



Performance Analysis of $2 \times n$ MIMO Systems in Rayleigh Channels with Maximum Likelihood Equalization Scheme

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Abstract— Channel equalizers are used to reduce the ISI arising from the delay spread or band limitation of the channel by adjusting the pulse shape so that it does not interfere with the neighbouring pulses. Equalization is a million intensive functions in cellular phone receivers. Channel equalization was first used in echo cancellation in telephone networks. The Maximum Likelihood equalizer achieves a much better performance where it determines the sequence of symbols that has most likely been transmitted and it is much similar to the decoding of Convolutional codes. In this paper the performance analysis of $2 \times n$ MIMO Systems in Rayleigh Fading Channels with Maximum Likelihood Equalization Scheme is carried out.

Keywords— Equalizers, ISI, MIMO, Rayleigh, Maximum Likelihood

I. INTRODUCTION TO THE CONCEPT OF EQUALIZATION

Delay spreads causes intersymbol interference (ISI), which can cause an irreducible error when the modulation symbol time is on the same order as the channel delay spread. Signal processing provides a powerful mechanism to counteract ISI. In a broader sense, equalization defines any signal processing technique used at the receiver to alleviate the ISI problem caused by delay spread. Signal processing can also be used at the transmitter to make the signal less susceptible to delay spread, spread-spectrum and multicarrier modulation fall in this category of transmitter signal processing techniques. Mitigation of ISI is required when the modulation symbol time T_s is on the order of the channel's rms delay spread σ_{T_m} . For example, cordless phones typically operate indoors, where the delay spread is small. Since voice is also a relatively low-data rate application, equalization is generally not needed in cordless phones. Higher-data-rate applications are more sensitive to delay spread and generally require high-performance equalizers or other ISI mitigation techniques. In fact, mitigating the impact of delay spread is one of the most challenging hurdles for high-speed wireless data systems.

Equalizer design must typically balance ISI mitigation with noise enhancement, since both the signal and the noise pass through the equalizer, which can increase the noise power. Nonlinear equalizers suffer less from noise enhancement than linear equalizers but typically entail higher complexity. Moreover, equalizers require an estimate of the channel impulse or frequency response to mitigate the resulting ISI. Since the wireless channel varies over time, the equalizer must learn the frequency or impulse response of the channel (training) and then update its estimate of the frequency response as the channel changes (tracking). The process of equalizer training and tracking is often referred to as adaptive equalization, since the equalizer adapts to the changing channel. Equalizer training and tracking can be quite difficult if the channel is changing rapidly. An equalizer can be implemented at baseband, the carrier frequency, or an intermediate frequency. Most equalizers are implemented digitally after A/D conversion because such filters are small, cheap, easily tunable, and very power efficient. The goal of equalization is to mitigate the effects of ISI. However, this goal must be balanced so that, in the process of removing ISI, the noise power in the received signal is not enhanced.

II. EQUALIZER TYPES

Equalization techniques fall into two broad categories: linear and non linear. The linear techniques are generally the simplest to implement and to understand conceptually. However, linear equalization techniques typically suffer from more noise enhancement than nonlinear equalizers and hence are not used in most wireless applications. Among non linear equalization techniques, decision-feedback equalization is the most common because it is fairly simple to implement and usually performs well. However, on channels with low Signal to Noise Ratio, the Decision Feedback Equalizer suffers from error propagation when bits are decoded in error, leading to poor performance. The optimal equalization technique is maximum likelihood sequence estimation. Unfortunately, the complexity of this technique grows exponentially with the length of the delay spread, so it is impractical on most channels of interest. However, the performance of the MLSE is often used as an upper bound on performance for other equalization techniques. Equalizers can also be categorized as symbol-by-symbol (SBS) or sequence estimators (SEs). SBS equalizers remove ISI from each symbol and then detect each symbol individually. All linear equalizers are SBS equalizers. Sequence estimators detect sequences of symbols, so the effect of ISI is a part of the estimation process. Maximum likelihood sequence estimation is

the optimal form of sequence detection, but it is highly complex. Linear and nonlinear equalizers are typically implemented using a transversal or lattice structure. The transversal structure is a filter with N-1 delay elements and N taps featuring tunable complex weights. The lattice filter uses a more complex recursive structure [1]. In exchange for this increased complexity relative to transversal structures, lattice structures often have better numerical stability and convergence properties and greater flexibility in changing their length [2]. In addition to the equalizer type and structure, adaptive equalizers require algorithms for updating the filter tap coefficients during training and tracking. Many algorithms have been developed over the years for this purpose. These algorithms generally incorporate trade-offs between complexity, convergence rate, and numerical stability.

III. MAXIMUM LIKELIHOOD EQUALIZATION SCHEME

Maximum-likelihood sequence estimation avoids the problem of noise enhancement because it doesn't use an equalizing filter, instead it estimates the sequence of transmitted symbols. Given the combined pulse-shaping filter and channel response $h(t)$, the MLSE algorithm chooses the input sequence $\{d_k\}$ that maximizes the likelihood of the received signal $w(t)$. Using a Gram-Schmidt orthonormalization process, $w(t)$ can be expressed on a time interval $[0, LT_s]$ as follows

$$w(t) = \sum_{n=1}^N w_n \phi_n(t)$$

where $\{\phi_n(t)\}$ form a complete set of orthonormal basis functions. The number N of functions in this set is a function of the channel memory, since $w(t)$ on $[0, LT_s]$ depends on d_0, \dots, d_L . With this expansion it can be expressed as follows

$$w_n = \sum_{k=-\infty}^{\infty} d_k h_{nk} + v_n = \sum_{k=0}^L d_k h_{nk} + v_n$$

$$\text{where } h_{nk} = \int_0^{LT_s} h(t - kT_s) \phi_n^*(t) dt$$

$$\text{and } v_n = \int_0^{LT_s} n(t) \phi_n^*(t) dt$$

Given a received signal $w(t)$ or equivalently, w^N , the MLSE decodes this as the symbol sequence d^L that maximizes the likelihood function $p(w^N | d^L, h(t))$. That is, the MLSE outputs the sequence as follows

$$\hat{d}^L = \arg \max [\log p(w^N | d^L, h(t))]$$

Thus the MLSE output can be expressed as follows

$$\hat{d}^L = \arg \max \left[2 \operatorname{Re} \left\{ \sum_k d_k^* y[k] \right\} - \sum_k \sum_m d_k d_m^* u[k - m] \right]$$

From the above equation that the MLSE output depends only on the sampler output $\{y[k]\}$ and the channel parameters $u[n-k]=u(nT_s-kT_s)$, where $u(t)=h(t)*h^*(-t)$. Because the derivation of the MLSE is based on the channel output $w(t)$ only prior to matched filtering, the derivation implies that the receiver matched filter with $g_m(t)=h(t)$ is optimal for MLSE detection (typically the matched filter is optimal for detecting signals in AWGN, but the above derivation shows that it is also optimal for detecting signals in the presence of ISI if MLSE is used. The Viterbi algorithm can be used for MLSE to reduce complexity of this equalization technique still grows exponentially with the channel delay spread. A non linear equalization technique with significantly less complexity is decision-feedback equalization.

IV. MIMO SYSTEMS

If the multiple antennas at the transmitter and receiver are present then it is called as multiple-input multiple-output (MIMO) systems. The multiple antennas can be used to increase data rates through multiplexing or to improve performance through diversity. In MIMO systems, the transmit and receive antennas can both be used for diversity gain. Multiplexing exploits the structure of the channel gains matrix to obtain independent signaling paths that can be used to send independent data. Indeed, the initial excitement about MIMO was sparked by the pioneering work of Winters [3], Foschini [4], Foschini and Gans [5], and Telatar [6] predicting remarkable spectral efficiencies for wireless systems with multiple transmit and receive antennas. These spectral efficiency gains often require accurate knowledge of the channel at the receiver- and sometimes at the transmitter as well. In addition to spectral efficiency gains, ISI and interference from other users can be reduced using smart antenna techniques. The cost of the performance enhancements obtained through MIMO techniques is the added cost of deploying multiple antennas, the space and circuit power requirements of these extra antennas (especially on small handheld units), and the added complexity required for multidimensional signal processing. When the MIMO channel bandwidth is large relative to the channel's multipath delay spread, the channel suffers from intersymbol interference; this is similar to the case of SISO channels. There are two approaches to dealing with ISI in MIMO channels. A channel equalizer can be used to mitigate the effects of ISI. However, the equalizer is

much more complex in MIMO channels because the channel must be equalized over both space and time. Moreover, when the equalizer is used in conjunction with a space-time code, the nonlinear and non causal nature of the code further complicates the equalizer design. In some cases the structure of the code can be used to convert the MIMO equalization problem to a SISO problem for which well-established SISO equalizer designs can be used [7, 8, and 9]. An alternative to equalization in frequency-selective fading is multicarrier modulation or orthogonal frequency division multiplexing (OFDM). OFDM techniques for SISO channels, the main premise is to convert the wideband channel into a set of narrowband sub channels that exhibit only flat fading. Applying OFDM to MIMO channels results in a parallel set of narrowband MIMO channels, and the space-time modulation and coding techniques just described for a single MIMO channel are applied to this parallel set. MIMO frequency-selective fading channels exhibit diversity across space, time, and frequency, so ideally all three dimensions should be fully exploited in the signaling scheme.

V. PERFORMANCE ANALYSIS OF BPSK MODULATION WITH 2 X N MIMO SYSTEMS IN RAYLEIGH FADING CHANNEL WITH MAXIMUM LIKELIHOOD SCHEME

The channel is assumed to be a flat fading, in simple terms, it means that the multipath channel has only one tap. So, the convolution operation reduces to a simple multiplication. The channel experienced by each transmit antenna is independent from the channel experienced by other transmit antennas. For the *i*th antenna to *j*th receive antenna, each transmitted symbol gets multiplied by a randomly varying complex number. As the channel under consideration is a Rayleigh channel, the real and imaginary parts are Gaussian distributed having mean and variance. The channel experienced between each transmitter to the receive antenna is independent and randomly varying in time. The random binary sequence of +1's and -1's are generated. Then it is grouped into pairs of 2 symbols and then the symbols are sent in one time slot. The symbols are multiplied with the channel and then white Gaussian noise is added. The minimum among the four possible transmit symbol combinations is found out. Based on the minimum the estimate of the transmit symbol is chosen. It is then repeated for multiple values of E_b/N_0 and then the simulation and theoretical results are plotted.

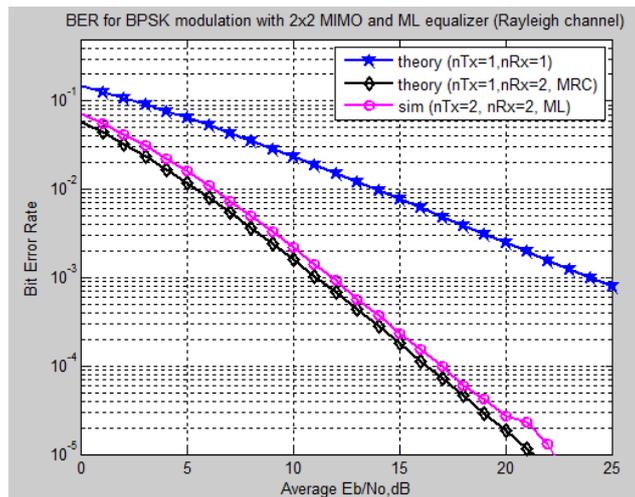


Fig. 1. BER plot for BPSK modulation in 2 x 2 MIMO System with Maximum Likelihood equalizer for Rayleigh Fading Channel

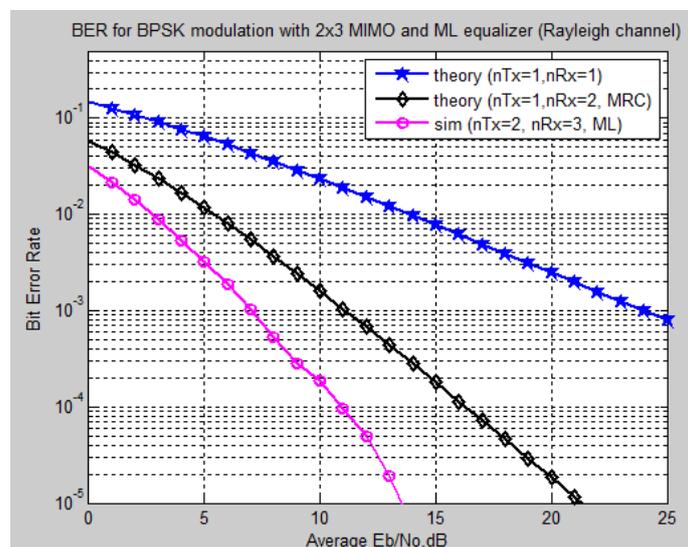


Fig. 2. BER plot for BPSK modulation in 2 x 3 MIMO System with Maximum Likelihood equalizer for Rayleigh Fading Channel

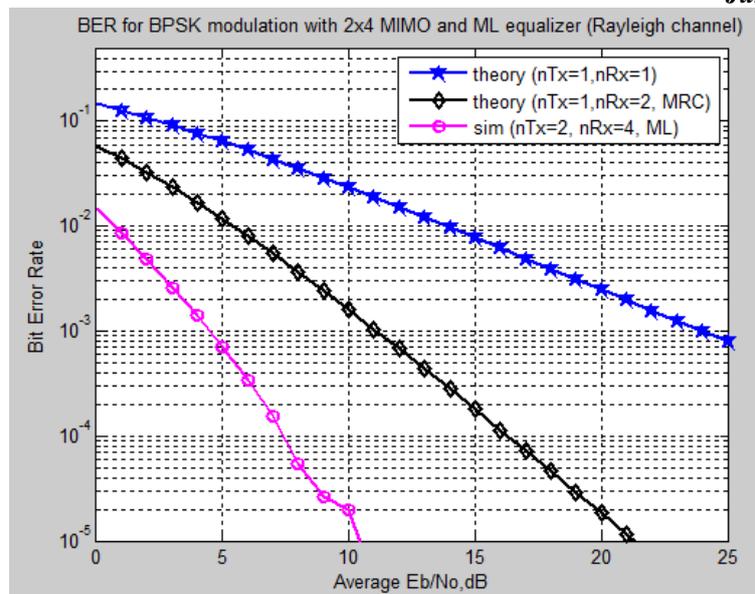


Fig. 3. BER plot for BPSK modulation in 2 x 4 MIMO System with Maximum Likelihood equalizer for Rayleigh Fading Channel

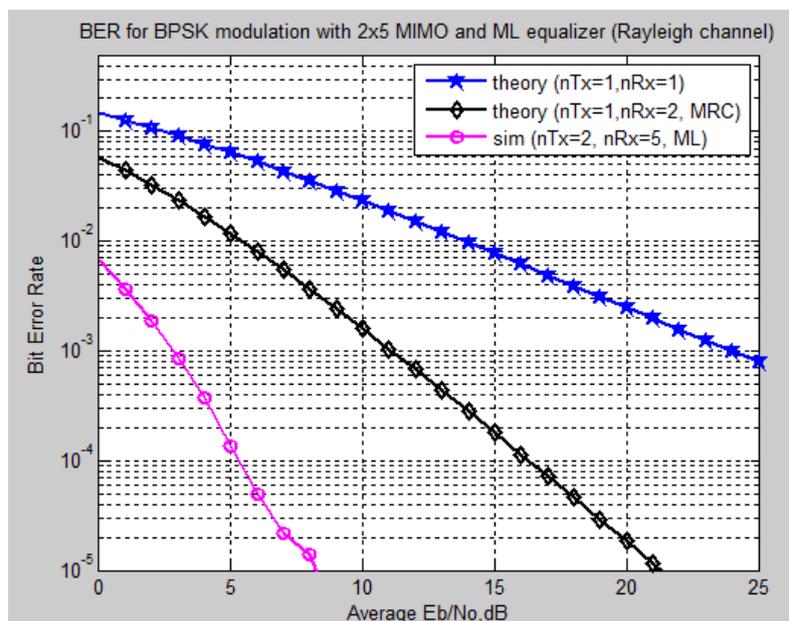


Fig. 4. BER plot for BPSK modulation in 2 x 5 MIMO System with Maximum Likelihood equalizer for Rayleigh Fading Channel

The results for 2 x n MIMO with Maximum Likelihood equalization helps to achieve a performance closely matching the 1 transmit 2 receive antenna Maximal Ratio Combining case (MRC) only for 2 x 2 MIMO Systems. When the signal to noise ratio is increased, then the Bit Error Rate decreases gradually. On observing the figures 1, 2, 3 and 4, it is apparent that when the number of receivers in a MIMO system increase then the Bit Error Rate gradually decreases. A very low Bit Error Rate can be achieved in the case of 2 x 5 MIMO systems when compared to the other 2 x n systems. Future works may include the analysis of different modulation schemes under different channel conditions to find out the lowest Bit Error Rate.

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