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Some Aspects of Fuzzy Boolean Algebras Formed by the Fuzzy Subsets of a Finite Set

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Abstract— The aim of this article is to study the behaviour of the fuzzy Boolean algebras formed by the fuzzy subsets of a finite set. The properties of homomorphism, isomorphism and automorphism of the fuzzy Boolean algebras have been investigated. Further, the ideal and filter of the fuzzy Boolean algebras have also been observed with their characteristics.

Keywords— Fuzzy Boolean algebra, scalar multiplication, homomorphism, ideal, filters

I. INTRODUCTION

It was accepted that the fuzzy sets cannot form Boolean algebra as it is unable to satisfy all the properties of Boolean algebra. The complement laws are not hold by the fuzzy sets. That is if $\underset{\sim}{A}$ is a fuzzy set and $\underset{\sim}{A}'$ is its complement then, $\underset{\sim}{A} \cup \underset{\sim}{A}' \neq the\ universal\ set,\ \underset{\sim}{A} \cap \underset{\sim}{A}' \neq the\ empty\ set.$

Hence, though the other laws of Boolean algebra are satisfied by fuzzy sets they cannot form Boolean algebra.

In this regard, in [11] an attempt was made by introducing a kind of family of fuzzy subsets of a finite set. After the redefinition of the complement operation, it was established that this family of fuzzy subsets hold the complement law of Boolean algebra and hence can form a fuzzy Boolean algebra. The properties like Homomorphism and Isomorphism are studied in Mathematics in order to extend to the insights from one phenomenon to others. If two objects are isomorphic, then any property that is preserved by an isomorphism and that is true of one of the objects is also true of the other. Isomorphism preserves all the structural properties of algebras. If two Boolean algebras are not isomorphic, it means that a structural property of one of the algebras that is not shared by the other. In mathematical order theory, an ideal is a special subset of a partially ordered set (poset). Although this term was derived from the notion of a ring ideal of abstract algebra, it has subsequently been generalized to different notion. Ideals are of great importance for many constructions in order and lattice theory. Filters and ideals play an important role in several mathematical disciplines like algebra, logic, measure theory etc. The Stone's theorem is fundamental to the deeper understanding of Boolean algebra that emerged in the first half of the 29th century. This theorem was first proved by Stone in 1936. The aim of this article is to continue the study of the characteristics of the fuzzy Boolean algebras formed by the fuzzy subsets of a finite set that was introduced in [11]. In order to make this article self-sufficient we recall some basic definitions and results at the beginning. The next section is composed of some subsections having the observations on the behaviours of the fuzzy Boolean algebras. Firstly, the scalar multiplication of the fuzzy Boolean algebras is defined. The properties of homomorphism and isomorphism have been investigated on the basis of the scalar multiplication. The Stone's representation theorem is also proved in case of the fuzzy Boolean algebras. The dual homomorphism of a fuzzy Boolean algebra has also been observed. The next it concerned with the observations of ideal and filters of a fuzzy Boolean algebra along with their properties. At the end of this article the automorphism property of the fuzzy Boolean algebra is investigated with the help of the concept of permutation of a set.

II. PRELIMINARIES

This section lists some basic definitions and concepts of Boolean algebra which have also used in case of fuzzy Boolean algebra in this article. They are as follows:

A. Homomorphism and Isomorphism

A Boolean homomorphism is a mapping f from a Boolean algebra B to a Boolean algebra A written as $f : B \rightarrow A$ such that:

a. $f(a \wedge b) = f(a) \wedge f(b),$

b. $f(a \vee b) = f(a) \vee f(b),$

c. $f(a') = (f(a))', \forall a, b \in B$

If the followings are hold instead of the above then it is called a dual homomorphism:

$$f(a \wedge b) = f(a) \vee f(b),$$

$$f(a \vee b) = f(a) \wedge f(b),$$

$$f(a') = \left(f(a)\right)'.$$

The image of f, written $\operatorname{Im} f$ is the set of image points in A known as a homomorphic image or range of homomorphism.

The Kernel of f, written $\ker f$ is the set of elements in B which maps into $0 \in A$, $\ker f : \{i \in B : f(i) = 0\}$.

Isomorphism: A one-to-one and onto homomorphism is called isomorphism.

Automorphism: An isomorphism from a Boolean algebra onto itself is called an automorphism.

Epimorphism: A onto homomorphism is called an epimorphism, i.e., every element of A is equal to $f(a)$ for some a in B.

B. Ideals and Filters of a Boolean algebra

An ideal in a Boolean algebra B is a subset I of B such that:

a. $0 \in I$,

b. if $p \in I$ and $q \in I$, then $p \vee q \in I$,

c. if $p \in I$ and $q \in B$, then $p \wedge q \in I$.

Every Boolean algebra B has a trivial ideal, namely the set $\{0\}$, consisting of 0 alone; all others ideals of B are called non-trivial. Every Boolean algebra B has an improper ideal, namely B itself; all other ideals are called proper.

A filter in a Boolean algebra B is a subset F of B such that:

i. $1 \in F$,

ii. if $p \in F$ and $q \in F$, then $p \wedge q \in F$,

iii. if $p \in F$ and $q \in B$, then $p \vee q \in F$.

A filter is also called the dual of an ideal.

C. The Stone's representation theorem

This theorem states that any Boolean algebra is isomorphic to a power set algebra $\left(P(S), \cap, \cup, \square, \varnothing, S\right)$ for some set S.

D. Permutation of a set

A one to one mapping f of the set $A = \{1, 2, \cdots, n\}$ onto itself is called a permutation. The permutation f is defined by:

$$f = \begin{pmatrix} 1 & 2 & . & . & . & n \\ p_1 & p_2 & . & . & . & p_n \end{pmatrix} \text{ or } f = (p_1 p_2 \ldots . p_n)$$

Where, $p_i = f(i)$ and $i \in A$.

III. SCALAR MULTIPLICATION OF THE FUZZY BOOLEAN ALGEBRAS

In this section, first we define the scalar multiplication of the fuzzy Boolean algebras of B, and then we shall try to establish the properties of homomorphism and isomorphism.

As introduced in the article [11], for a finite set $E = \{x_0, x_1, x_2 \ldots, x_{n-1}\}$ with n elements with the set M of membership values, such that,

$$M = \left\{0, \frac{1}{p}, \frac{2}{p}, \frac{3}{p}, \ldots, \frac{p-1}{p}, \frac{p}{p} = 1\right\}$$

$$= \{0, h, 2h, 3h, \ldots, (p-1)h, ph\}, \text{ where, } h = \frac{1}{p} \text{ and } p \text{ is any number}$$

Then for any mapping $E \to \{0, kh\}$, where $1 \le k \le p$, forms a Boolean algebra. This is called fuzzy Boolean algebra, as it formed by fuzzy subsets.

Hence, for the mappings $E \to \{0,h\}$, $E \to \{0,2h\},...E \to \{0,ph\}$ we can obtain p -numbers of Fuzzy Boolean algebras denoted by $B_1, B_2, B_3,...B_p$ respectively. Let us denote this set of fuzzy Boolean algebras as B , which is: $B = \{B_1, B_2, B_3,....B_p\}$.

Now, we define the scalar multiplication among the fuzzy Boolean algebras based on the membership values of the elements of the fuzzy subsets. For, any two fuzzy Boolean algebras $B_r, B_s \in B,$ where, $1 \le r \le p, 1 \le s \le p$ and $r < s$, the scalar multiplication is defined as:

$$\mu_{B_r}(x_i) = \frac{r}{s}\mu_{B_s}(x_i), \forall x_i \in E,$$

Where, $i = 0,1,...n.$ again, $\mu_{B_r}(x_i)$ and $\mu_{B_s}(x_i)$ are the membership values of the i^{th} element of the fuzzy subsets of B_r and B_s respectively.

A. Homomorphism and Isomorphism of the fuzzy Boolean algebras

As in Boolean algebra, the same definition of homomorphism will be used in case of fuzzy Boolean algebra except that in this case it is based on the membership values.

1. Example: A Let B_k be a fuzzy Boolean algebra and a_0 be an arbitrary element of B_k. Let, A be the set of elements, where $A = \{a \wedge a_0 : \forall a \in B\}$. Then the mapping f defined by $f(a) = a \wedge a_0$ is a fuzzy Boolean homomorphism from B onto A , if the top element (g) in A is defined to be the elements a_0 and a' in A is defined by: $\mu_{a_0}(x) - \mu_a(x), \forall x \in E$.

The homomorphic image A of f is itself a fuzzy Boolean algebra with the top (g) and the bottom (0):

$f(0) = 0 \wedge a_0$ and $f(g) = g \wedge a_0 = a_0$.

Now, based on the scalar multiplication of fuzzy Boolean algebras which have defined in above, we can obtain the following theorems:

B. Theorem: The fuzzy Boolean algebras of B are isomorphic to each other.

Proof: Let any two fuzzy Boolean algebras $B_r, B_s \in B,$ where, $1 \le r \le p, 1 \le r \le p$ and $r < s$, which are written as:

$$B_r = \{I_0, I_1, I_2......I_{n-2}, I_{n-1}\} \;\; and \;\; B_s = \{I'_0, I'_1, I'_2......I'_{n-2}, I'_{n-1}\}$$

Both the fuzzy Boolean algebras have the same numbers of elements which is 2^n.

Now from the definition of scalar multiplication, we can define a function f from F_r to F_s such that $f(I_0) = I'_0, f(I_1) = I'_1,.....f(I_{n-1}) = I'_{n-1}$; which implies that f is one to one and onto function.

Now, for any two elements $I_a, I_b \in B_r$, we get:

$f(I_a \wedge I_b) = f(I_c) = I'_c, I_c \in B_r$ and $I'_c \in B_s$ and $f(I_a) \wedge f(I_b) = I_c \in B_s$.

Hence, $f(I_a \wedge I_b) = f(I_a) \wedge f(I_b)$

Again, for any two elements $I_a, I_b \in B_r$, we get:

$f(I_a \vee I_b) = I_c \in B_s$ and $f(I_a) \vee f(I_b) = I_c \in B_s$.

Hence, $f(I_a \vee I_b) = f(I_a) \vee f(I_b)$

Also, $(f(a')) = f(a')$. Hence, it is proved that the two fuzzy Boolean algebras are isomorphic to each other, written as $B_r \cong B_s$.

Similarly, it can be proved that: $B_1 \cong B_2 \cong B_3.... \cong B_p$.

C. Theorem: The relation isomorphism of the fuzzy Boolean algebras forms an equivalence relation.

Proof: For any three fuzzy Boolean algebras $B_l, B_m, B_n \in B$ and from the definition of scalar multiplication among them we get –

Reflexivity: for all B_l, $B_l \cong B_l, \forall B_l \in B$

Symmetric: if $B_l \cong B_m$ then $B_m \cong B_l$

Transitivity: if $B_l \cong B_m$ and $B_m \cong B_n \Rightarrow B_l \cong B_n$

Hence, the isomorphism relation satisfies all the properties of an equivalence relation. So, it is an equivalence relation.

D. The Stone's representation theorem

The fuzzy Boolean algebra also satisfies the Stone's representation theorem. That is, any fuzzy Boolean algebra is isomorphic to the ordinary power set of universal set of the fuzzy Boolean algebra. This is proved in the following example:

1) Example

Let, $E=\{x_0,x_1,x_2\}$ be the universal set. Now, considering the fuzzy Boolean algebra B_1, written as follows:

$$B_1=[0=\{(x_0,0),(x_1,0),(x_2,0)\},$$
$$1=\{(x_0,0),(x_1,0),(x_2,h)\},$$
$$4=\{(x_0,0),(x_1,h),(x_2,0)\},$$
$$5=\{(x_0,0),(x_1,h),(x_2,h)\},$$
$$16=\{(x_0,h),(x_1,0),(x_2,0)\},$$
$$17=\{(x_0,h),(x_1,0),(x_2,h)\},$$
$$20=\{(x_0,h),(x_1,h),(x_2,0)\},$$
$$21=\{(x_0,h),(x_1,h),(x_2,h)\}]$$

The ordinary power set of E is denoted by $P(E)$, where:

$$P(E)=\{\varnothing,\{x_0\},\{x_1\},\{x_2\},\{x_0,x_1\},\{x_0,x_2\},\{x_1,x_2\},\{x_0,x_1,x_2\}\}$$

The Hass diagram of $P(E)$ is shown in Fig. 1. bellow:

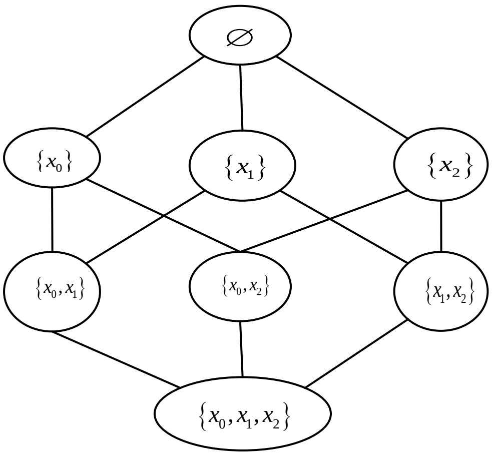

Fig. 1 The Hass diagram of $P(E)$

The Hass diagram of B is shown in the Fig. 2. bellow:

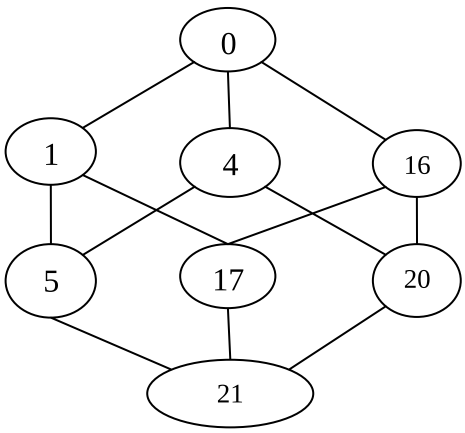

Fig. 2 The Hass diagram of B

If we consider the mapping $f:B\rightarrow P(E)$ defined by:

$f(0)=\varnothing$, $f(1)=\{x_0\}$, $f(4)=\{x_1\}$, $f(5)=\{x_0,x_1\}$, $f(16)=\{x_2\}$, $f(17)=\{x_0,x_2\}$, $f(20)=\{x_1,x_2\}$,
$f(21)=\{x_0,x_1,x_2\}$.

This mapping is one-to-one and onto. All the conditions of homomorphisms are satisfied. Hence, f is an isomorphism. That is the fuzzy Boolean algebra B is isomorphic to the ordinary power set of universal set E of the fuzzy Boolean algebra B.

E. Dual homomorphism of the fuzzy Boolean algebras

If a Boolean homomorphism satisfies the dual properties of homomorphism then it is called a dual homomorphism. Applying the same concept in the fuzzy Boolean algebras, we obtain the following result:

F. Theorem: For any fuzzy Boolean algebra $B_k \in B$, the mapping $f: B_k \to B_k$ defined by: $f(p)=p', \forall p \in B_k$, is a dual homomorphism.

Proof: Let, p and q be any two elements (i.e. fuzzy subsets) of B_k then,

$$f(p \wedge q) = (p \wedge q)' = p' \vee q' \text{ [by De-Morgan's Law]}$$
$$= f(p) \vee f(q).$$

Similarly,

$$f(p \vee q) = (p \vee q)' = p' \wedge q'$$
$$= f(p) \wedge f(q).$$

Again, $f(p') = f(f(p)) = (f(p))'$.

Hence, all the properties of dual homomorphism are satisfied by f. So, f is a dual homomorphism. Again, since it is one-to-one and onto itself, so it is also a dual automorphism.

G. Remark: The fuzzy Boolean homomorphism is order preserving. On the other hand, the dual homomorphism is reverse order preserving.

H. Ideals and Filters of the fuzzy Boolean algebras

Now, we observe the ideal and filter of the fuzzy Boolean algebras.

1. Ideal

If B and C are two fuzzy Boolean algebras and $f: B \to C$ is an epimorphism, then C is an ideal if the following properties are satisfied:

a. $f(0) = 0 \in C$,

b. if $p \in C$ and $q \in C$, then $p \vee q \in C$,

c. if $p \in C$ and $q \in B$, then $p \wedge q \in C$.

2. Example

Considering the fuzzy Boolean algebra B_1 as written in the example of the section D. where:

$$B_1 = \{0,1,4,5,16,17,20,21\}$$

Again, B_2 is a fuzzy Boolean algebra as follows:

$$B_2 = [0 = \{(x_0,0),(x_1,0),(x_2,0)\},$$
$$1 = \{(x_0,0),(x_1,0),(x_2,h)\},$$
$$4 = \{(x_0,0),(x_1,h),(x_2,0)\},$$
$$5 = \{(x_0,0),(x_1,h),(x_2,h)\}],$$

Let $f: B_1 \to B_2$ is an epimorphism defined by $f(B_1) = \alpha \wedge B_1 = B_2$, where $\alpha = 5$ is a fixed element of B_1.

$$f(0) = 5 \wedge 0 = 0,\ f(1) = 5 \wedge 1 = 1,\ f(4) = 4,$$
$$f(5) = 5,\ f(16) = 0,\ f(17) = 1,\ f(20) = 4, \quad \backslash$$
$$f(21) = 21.$$

Here,

i. $0 \in B_2$

ii. $4,5 \in B_2 \Rightarrow 4 \vee 5 = 5 \in B_2$

iii. $4 \in B_2, 16 \in B_1 \Rightarrow 4 \wedge 16 = 0 \in B_1$

Hence, B_2 is an ideal.

3. Kernel

If B and C are two fuzzy Boolean algebras and $f : B \to C$ is an epimorphism, then $\ker f$ is an ideal if the following properties are satisfied:

a. $0 \in \ker f$,

b. if $p \in \ker f$ and $q \in \ker f$, then $p \vee q \in \ker f$,

c. if $p \in \ker f$ and $q \in B$, then $p \wedge q \in C$.

4. Example of Kernel

In the above example, $\ker f = \{0, 16\}$ is an ideal because:

i. $0 \in \ker f$

ii. $0, 16 \in \ker f \Rightarrow 0 \vee 16 = 16 \in \ker f$

iii. $16 \in \ker f$ and $5 \in B_1$

$\Rightarrow 16 \wedge 5 = 0 \in \ker f$

Hence, $\ker f$ is an ideal.

I. Theorem: The intersection of two ideals of a fuzzy Boolean algebra is again an ideal.

Proof: If I_1 and I_2 be two ideals of a fuzzy Boolean algebra B. Then, $0 \in I_1$ and $0 \in I_2$

Since, I_1 and I_2 are non-empty subset of B, so $I_1 \cap I_2$ is also a non-empty subset of B.

Suppose, $p, q \in I_1 \cap I_2$, which implies that:

$p, q \in I_1$ and $p, q \in I_2 \Rightarrow p \vee q \in I_1$ and $p \vee q \in I_2$ [as I_1 and I_2 are ideals of B]

$\Rightarrow p \vee q \in I_1 \cap I_2 \longrightarrow$ (1)

Again, as $p \in I_1 \cap I_2 \Rightarrow p \in I_1$ and $p \in I_2$

If $s \in B_i$ then $p \wedge s \in I_1$ and $p \wedge s \in I_2 \Rightarrow p \wedge s \in I_1 \cap I_2 \longrightarrow$ (2)

Hence, from (1) and (2), it is proved that $I_1 \cap I_2$ is an ideal of B.

J. Theorem: The union of two ideals of a fuzzy Boolean algebra is not necessarily is an ideal.

Proof: The proof is obvious, can be seen from the above example that B_2 and $\ker f$ are two ideals of B_1, but their union is not an ideal.

K. Filter of a fuzzy Boolean algebra

Now, we discuss the filter of a fuzzy Boolean algebra.

1. Filter

Considering B and C are two fuzzy Boolean algebras $f : B \to C$ is a mapping then the range F is a filter if the following properties are satisfied:

i. $g \in F$,

ii. if $p \in F$ and $q \in F$, then $p \wedge q \in F$,

iii. if $p \in F$ and $q \in C$, then $p \vee q \in F$.

2. Example of a filter

Let B_2 is a fuzzy Boolean algebra defined by:

$$B_2 = [0 = \{(x_0, 0), (x_1, 0), (x_2, 0)\},$$
$$2 = \{(x_0, 0), (x_1, 0), (x_2, 2h)\},$$
$$8 = \{(x_0, 0), (x_1, 2h), (x_2, 0)\},$$
$$10 = \{(x_0, 0), (x_1, 2h), (x_2, 2h)\},$$
$$32 = \{(x_0, 2h), (x_1, 0), (x_2, 0)\},$$
$$34 = \{(x_0, 2h), (x_1, 0), (x_2, 2h)\},$$
$$40 = \{(x_0, 2h), (x_1, 2h), (x_2, 0)\},$$
$$42 = \{(x_0, 2h), (x_1, 2h), (x_2, 2h)\}]$$

Then, the mapping $f : B_2 \rightarrow B_2$ is defined by $f(x) = \alpha \vee x, \forall x \in B_2$ and α is a fixed element of B_2 If $\alpha = 8$, then the range of the mapping is $F = \{8, 10, 40, 42\}$. Here F is a filter since:

i. $f(g) = f(42) = 42 \in F$

ii. $8 \in F, 10 \in F \Rightarrow 8 \wedge 10 = 8 \in F$

iii. $8 \in F, 34 \in B \Rightarrow 8 \vee 34 = 42 \in F$

L. Theorem: The intersection of two filters of a fuzzy Boolean algebra is again a filter.

Proof: The proof is obvious.

M. Theorem: The union of two filters of a fuzzy Boolean algebra is not necessarily a filter.

Proof: The proof is obvious can be derived from example.

N. Automorphism of the fuzzy Boolean algebras

This section observes the automorphism property of a fuzzy Boolean algebra. An automorphism of a fuzzy Boolean algebra is an isomorphism from a Boolean algebra into itself. Before going to automorphism, we discuss the permutations of a set.

1. *Example: If* $E = \{x_0, x_1, x_2\}$, then the permutations of the set E are:

$$\delta = \begin{pmatrix} x_0 & x_1 & x_2 \\ x_0 & x_1 & x_2 \end{pmatrix}, \ \delta_1 = \begin{pmatrix} x_0 & x_1 & x_2 \\ x_1 & x_2 & x_0 \end{pmatrix}$$

$$\delta_2 = \begin{pmatrix} x_0 & x_1 & x_2 \\ x_2 & x_0 & x_1 \end{pmatrix}, \ \delta_3 = \begin{pmatrix} x_0 & x_1 & x_2 \\ x_0 & x_2 & x_1 \end{pmatrix}$$

$$\delta_4 = \begin{pmatrix} x_0 & x_1 & x_2 \\ x_2 & x_1 & x_0 \end{pmatrix}, \ \delta_5 = \begin{pmatrix} x_0 & x_1 & x_2 \\ x_1 & x_0 & x_2 \end{pmatrix}$$

Considering the fuzzy Boolean algebra B_1 as given in the example of section D, then with respect to the permutation δ as illustrated in above, the isomorphism $F_\delta : B_1 \rightarrow B_1$ defined by:

$$F_\delta(0) = 0, F_\delta(1) = 1, F_\delta(4) = 4, F_\delta(5) = 5, F_\delta(16) = 16, F_\delta(17) = 17, F_\delta(20) = 20, F_\delta(21) = 21.$$

-is an automorphism.

With respect to the permutation δ_1, the isomorphism $F_{\delta_1} : B_1 \rightarrow B_1$ defined by:

$$F_{\delta_1}(0) = 0, F_{\delta_1}(1) = 4, F_{\delta_1}(4) = 16, F_{\delta_1}(5) = 20, F_{\delta_1}(16) = 1, F_{\delta_1}(17) = 5, F_{\delta_1}(20) = 17, F_{\delta_1}(21) = 21.$$

-is an automorphism.

With respect to the permutation δ_2, the isomorphism $F_{\delta_2} : B_1 \rightarrow B_1$ defined by:

$$F_{\delta_2}(0) = 0, F_{\delta_2}(1) = 16, F_{\delta_2}(4) = 1, F_{\delta_2}(5) = 17, F_{\delta_2}(16) = 4, F_{\delta_2}(17) = 20, F_{\delta_2}(20) = 5, F_{\delta_2}(21) = 21.$$

-is an automorphism.

With respect to the permutation δ_3, the isomorphism $F_{\delta_3} : B_1 \rightarrow B_1$ defined by:

$$F_{\delta_3}(0) = 0, F_{\delta_3}(1) = 4, F_{\delta_3}(4) = 1, F_{\delta_3}(5) = 5, F_{\delta_3}(16) = 16, F_{\delta_3}(17) = 20, F_{\delta_3}(20) = 17, F_{\delta_3}(21) = 21.$$

-is an automorphism.

With respect to the permutation δ_4, the isomorphism $F_{\delta_4} : B_1 \rightarrow B_1$ defined by:

$$F_{\delta_4}(0) = 0, F_{\delta_4}(1) = 16, F_{\delta_4}(4) = 4, F_{\delta_4}(5) = 20, F_{\delta_4}(16) = 1, F_{\delta_4}(17) = 17, F_{\delta_4}(20) = 5, F_{\delta_4}(21) = 21.$$

-is an automorphism.

We observe that the number of automorphisms of the fuzzy Boolean algebra B_1 is equal to the number of permutations of the elements of the universal set of B_1.

O. Theorem: The number of automorphisms of a fuzzy Boolean algebra $B_k \in B$*, defined by the mapping from* $E = \{x_0, x_1, x_2, ...x_{n-1}\}$ *to* $M = \{0, kh\}$ *is* $\lfloor p$*, where,* $1 \leq k \leq p$*,* $h = \frac{1}{p}$ *and p is any number.*

IV. CONCLUSIONS

In the study of the properties common to all algebraic structures (such as groups, lattices, rings, etc.) and even some of the properties that distinguish one class of algebras from another, Boolean algebras in an essential and natural way. To investigate the structure of a Boolean algebra, it is clear that homomorphism and isomorphism play an important role. In this article, we have studied those characteristics of the fuzzy Boolean algebra introduced in [11]. We have also established an important result for automorphism. We have given some examples in support of our results. It is our clear hope that this work would serve as a foundation for further study of the introduced fuzzy Boolean algebra.

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REFERENCES

[1] Byliński, C. (1990). Functions from a set to a set. *Formalized Mathematics*, *1*(1), 153-164.
[2] Dubois, Didier J. *Fuzzy sets and systems: theory and applications*. Vol. 144. Access Online via Elsevier, 1980.
[3] Frink, O. (1954). Ideals in partially ordered sets. The American Mathematical Monthly, 61(4), 223-234.
[4] Givant, S., Halmos, P.: Introduction to Boolean Algebras. Undergrad. Texts Math. Springer, New York (2009).
[5] Klir, George J., and Bo Yuan. *Fuzzy sets and fuzzy logic*. New Jersey: Prentice Hall, 1995.
[6] Monk, J. D., & Bonnet, R. (1989). *Handbook of Boolean algebras* (Vol. 3). North Holland.
[7] Rosen, K. H., & Krithivasan, K. (1999). *Discrete mathematics and its applications* (Vol. 6). New York: McGraw-Hill.
[8] Sikorski, R., Sikorski, R., Sikorski, R., Sikorski, R., & Mathematician, P. (1969). *Boolean algebras* (Vol. 2). New York: Springer.
[9] Squires, R. J. An introduction to Boolean algebra. Journal of the Institute of Actuaries Students' Society 17 (1964):314.
[10] Stone, M. H. (1936). The theory of representation for Boolean algebras. *Transactions of the American Mathematical Society*, *40*(1), 37-111.
[11] Dwiraj Talukdar, and Sisir Kumar Rajbongshi. "An Introduction to a Family of Fuzzy Subsets forming Boolean algebra." *International Journal of Computer Applications* 68.24 (2013): 1-6.

