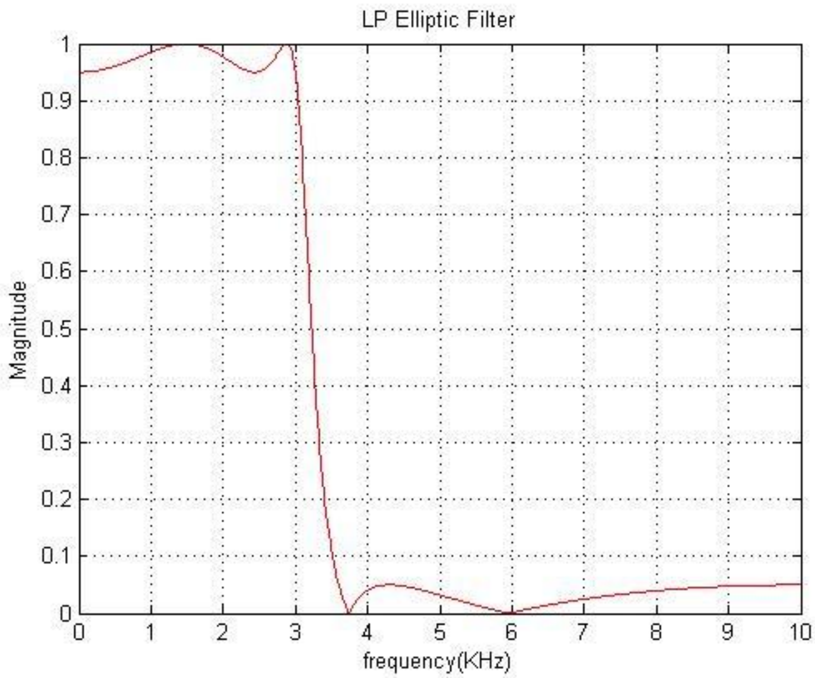

Fig. 5 Low pass Elliptic filter with cut off frequency 3 KHz

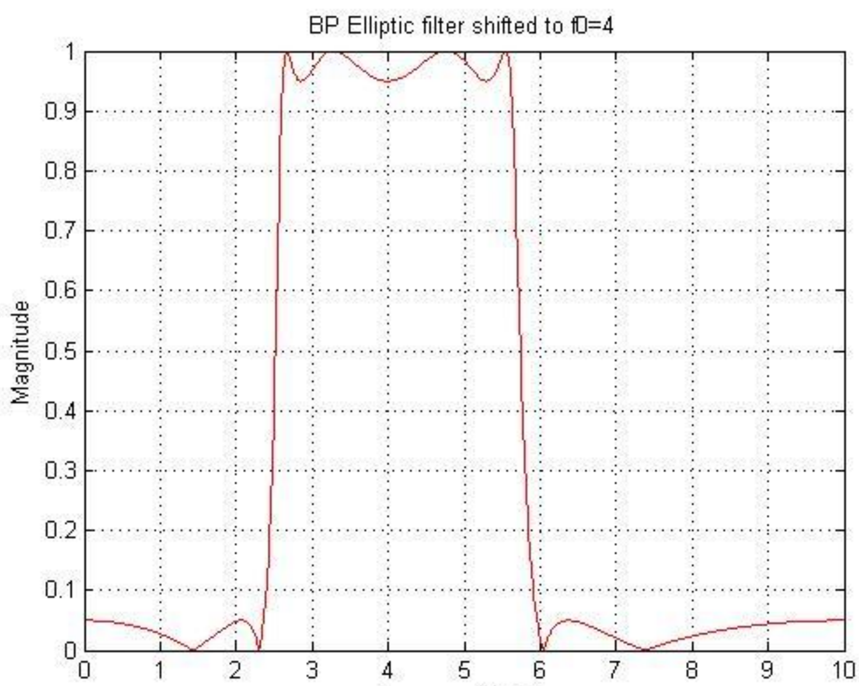

Fig. 6 Band pass Elliptic filter with center frequency f_0 = 4 KHz

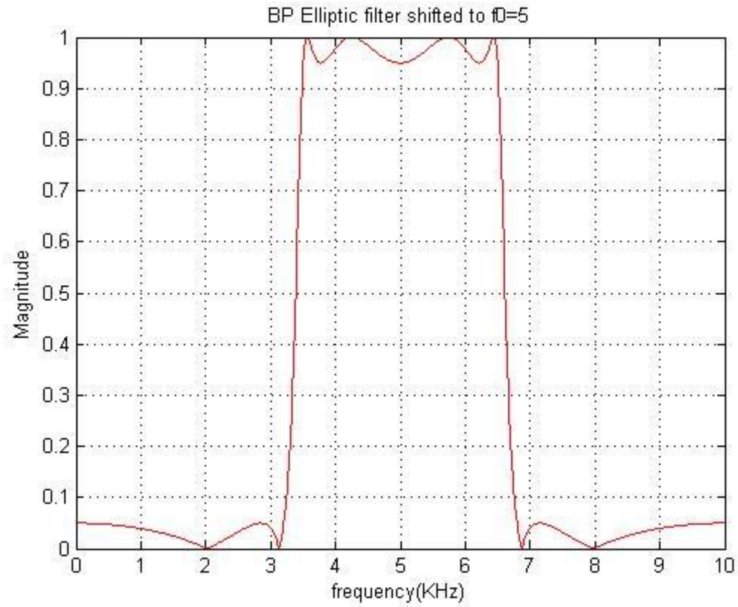

Fig. 7 Band pass Elliptic filters with center frequency $f_0 = 5$ KHz

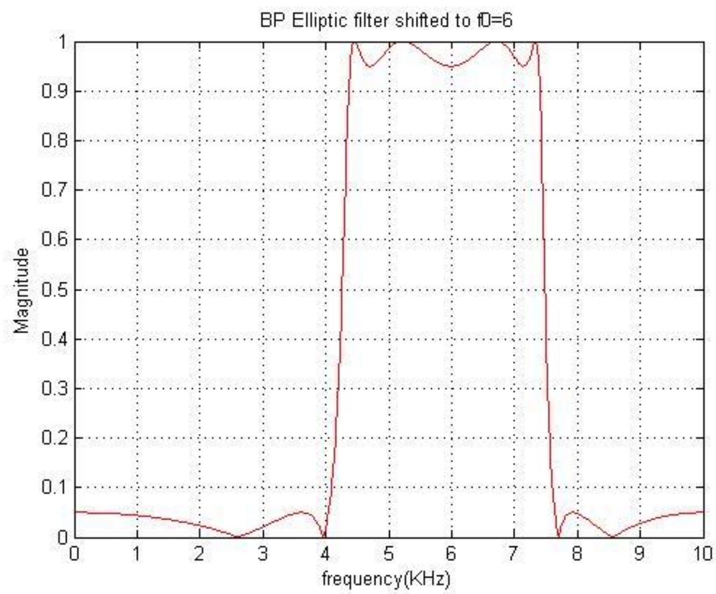

Fig. 8 Band pass filter with center frequency $f_0 = 6$

VI. EFFECT OF TUNABILITY ON THE ORDER OF THE FILTERS

The characteristics of tunable filters were studied as shown below. The relation of the order of the filter with the tuning element of the filter was studied. As it was said earlier in band pass and band stop filters, center frequency is the tuning element. So a table of center frequency with the order of the filter has been made as shown below.

Table. 2 Effect of center frequency (Tuning element) on Order of Butterworth filter

Sr. no.	Center frequency (f_0 KHz)	Order of Butterworth filter (N)
1	0	12
2	0.5	12
3	1	12
4	1.5	12

5	2	12
6	2.5	12
7	3	12
8	3.5	12
9	4	12
10	4.5	12
11	5	12
12	5.5	12
13	6	12
14	6.5	12
15	7	12
16	7.5	12
17	8	12
18	8.5	12
19	9	12
20	9.5	12
21	10	12

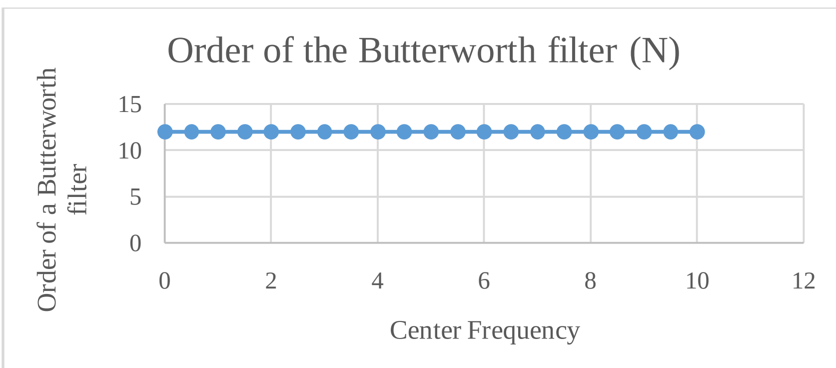

Fig. 9 Order of Butterworth filter for different center frequencies (Tuning element)

Table. 2 shows the effect of center frequency on the order of the Butterworth filter. As it is shown in the table and figure, the center frequency or tuning element of the filter is changing but order of the filter remains same. The effect of center frequency on the order of the filter can also be studied by using different filters like Chebyshev or Elliptic filters.

The effect of the center frequency on the order of Elliptic filters can also be studied. Table. 3 and Fig. 10 shows the effect of center frequencies on the order of Elliptic filters. Here again it can be said that order of the filter remain same for different center frequencies.

Table. 3 Effect of center frequency (Tuning element) on the order of Elliptic filters

Sr. no.	Center frequency	Order of Elliptic filters
1	0	4
2	1	4
3	2	4
4	3	4
5	4	4
6	5	4
7	6	4

8	7	4
9	8	4
10	9	4
11	10	4

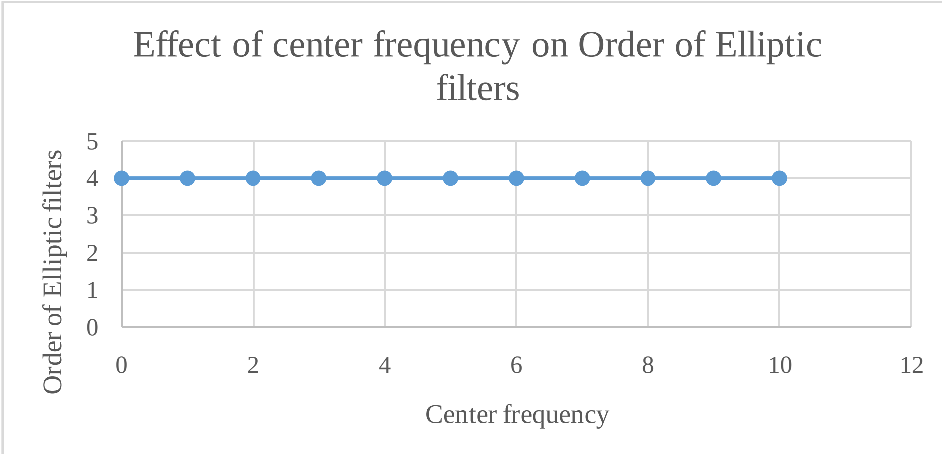

Fig. 10 Order of Elliptic filter for different center frequencies (Tuning element)

VII. Conclusion

So order of the filters for different center frequencies were investigated for different filters. And it can be concluded that for different center frequencies, order of the filter always remains same. And by changing its center frequencies filters are being tuned to different frequencies. So in other words it can be said that in tunable filters, by tuning it to different frequencies order of the filter does not change. It means in tunable filters, pass band and stop band frequencies are being changed without changing the design procedure of the filter.

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