Various Genetic Approaches for Solving Single and Multi-Objective Optimization Problems: A Review

Priti Punia
Thapar University, Patiala,
India

Maninder Kaur
Assistant Professor,Thapar University, Patiala,
India

Abstract -Optimization is a discipline of dealing with those kinds of problems where one has to minimize or maximize one or more objectives that are functions of some integer or real variables without exploiting the given constraints. The Single objective and multi-objective optimization problem is to optimize a problem consisting of single objective function only and multi-objective functions. The optimization techniques have a major role in enhancing the efficiency of any domain in science and engineering. For decades scientists are researching to emerge out with modified and faster techniques. According to the literature it has been seen that various numerical and search techniques have played a major role in solving optimization problems. The paper presents a survey on various variations of genetic approaches that have been applied to solve single-objective and multi-objective optimization problems.

Keywords- single objective optimization problem, genetic algorithm, multi-objective optimization problem, numerical methods, objectivization.

1.   Introduction

The optimization is a significant tool in analysis of physical system and decision science. This is very much related to our real life problems. For instance, airline companies does scheduling in order to minimize the cost. Manufacturers aim for maximizing the efficiency in the design and various operations of their production sequence. Nature optimizes. Physical systems tend to reach a state of minimum energy. The molecules in any isolated chemical system tend to react with each other till total potential energy of their electrons is   minimized. The rays of light follows path through which the travelling time gets minimized. Optimization came into existence in 1940s, when George Dantzig used some mathematical techniques and generated programs for scheduling timetables for military application. Today, optimization consists of wide variety of techniques from Artificial Intelligence, fuzzy math, operations research and computer science. In optimization problems, solutions are needed to be found which are optimal or near-optimal with respect to some desired aim. Usually, the optimization problems are not solved in just one step, rather a sequence of steps are to be followed for problem solving. Commonly used steps are to recognize and define problems, construct and to solve models, and evaluate and to implement solutions. The steps include a) to check for the need of optimization b) Choosing design variables c) Formulating constraints d) Formulation of objective function e) setting up variable bounds f) choosing an optimization algorithm g) obtaining solution or solutions.

The objective [2] depends on certain characteristics of the system, called variables or unknowns. The aim is to find values of the variables that will optimize the objective. Generally the variables are restricted, or constrained, in these optimization problems. For instance, quantities such as electron density in a molecule and the interest rate on a loan can never be negative [2]. The decision variables may get values from discrete sets, bounded and additional constraints on basic resources, such as capital, labour, or supplies, restricting the possible alternatives that are taken feasible [3].

Combinatorial optimization problems are concerned with the efficient allocation of limited resources to meet goals. Possible objectives of a planning or optimization process are either to find an optimal solution of the problem or to find out a solution that is better than some predefined threshold (the current solution). In this paper the optimization problems which are relevant for modern heuristics are considered. Two important properties of optimization are considered 1) locality and 2) decomposability. The locality of a problem is exercised by local search methods, whereas the decomposability is taken care of by recombination-based search methods. There are mainly two types of classical optimization techniques namely: 1) Single-variable optimization 2) Multi-variable optimization. The multi-variable optimization is further divided into three parts named as: i) with no constraints ii) with equality constraints iii) with inequality constraints.

In single-objective optimization (SOO) problem deals with the maximization or minimization of the objective function based upon a single variable given a constraint or an unconstrained problem. The SOO problems have a single variable in the given objective function. The function may vary according to the different values of that variable. The function may have i) Relative or Local Minimum ii) Relative or Local Maximum iii) Absolute or Global Minimum iv) Absolute or Global Maximum. The applications of SOO are related to less complex real time problems. However, at small levels too optimization is needed.

In Multi-objective (or multi-criteria or multi-attribute) optimization (MOO), two or more conflicting objectives are simultaneously optimized with respect to a given set of constraints. Although, in real-world problems many times improvement in one objective leads to the degradation of another. The applications of MOO can be easily seen in the field of network analysis, aircraft design, bioinformatics, oil and gas industry, automobile design, product and process design and many more fields. Summing up, the optimization problems have the following characteristics [3]:

- Availability of different decision alternatives.
- Number of available decision alternatives limited by additional constraints.
- Different effect by each decision alternative on the evaluation criteria.

This paper is categorized into the following three sections:
Section I)Problem Formulation. Section II) Description of various approaches on single objective and multi-objective optimizarion.
Section III) Conclusions.

2. Problem Formulation

The optimization problems may be categorized into Single Variable and Multi-Variable Optimization problems.

2.1 Single objective optimization problem
A single-objective optimization problem (SOOP) has the objective function ($f(x')$), which must be minimized or maximized and a number of constraints ($g(x')$). Equation (1) shows the formula of the SOOP in its general form.

$$\text{minimize } f(x')$$
$$\text{s.t.} \quad g_j(x') \geq 0 \qquad \text{where } (j = 1,.....,m) \tag{1}$$
$$x' \in X \subset R^n$$

where x' is a vector of n decision variables, $x' = (x_1, x_2,........,x_n)^T$, and X represents a feasible region.

2.2 Multi-objective optimization problem
Similarly, multi-objective optimization problems (MOOP) with a number of objective functions are shown in equation (2).

($\mathbf{f}(x') = (f1(x'); f2(x'),......, fk(x'))T$) can be stated as follows:

$$\text{minimize } f(x') = (f_1(x'), f_2(x'),......, f_k(x'))^T$$
$$\text{s. t.} \quad g_j(x') \geq 0 \qquad \text{where } (j = 1,......,m)$$
$$\tag{2}$$
$$x' \in X \subset R^n$$

The scalar concept of "optimality" cannot be applied directly in the multi-objective model. Thus the notion of Pareto optimality has to be entertained. Essentially, a vector $x^* \in S$ is said to be Pareto optimal for a multi-objective problem if all other vectors $x \in S$ have a higher value for at least one of the objective functions f_i, with $i = 1, . . . ,n$, or have the same value for all the objective functions.

The formal definitions for multi-objective optimization problem are as following [4]:
- A point x^* is said to be a *weak* Pareto optimum or a *weak* efficient solution for the multi-objective problem if and only if there is no $x \in S$ such that $f_i(x) < f_i(x^*)$ for all $i \in \{1, . . . ,n\}$. Function $f(x)$ is said to have local or relative minimum at x as in figure 1.
- A point x^* is said to be a *strict* Pareto optimum or a *strict* efficient solution for the multi-objective problem if and only if there is no $x \in S$ such that $f_i(x) \leq f_i(x^*)$ for all $i \in \{1, . . . ,n\}$, with at least one strict inequality. Function $f(x)$ is said to have relative or local maximum at point x as in figure 1.
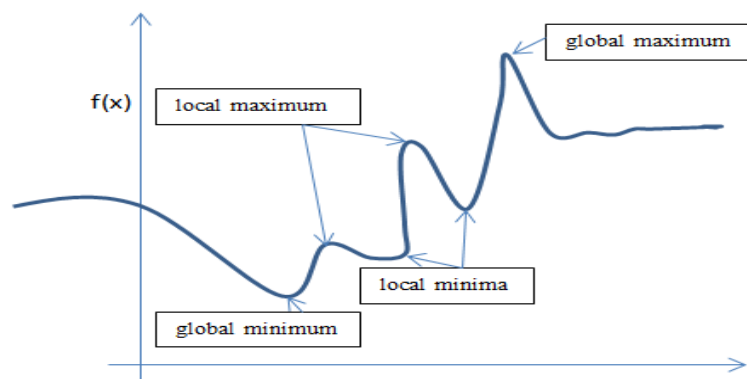

Fig 1: Function *f(x)* having local, global maximum and local, global minimum at point *x*.

3.   Techniques for Optimization

There are a few common techniques which are common to both single and multi- objective optimization problems. However there are some advanced techniques which are applied to multi-objective optimization problems as these problems contain multi-dimensional objectives to be satisfied. As found in literature (in figure 2) the different optimization techniques can be broadly classified into following three categories [4]:

• Calculus-based techniques or Numerical methods.
• Enumerative techniques.
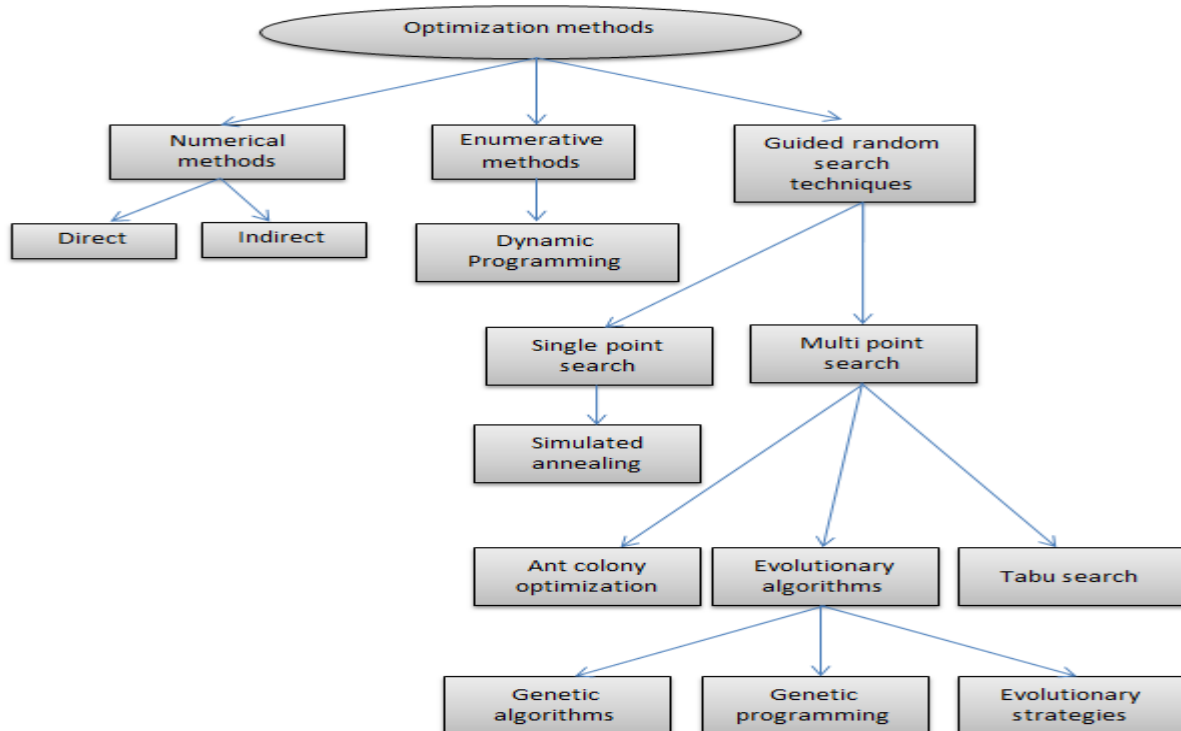• Random techniques.
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Fig. 2 The different search and optimization techniques [4]

Calculus methods, also known as numerical methods use a set of necessary and sufficient conditions which must be satisfied by the solution of the optimization problem [4].Numerical methods further divided into direct and indirect methods. Direct search methods deals with hill climbing in the function space by moving in local gradient direction. Whereas in indirect methods the gradient of the objective function is set to zero and thus solution is get by solving these set of equations. All the calculus based methods assume strictly the existence of derivatives and are local in scope too. These constrains limit their application in real-world problems; however in small class of unimodal problems these can be efficiently used. Enumerative techniques tends to evaluate each and every point of the finite, or discrete infinite, search space to sought optimal solution[4]. A well-known example of enumerative search technique is dynamic programming. Thus in order to search each and every point enumerative needs to break down the problems even of moderate size and complexity into smaller divisions.

        Guided random search techniques are based on the concept of enumerative methods only but with the use of additional information about the search space in order to seek the potential regions faster [4].Guided is further categorized into single-point and multi-point search, means whether it is searching just with one point or with several points at a given time. For single-point search technique, simulated annealing is widely used. It uses thermodynamic evolution in order to find states of minimum energy. For multi-point search, where random choice is used as a tool to guide through a highly explorative search space, genetic algorithms are in trend. They are basically used assuming that a near-optimal solution will be accepted; given the search space is huge, noisy, multimodal as well as discontinuous.

3.1 Overview of Genetic Algorithms (GAs)

Gas are efficient, self-adaptable, self-repairable and robust, nature inspired search and optimization tool. GAs performs well in large, complex and multimodal search space. GAs are modelled based upon the natural genetic principles where the potential solution is encoded in structures known as chromosomes. These make use of problem or domain dependent knowledge to search potential and promising areas; also called *fitness function,* in search space. Each individual or chromosome has a fitness value associated with it, which describes its goodness compared to other individuals in the current population with respect to the solution. The genetic operators such as selection, crossover and mutation are also inspired by the nature and are applied to chromosomes in order to yield better and potential solutions. The sequence of steps taken in a GA to solve any optimization problem is shown in figure 3. GAs are adaptive computational tools

modelled on the mechanics of nature. These efficiently exploit historical information to guess newly upcoming offspring with improved performance. Genetic algorithms are heuristic search methods means it estimates the solution, which can be used for both solving problems and modelling evolutionary systems.

GAs are preferred when the search space is huge, discontinuous, multi-dimensional, multi-modal and noisy. Whereas the classical gradient search techniques are applied where there are tight constraints associated with the given problem. GAs have been found to outperform both the gradient descent method and various forms of random search as literature shows [4, 9, 12, 13].
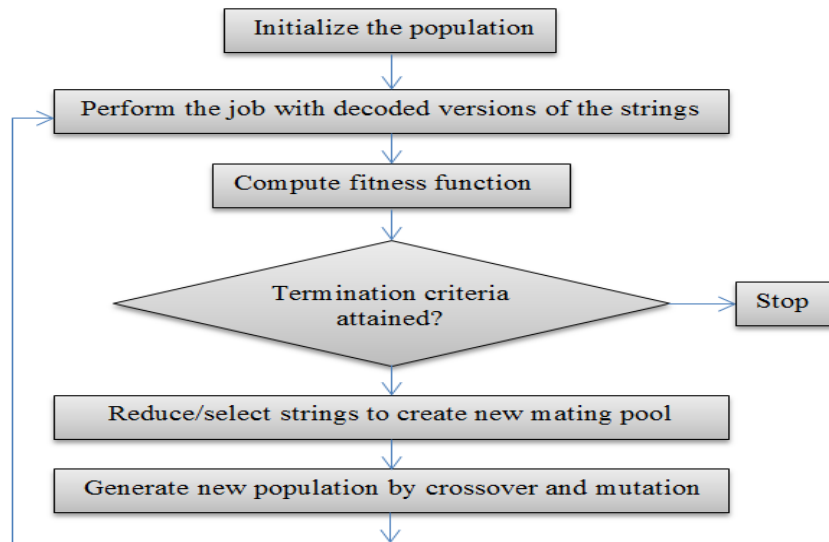
Fig 3: Basic steps of GA

3.1.1 Genetic Algorithms: Basic Principles and Features-Based on the theory of genetics, the GA encodes each individual in the population with a chromosome [8]. This encoding represents the parameters for the objective function being optimized. There are several different techniques for encoding parameters, performing the selection and the alteration stages of the algorithm. The alteration stage is separated into Crossover and Mutation [5]. A GA uses a highly abstract version of evolutionary processes to evolve solutions to some given problems. Each GA operates on a population of given problems. Each GA operates on a population of artificial chromosomes. These are strings in a finite alphabet (usually binary). Each chromosome represents a solution to a problem and has fitness, a real number, which is a measure of the quality of the solution to the particular problem. GA starts with a randomly generated population of chromosomes then carries out a process of fitness-based selection and recombination to produce a successor population, the next generation [6]. During recombination, selecting parent chromosomes and their genetic material is recombined to produce offspring chromosomes. Then these are then passed to the successor population. As this process is iterated, a sequence of successive generations evolves and the average fitness of the chromosomes tends to increase until some stopping criterion is reached. In this way, a GA "evolves" a best solution to the given problem [6]. The information transfer is done from one generation to next through the breeding of the trial solutions selected on the basis of their fitness, and that is why the crossover i defines the defining feature of any GA [8]. The combined effect of crossover and fitness based selection on any population of strings encoded trial solutions basically helps to enhance the occurrence frequency of substrings to further convey their decoded trial solution above-average fitness value, at a rate proportional to difference between the average fitness of all the trial solutions and the average fitness of whole population.
The features of GA which differentiate it from other search methods are as given below:
- GA works with coding of the parameter set, and not with the parameters itself.
- GA starts searching from a population consists of possible solutions, and does not iterate on single solution.
- GA does not use derivatives or other auxiliary information rather uses objective function information.
- GA does not use deterministic rules rather probabilistic transition rules.

A system of nonlinear equations are solved using [5] genetic algorithm techniques. To achieve this propose Gauss-Legendre integration technique is used first to solve the system of nonlinear equations and then GA is used to find the results without converting the nonlinear equations to linear equations. The standard coding scheme is used to accomplish the goal. Hence, the parameters of the search problem are represented as bit strings. The obtained results are confirmed with the results obtained from numerical methods and hence it is shown that GA is an efficient and effective approach to solve the systems of nonlinear equations that arise in the implementation of Gauss-Legendre numerical integration.
        The paper [9] presented a two-space genetic algorithm and also suggested that there is a general technique to solve minimax and robust discrete optimization problems. Robust discrete optimization is a technique for structuring the uncertainty in decision-making process. The goal is to find out a robust solution that has the best worst-case performance over a set of possible scenarios. The proposed algorithm maintains two populations where the first population represents

solution and the second population represents scenarios. Here the individual in one population is evaluated with respect to individuals in the other population. Both the populations evolve simultaneously and they converge to a robust solution and its worst-case scenario. The minimax problems occur in many domains thus the given algorithm has a wide variety of applications. The GA is quite useful in solving two-space minimax problem in a wide-case scenario. In this paper to illustrate the potential of two-space GA, a parallel machine scheduling problem with uncertainty in processing time is solved. For this particular problem, good lower bounds are found and thus algorithm's performance is evaluated. The results confirmed that a two-space genetic algorithm is a very suitable technique for robust and discrete optimization problems. Turkcan [10] used PSGA (Problem search genetic algorithm) for multi-objective optimization. In multi-objective search, the key issues are guiding the search towards the global Pareto set and maintaining diversity. Here a new fitness assignment method is proposed to find a uniformly distributed, well-diversified set of solutions that are very close to the global Pareto set. A multi-objective optimization (MOP) problem formulation is stated as:

$$\text{Min } f(x) = (f_1(x), f_2(x), ..., f_n(x))$$
$$\text{s.t. } x \in X$$

where *x* is a vector of discrete decision variables and X is a set of feasible solutions. As the objectives conflict with each other, a number of solutions known as Pareto-optimal or efficient solutions are found. A real world application of solving tool management and scheduling problems simultaneously in flexible manufacturing systems (FMS) is taken as the problem definition. The proposed fitness assignment method is taken as a combination of non-dominated sorting based method which is mostly used in multi-objective optimization literature and aggregation of objectives method which is popular in the literature of operation research. With the use of PSGA there is no need to do feasibility check hence reducing the significant amount of computation time. The PSGA is applied to single objective optimization problems. The aim in single objective optimization problems is to find a single solution giving the minimum objective function. PSGA was proposed by Storer *et al.*(1992). It is a local search method which provides a new neighbourhood structure defined in the space of possible problem data perturbations. The proposed method, NSAPV, is a composite measure and gives higher fitness values to the non-dominating solutions which are closer to the global Pareto-optimal set, have better aggregated objective function value and less number of neighbors in objective space [9]. A system of linear equations is solved using GA [11] since it is difficult to describe the solution set of a linear system with infinitely many solutions. A system of linear equations is a collection of two or more equations with the same set of unknowns. To avoid the disadvantages of solving large system of linear equations such as inversion of large matrixes, rounding errors, GA is effective and presents an efficient approach to solve the system of linear equations. The coding scheme used is standard one and the parameters of the search space are represented as bit strings. The solution obtained is similar to analytical one. The concept of Hydroinformatics is discussed in paper [12]. This field includes water supply management, design of water distribution networks and systems, water resources, water supply management, watershed water quality management, waste water management, irrigation scheduling. In paper the evolutionary algorithms have been described as a special case of a population based approach. The efficiency of EAs in solving optimization problems has been outlined. The ability to handle mixed type of variables, non-linear constraints, customizing for solving different classes of problems efficiently, and finding multiple trade-off optimal solutions in the presence of multiple conflicting objectives are some of the commonplace in the field of hydroinformatics which have been discussed in the paper.

In paper [13] the focus is on the study of evolutionary algorithms for solving multi-objective optimization problems with a large number of objectives. The proposed algorithm dynamical multi-objective evolutionary algorithm (DMOEA) is compared with the already existing algorithms for solving multi-objective optimization problems. A new definition of optimality (named as L-optimality) is also proposed which not only considers the number of improved objective values but also considers the values of improved objective functions but also takes into account the value of improved objective functions if all the objective functions have the same importance. The Simulations and comparative experiments indicated that the newly developed algorithm MDMOEA can converge to the true L-optimal front and it maintains a widely distributed set of solutions. However, even if it is proved that L-optimal solutions are subsets of Pareto-optimal solutions; even then L-optimal solutions cannot be obtained only by choosing from Pareto-optimal solutions, which utilize MOEAs based on the Pareto-dominance concept.

The paper [14] presents two new approaches for transforming single-objective problem into a multi-objective problems. The multi-objectivization approach is used to translate SOOP into MOOP and then applies EMO. The advantages of multi-objectivization such as reduction of the effect of local optima, increasing the search path to global optimum, or making the problem easier. The two new multi-objectivization approaches based on addition of new objectives are as: 1) Relaxation of the constraints of the problem. 2) Addition of noise to the objective value or the decision variables. These new approaches give more freedom to explore and a reduced likelihood of getting trapped into local optima. The characteristics and effectiveness of the proposed approaches are investigated by comparing the performance on single-objective problems and multi-objective versions of those same problems. Using numerical examples, it is showed that the multi-objective versions produced by relaxing constraints are providing good results and the addition of noise can obtain better solutions when the function considered is multimodal and separable.

The authors [15] proposed a new algorithm for multi-objective optimization called "Neighborhood Cultivation GA (NCGA)". The recent studies such as SPEA2 or NSGA-II, demonstrated that some of the mechanisms are important such as the mechanisms of placement in an archive of excellent solutions, assign of fitness, sharing without parameters, selection and reflection the archived solutions to the search population. Not only NCGA includes these mechanisms but also the neighborhood crossover. NCGA is compared with SPEA2 and NSGA-II with some test functions and it shows that NCGA is a robust algorithm to find Pareto-optimum solutions. The effect of neighbourhood crossover is made clear

through the comparison between the case of using neighborhood crossover and the case of using normal crossover in NCGA. The authors [16] have suggested a nondominated sorting-based multi-objective EA (MOEA), called nondominated sorting genetic algorithm II (NSGA-II). Multi-objective evolutionary algorithms (EAs) which use nondominated sorting and sharing have been criticized mainly for the mentioned reasons such as: 1) computational complexity $O(MN^3)$ (where is the number of objectives and is the population size); 2) the need for specifying a sharing parameter, and 3) nonelitism approach. The proposed algorithm alleviates all the above three difficulties. Particularly, a fast nondominated sorting approach with $O(MN^2)$ computational complexity is presented. A selection operator is also presented that creates a mating pool by combining the parent and offspring populations and selecting the best solutions. Simulation results from difficult test problems show that the proposed NSGA-II, in most of the problems, is able to find much better spread of solutions and better convergence near the true Pareto-optimal front as compared to Pareto-archived evolution strategy and strength-Pareto EA—the two other elitist MOEAs that pay special attention to creating a diverse Pareto-optimal front. Furthermore, the definition of dominance is modified in order to solve constrained multi-objective problems efficiently and effectively. The simulation results obtained from the constrained NSGA-II on a number of test problems, which includes a five-objective seven-constraint nonlinear problem, are matched up with another constrained multi-objective optimizer and NSGA-II offered much better performance.

The authors [17] have presented a new distributed genetic algorithm for multi-objective optimization problems. The proposed approach uses island model with a distributed genetic algorithm and an operation is performed for sharing Pareto-optimum solutions with the total population. The Pareto-optimum solutions are needed to be derived for designers in multi-objective optimization problems. Not only the accuracy but also the diversity of the solutions is needed to be high as the Pareto-optimum solutions are the set of optimum solutions that are in relationship of trade-off. Indexes are introduced that can evaluate the performance of the algorithm. The indexes taken are population size, error, coefficient of variation and cover rate. These can be applied the problems that have more than three objectives to be achieved. High accuracy is achieved by the effect of the distributed population and the high diversity of solutions is achieved by the sharing effect. The numerical examples which have more than three functions are taken as test problems to examine the effects. In paper [18] proposed a novel parallel hybrid algorithm which combines multi-objective and single objective genetic algorithm. The results confirmed that this approach outperforms traditional parallel versions of multi-objective genetic algorithm. This algorithm is proposed as the literature shows that the majority of the multi-objective genetic algorithms are computationally expensive, thus they are often parallelized. In this paper the single objective (SOGA) evolutionary algorithm is combined with multi objective evolutionary algorithm (MOGA) in heterogeneous island model and has it has outperformed the traditional island model. The experiments showed that adding SOGA with island model can be more effective than adding MOGA island, thus it leads to better utilization of computational resources. Also in some of the cases it has reduced the need for function evaluation during evolution, and it leads to the reduction in run-time of the optimizer.

4. Conclusion

The literature shows that the genetic algorithms have grown in popularity to solve various single as well as multi-objective optimization problems in diverse scientific research subjects. This paper reports few selected examples of great optimization work simplification with quite acceptable results. In each and every case, the genetic algorithm is well adapted to the considered problems. Various new approaches have been proposed by many authors [12, 13, 14, 15, 16, 17] to solve the single as well as multi-objective optimization problems. Sometimes single objective problem itself is converted to multi-objective problems by adding additional objectives in order to apply multi-objective techniques to the translated problem in order to gain better and efficient results [14]. In some cases it is necessary to make an effort to parallelize GA (like in [17]) in order to obtain the results of complex problems in less amount of time and cost. However, GAs do not demand a previous or additional knowledge (derivatives) of the function being optimized, but it is necessary to have an idea that a global optimal exists. Generally, the genetic algorithms are shown as an excellent option for the global robust search of an optimal value in non-linear and multi-dimensional functions.

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