



## A Recent Concept of Transmission Loss Allocation in a Deregulated Power System

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**Abstract**— This paper presents a new concept of allocation of loss in every transmission line to generators and loads of a deregulated power system. The proposed method uses orthogonal projection of current due to a generator (load) on net branch current for loss allocation and real power losses due to mutual term of current projections are shared according to the ratio of magnitude of the current projection components leading to minimization of cross-subsidies. A sample 3-bus radial and 4-bus cyclic transmission systems have been studied for base case loading and with variation in loading conditions and the numerical results are analyzed to test the performance and fairness of the proposed method compared to others and is shown to be superior to existing methods.

**Keywords** -Circuit theory, deregulated power system, loss allocation, orthogonal current projection, Mutual term, superposition theorem.

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### I. INTRODUCTION

Deregulated or restructured power system is now most preferable choice in all over the world. The rapid change in economic background of electric power utility sector has injected competition in electricity market. The real power loss in transmission is of the order of 5-10% of total real power generation worthing several million dollars per year, so cost of lost power is recovered in terms of loss allocation to market participants (generators and loads). Under deregulation, transmission loss allocation is the key issue for all market participants, as a slight difference in loss allocation may result in significant loss or gain to the participants. Loss should be allocated equitably between all parties considering minimization of cross-subsidies [1]. Loss allocation is not trivial due to nonlinearity and investigations are still going on [2]. Several approaches for loss allocation have been proposed in previous research [3-4]. These approaches can be categorized as pro rata, incremental transmission loss (ITL), proportional sharing, circuit theory based approaches. In pro rata method [4-6] loss allocated to each generator or load depends on the active power or current level, disregarding the relative location within network. As a result generators or loads situated far away get benefit at cost of the others [6]. Incremental transmission loss based (ITL)[7-9] approaches do not allocate loss to slack bus and therefore normalization procedure is required to calculate final loss allocation[10] and these allocation techniques require to run conventional or optimal power flow large number of times to perform the desired integration thus making them computationally burdensome. Proportional sharing technique[11] gives a computationally efficient method for loss allocation. This method uses the linear proportional sharing principle which states that the power flow reaching a bus through any line splits among the lines evacuating power from the bus proportionally to their corresponding power flows, which is neither provable nor disprovable[10]. Circuit theory based techniques have attracted the attentions of researchers presently. Conejo *et al* [12] proposed Z bus loss allocation where total system loss is expressed as function of the Z-bus matrix and bus current injections and this method can yield negative loss allocation to 'reward' the participants who contribute to reduce system losses due to their strategically well positioned location within network. But, if the network has no shunt element or low value of shunt element leading to nearly singular Y-bus matrix, then this method fails to allocate loss. Ding *et al* [13] proposed that bus current injections are to be reconverted to bus power injections. The above methods allocate total system losses to generators and loads current injections. But for transparency and fairness in loss allocation the loss at each line should be allocated to agents [2]. Daniel *et al* [14] proposed a loss allocation method based on modified Y-bus. In this method loads are converted into equivalent admittances when generators are converted into current injections and loss is allocated to generators only or generators are converted into equivalent admittances when loads are converted into current injections and loss is allocated to loads only. But power transmission is due to mutual effect of generators and loads and both the generators and loads are benefitted by power transmission, so half of the real power loss at every line should be allocated to generators and half to loads [4,15,16].

In circuit-based methods, the choice of equivalence for power injections is important. Using two radial network, Wang *et al* [15] have shown that the CE equivalence mode where loads (generators) are converted into equivalent admittances when generators (loads) are converted into current injections (equivalence) and 50% of loss in each line is allocated to generators and rest 50% to loads is better than the CC equivalence mode where all generators and loads are converted to current injections simultaneously and loss is allocated to all the injections.

Wang *et al* [15] have presented a very interesting method for branch power flow decomposition and branch loss allocation based on circuit theory and the concept of orthogonal projection. They have used CE equivalence mode. At first they calculated orthogonal projection of current through a line due to a generator (load) on net current through the line. They have tried to show that the share of power injection at any bus on the power flow through any branch equals to the ratio of its current projection component on the total branch current to the total branch current. In view of this they allocate loss in any branch to generators (loads) proportional to their current projection components. But there is something wrong. To show the share of current injection at a bus on the complex power flow through a line they have used the bus voltages due to all the injections but current due to one generator and this is wrong. Instead of that bus voltage also should be due to one injection and test for the share of one generator on net power flow through a line may be done then. Superposition theorem can be applied for a linear bilateral circuit for calculating net voltage of a node or net current through a branch, but it cannot be used for power calculation as power is a nonlinear function of voltage or current. We think this is not a right approach. Orthogonal projection concept proposed by Wang *et al* is a very convincing one. But it should be used in another way. In any case loss is proportional to square of net branch current. So loss parcel due to square term of individual current projection should be allocated to the responsible generator(load) and loss parcel due to mutual term of two current projections may be shared equally as proposed in ref.[14,15,18] or in proportion to their magnitude as proposed in ref. [19,20] as some of the projections may be negative[18]. Here we see sharing the proportional sharing is justified. Here in this paper we have shared the loss parcel due to mutual term of two current projection in proportion to their magnitude and get a fairer way of loss allocation.

## II. EQUIVALENCE MODE

At first all buses are distinguished. A bus is considered to be a generator bus if it's net real power injection is nonnegative otherwise it is a load bus. Then convert the generators into current injections and loads into admittances as in paper [14,15]. Equivalent admittance matrix ( $Y_d$ ) due to loads is added to the original bus admittance matrix ( $Y_{bus}$ ) to get new admittance matrix ( $Y_G$ ) as in paper [15]. For all the buses,  $k=1$  to  $n_{bus}$

$$I_k = \left(\frac{S_k}{V_k}\right)^* \text{ when } P_{Gk} - P_{Dk} > 0 \quad (1)$$

$$y_{d\ k\ k} = -\frac{S_k^*}{|V_k|^2} \text{ when } P_{Gk} - P_{Dk} \leq 0 \quad (2)$$

$$\text{and } Y_G = Y_{bus} + \text{diag}(y_{d\ k\ k}) \quad (3)$$

Then invert the new bus admittance matrix( $Y_G$ ) to get the bus impedance matrix( $Z$ ) to calculate the current due to one generator as calculated in Wang's method[15].

## III. CURRENT PROJECTION COMPONENT

Orthogonal Current projection component is the key technique proposed by Wang *et al* [15]. Let, net current through any branch(say,  $r$ -th branch) due to all current injections at different buses and due to injection at  $i$ -th generator bus is  $I_r$  and  $I_r^{Gi}$  respectively and let,  $I_{rp}^{Gi}$  denotes the orthogonal projection vector of  $I_r^{Gi}$  on  $I_r$  and is expressed as

$$I_{rp}^{Gi} = \frac{I_r^{Gi} \cdot I_r}{|I_r|} e^{j\phi_r} = |I_r^{Gi}| \cos(\phi_r^{Gi} - \phi_r) e^{j\phi_r} \quad (4)$$

If slack bus changes then angle of different buses changes and angle of  $I_r$  also changes. But the ratio of orthogonal projection of current due to each generator or load on the net branch current is real positive or negative no. So this method is independent of choice of slack bus.

## IV. PROPOSED METHOD

Proposed method will give an idea for real power loss allocation in a justifiable way

### A. Real Power Loss Allocation Considering Mutual Term

For fair and transparent loss allocation, loss in each line should be decomposed. 50% of total real power loss in any line is allocated to generators and rest 50% to loads which is well accepted by the network participants [4,15,16]. If only for loss allocation of  $r$ -th line, we consider net current in line  $r$  ( $I_r$ ) as reference, loss allocation does not change but calculations become simplified. Now to make proposed method clear, we consider a power system with 2 generator buses 1 and 2. Let,  $I_{rp}^{G1}$  and  $I_{rp}^{G2}$  are the projections due to injections at generator bus 1 and generator bus 2 respectively on  $I_r$ , and for real loss of line  $r$ (having series resistance  $R_r$ ),  $P_{lr}$ ,  $P_{lr}^{GT}$ ,  $P_{lr}^{G1}$  and  $P_{lr}^{G2}$  are total real loss in line  $r$ , real loss allocated to all generators, real loss allocated to generator bus 1 and 2 respectively then we can write

$$\begin{aligned} P_{lr}^{GT} &= 0.5 * P_{lr} = 0.5R_r \left| I_{rp}^{G1} + I_{rp}^{G2} \right|^2 = 0.5R_r (I_{rp}^{G1} + I_{rp}^{G2})^2 \\ &= 0.5R_r \left\{ (I_{rp}^{G1})^2 + (I_{rp}^{G2})^2 + 2I_{rp}^{G1} I_{rp}^{G2} \right\} \end{aligned} \quad (5)$$

The term  $2R_r I_{rp}^{G1} I_{rp}^{G2}$  is loss parcel due to *mutual* term of two orthogonal projections of current due to two generator bus 1 and 2. Wang *et al* [6] proposed loss allocation should be proportional to the current projection. That means this

mutual term is equally divided. But here we want to divide it proportionally to reduce cross subsidy and as a projection may be negative also so we share it in proportion to their magnitude. Now,

$$P_{lr}^{G1} = 0.5R_r (|I_{rp}^{G1}|^2 + 2 I_{rp}^{G1} I_{rp}^{G2} \frac{|I_{rp}^{G1}|}{|I_{rp}^{G1}|+|I_{rp}^{G2}|}) \quad (6)$$

$$P_{lr}^{G2} = 0.5R_r (|I_{rp}^{G2}|^2 + 2 I_{rp}^{G1} I_{rp}^{G2} \frac{|I_{rp}^{G2}|}{|I_{rp}^{G1}|+|I_{rp}^{G2}|}) \quad (7)$$

Hence ratio of loss allocation will be equal to

$$\frac{P_{lr}^{G1}}{P_{lr}^{G2}} = \frac{|I_{rp}^{G1}|^2 + 2 I_{rp}^{G1} I_{rp}^{G2} \frac{|I_{rp}^{G1}|}{|I_{rp}^{G1}|+|I_{rp}^{G2}|}}{|I_{rp}^{G2}|^2 + 2 I_{rp}^{G1} I_{rp}^{G2} \frac{|I_{rp}^{G2}|}{|I_{rp}^{G1}|+|I_{rp}^{G2}|}} \neq \frac{I_{rp}^{G1}}{I_{rp}^{G2}} \quad (8)$$

If the mutual term would be shared equally we can see from equation 8 that loss allocation would be proportional to the current projection. Now in general, for a power system having  $n_g$  no. of generator buses and  $n_l$  no. of load buses, real power loss in line no r allocated to  $i^{th}$  generator bus is calculated as

$$P_{lr}^{Gi} = 0.5R_r |I_{rp}^{Gi}| (|I_{rp}^{Gi}| + 2I_{rp}^{Gi} \sum_{j=1, j \neq i}^{n_g} \frac{I_{rp}^{Gj}}{|I_{rp}^{Gi}|+|I_{rp}^{Gj}|}) \quad (9)$$

where,  $i=1,2,\dots,n_g$  and similarly real power loss in line r allocated to  $m^{th}$  load bus is calculated as

$$P_{lr}^{Lm} = 0.5R_r |I_{rp}^{Lm}| (|I_{rp}^{Lm}| + 2I_{rp}^{Lm} \sum_{j=1, j \neq m}^{n_l} \frac{I_{rp}^{Lj}}{|I_{rp}^{Lm}|+|I_{rp}^{Lj}|}) \quad (10)$$

where,  $m=1,2,\dots,n_l$ .

### V. NUMERICAL RESULTS AND ANALYSIS

Wang et al [15] has shown that individual branch loss allocation among participants by their method is more effective and fair than existing methods(ITL,CCI,PS)[6].So here we will compare the proposed method with Wang’s method only. We consider two small sample power systems, i) Three Bus Radial System ii) Four Bus Ring Main System for illustration and comparison.

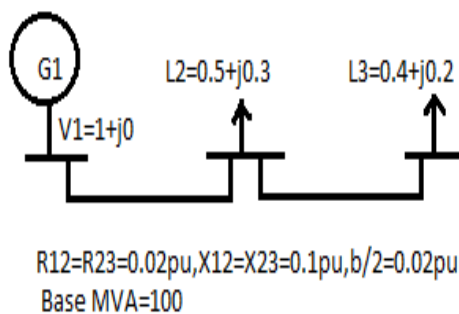


Figure 1: Three-bus sample radial system.

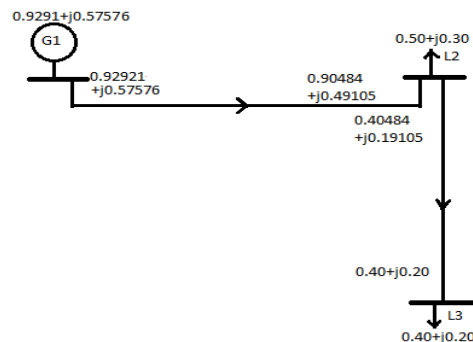


Figure 2: Power flow data of three bus radial system in nominal case

TABLE-1: CURRENT PROJECTIONS OF 3-BUS RADIAL SYSTEM

Line	Bus2 (load2)		Bus3(load3)	
	0.5+j0.3	0.6+j0.3	0.4+j0.2	0.4+j0.2
	pu(case1)	pu(case2)	pu(case1)	pu(case2)
1-2	0.6151	0.7130	0.4887	0.4925
2-3	-0.0046	-0.0052	0.4965	0.4992

TABLE-2: ACTIVE LOSS ALLOCATED TO LOADS FOR 1-2 LINE IN MW

Method	Bus2 (load2)		Bus3(load3)	
	0.5+j0.3	0.6+j0.3	0.4+j0.2	0.4+j0.2
	(case1)	(case2)	(case1)	(case2)
Method[6]	0.6789	0.8595	0.5394	0.594
Proposed method	0.7134	0.9238	0.5050	0.5295

A. Three-Bus Radial System

Consider the 3 bus radial system whose one line diagram, line and bus data are given in the fig. 1 having base MVA=100. We use a convergence tolerance of 0.001 pu for load flow study. The load flow data (power flow through lines and bus power injections) are given in fig 2. As there is only one generator at bus 1, so half of real power loss in each line is allocated to generator and half is allocated to loads in bus 2 and 3[4,15,16]. From load flow solution we get, the magnitude of bus current injections at load buses 2 and 3 at base case are 0.6301pu and 0.4997pu respectively and at a loading when real demand at bus 2 alone increases by 0.1 pu(case 2) are 0.7277pu and 0.5018pu respectively. So, it is observed that 0.1 pu increase in real power demand at bus 2 alone leads to 0.0976 pu 0.0021pu increase in magnitude of bus current injections at bus 2 and 3 respectively. Current projections on net current in the branches for load current injections at base case load(case 1) and at a increased real power loading by 0.1 pu in bus 2 alone (case 2) is given in table 1 and it is observed that current projections in line1-2 due to load at bus 2 and 3 increases by 0.0979 pu and 0.0038 pu respectively. Real power loss in line 1-2 allocated to loads using the equal sharing method by Wang et al [6] and proposed method is given in table 2 for comparison and it is observed that real power loss in line 1-2 allocated to load bus 2 increases by 0.1806MW (0.001806 pu) and to load bus 3 increases by 0.0543MW (0.000543pu) by Wang’s method. Whereas by our proposed method the real power loss in line 1-2 allocated to bus 2 and 3 increases by 0.2104 MW (0.002104pu) and 0.0245MW (0.0245pu). If power injection of one participant (generator or load) of a transmission system is changed by certain amount keeping all other’s injection fixed, the loss allocated to that particular participant should change prominently whereas loss allocation to other participants should change negligibly to have low cross subsidies.

As load 2 alone is changed, so loss allocated to load 2 should change prominently which is met by the proposed method.

In table 1, the projection of current due to load bus 2 on current in line 2-3 is shown negative due to the following fact. When generator is represented as equivalent admittance and only load 2 as injection for calculating line current due to load bus 2 alone, the charging capacitance present in line 2-3 supplies reactive power to load bus 2 (as there is no load at bus 3) driving a current of small magnitude in 3-2 direction which is of opposite direction of resultant current in line 2-3. Due to negative current projection, loss in line 2-3 allocated to bus 2 is negative(table 2) and is very small and the half of loss in line 2-3 (which is to be allocated among load buses) is allocated to load at bus 3 totally, as it alone is responsible for the loss proving the superiority of the method over existing methods. If there would not be any line charging capacitor in line 2-3 then projection of current due to load at bus 2 on line current 2-3 would be zero and loss allocation to bus 2 would be zero and the half of loss in line 2-3 (which is to be allocated among load buses) would be allocated to load bus 3.

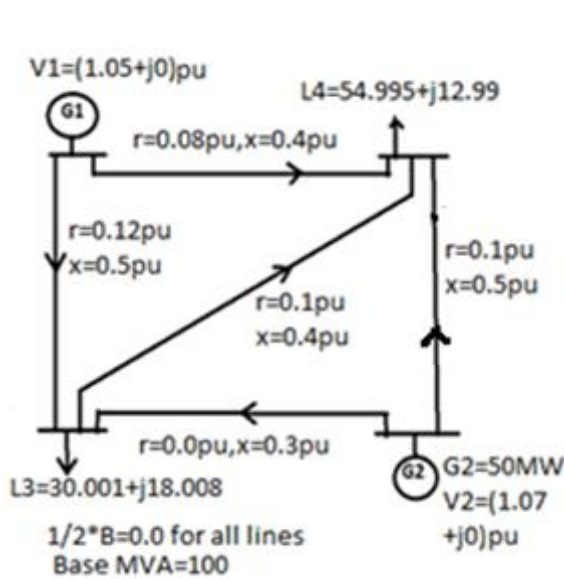


Figure 3: Four-bus sample ring main system

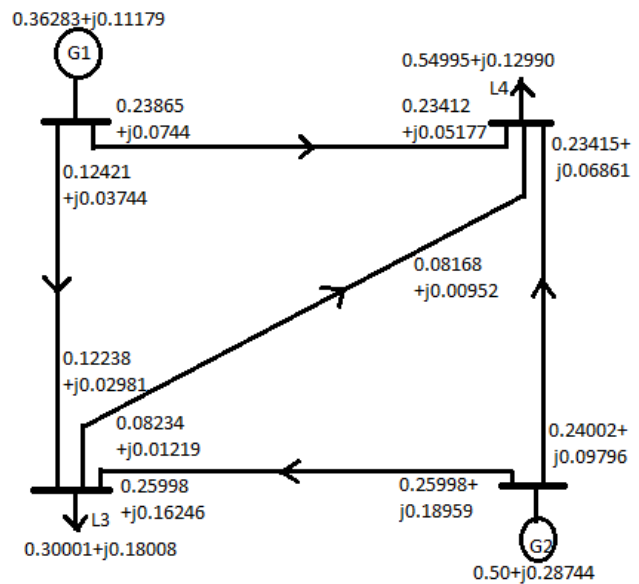


Figure 4: Power flow data of four bus system ring main system base case

TABLE-3:CURRENT PROJECTION IN NOMINAL CASE.

Lines	Gen1	Gen2	Load3	Load4
Line 1-4	0.2073	0.0308	0.0423	0.1958
Line 1-3	0.1543	-0.0307	0.0957	0.0279
Line 3-4	0.0137	0.0680	-0.0758	0.1575
Line 4-2	0.0067	0.2356	0.0358	0.2065
Line 2-3	-0.0047	0.3054	0.1689	0.1319

TABLE-4: ALLOCATED LOSS IN MW BY WANG'S METHOD [15]FOR BASE CASE.

<i>Lines</i>	<i>Gen1</i>	<i>Gen2</i>	<i>Load3</i>	<i>Load4</i>	<i>Total</i>
1-4	0.1974	0.0293	0.0403	0.1865	0.453
1-3	0.1144	-0.0229	0.0709	0.0206	0.183
3-4	0.0056	0.0277	-0.0309	0.0643	0.067
4-2	0.0081	0.2854	0.0433	0.2502	0.587
Total	0.3255	0.3195	0.1236	0.5216	1.290

TABLE-5: ALLOCATED LOSS IN MW BY PROPOSED METHOD IN BASE CASE.

<i>Lines</i>	<i>Gen1</i>	<i>Gen2</i>	<i>Load3</i>	<i>Load4</i>	<i>Total</i>
1-4	0.2163	0.0104	0.0189	0.2078	0.453
1-3	0.0953	-0.0038	0.0797	0.0119	0.183
3-4	0.0025	0.0308	-0.0101	0.0434	0.067
4-2	0.0007	0.2928	0.0173	0.2762	0.587
Total	0.3148	0.3302	0.1059	0.5400	1.290

TABLE-6: CURRENT PROJECTIONS INCREASING L3 BY 0.05PU

<i>Lines</i>	<i>Gen1</i>	<i>Gen2</i>	<i>Load3</i>	<i>Load4</i>
Line 1-4	0.2311	0.0268	0.0539	0.2040
Line 1-3	0.1766	-0.0265	0.1135	0.0367
Line 3-4	0.0080	0.0599	-0.0887	0.1566
Line 4-2	0.0039	0.2330	0.0361	0.2008
Line 2-3	-0.0021	0.3121	0.1853	0.1247

TABLE-7: ALLOCATED LOSS IN MW BY WANG'S METHOD [15] FOR REAL POWER DEMAND INCREASED AT LOAD BUS L3 BY 0.05PU

<i>Lines</i>	<i>Gen1</i>	<i>Gen2</i>	<i>Load3</i>	<i>Load4</i>	<i>Total</i>
1-4	0.2384	0.0276	0.0556	0.2104	0.532
1-3	0.1592	-0.0239	0.1023	0.0331	0.271
3-4	0.0027	0.0203	-0.0301	0.0531	0.046
4-2	0.0046	0.2760	0.0427	0.2379	0.561
Total	0.405	0.30	0.1705	0.5345	1.410

TABLE-8: ALLOCATED LOSS IN MW BY PROPOSED METHOD FOR REAL POWER DEMAND INCREASED AT LOAD BUS L3 BY 0.05PU.

<i>Lines</i>	<i>Gen1</i>	<i>Gen2</i>	<i>Load3</i>	<i>Load4</i>	<i>Total</i>
1-4	0.2579	0.008	0.0300	0.2360	0.532
1-3	0.1384	-0.0031	0.1150	0.0203	0.271
3-4	0.0009	0.0222	-0.0109	0.0340	0.046
4-2	0.0002	0.2803	0.0175	0.263	0.561
Total	0.4050	0.3000	0.1705	0.5345	1.410

TABLE- 9:CURRENT PROJECTIONS INCREASING L4 BY 0.05PU

<i>Lines</i>	<i>Gen1</i>	<i>Gen2</i>	<i>Load3</i>	<i>Load4</i>
Line 1-4	0.2369	0.0332	0.0478	0.2224
Line 1-3	0.1730	-0.0331	0.1003	0.0397
Line 3-4	0.0192	0.0736	-0.0769	0.1697
Line 4-2	0.0093	0.2401	0.0314	0.2180
Line 2-3	-0.0067	0.3048	0.1636	0.1345

TABLE10: ALLOCATED LOSS IN MW BY WANG’S METHOD [15] FOR REAL POWER DEMAND INCREASED AT LOAD BUS L4 BY 0.05PU.

<i>Lines</i>	<i>Gen1</i>	<i>Gen 2</i>	<i>Load3</i>	<i>Load4</i>	<i>Total</i>
1-4	0.2560	0.0359	0.0516	0.2404	0.584
1-3	0.1452	-0.0278	0.0842	0.0333	0.235
3-4	0.0089	0.0342	-0.0357	0.0788	0.086
4-2	0.0116	0.2995	0.0392	0.2720	0.622
Total	0.4217	0.3418	0.1393	0.6245	1.528

TABLE11: ALLOCATED LOSS IN MW BY PROPOSED METHOD FOR REAL POWER DEMAND INCREASED AT LOAD BUS L4 BY 0.05PU

<i>Lines</i>	<i>Gen1</i>	<i>Gen 2</i>	<i>Load3</i>	<i>Load4</i>	<i>Total</i>
1-4	0.2798	0.0122	0.0242	0.2680	0.584
1-3	0.122	-0.005	0.0945	0.0230	0.235
3-4	0.0047	0.0383	-0.011	0.0543	0.086
4-2	0.0013	0.3098	0.0138	0.2976	0.622
Total	0.4078	0.3553	0.1214	0.6429	1.528

TABLE-12 :CURRENT PROJECTIONS INCREASING G2 BY 0.05PU

<i>Lines</i>	<i>Gen1</i>	<i>Gen2</i>	<i>Load3</i>	<i>Load4</i>
Line 1-4	0.1822	0.0333	0.0333	0.1822
Line 1-3	0.1356	-0.0333	0.0882	0.0141
Line 3-4	0.0129	0.0737	-0.0737	0.1604
Line 4-2	0.0066	0.2530	0.0429	0.2167
Line 2-3	-0.0051	0.3279	0.1771	0.1456

TABLE-13: ALLOCATED LOSS IN MW BY WANG’S METHOD [15] FOR REAL POWER GENERATION INCREASED AT P-V BUS G2 BY 0.05PU

<i>Lines</i>	<i>Gen1</i>	<i>Gen 2</i>	<i>Load3</i>	<i>Load4</i>	<i>Total</i>
1-4	0.1571	0.0287	0.0287	0.1571	0.372
1-3	0.0832	-0.0204	0.0542	0.0086	0.126
3-4	0.0056	0.0319	-0.0319	0.0694	0.075
4-2	0.0085	0.3285	0.0557	0.2814	0.674
Total	0.2544	0.3687	0.1067	0.5165	1.246

TABLE-14: ALLOCATED LOSS IN MW BY PROPOSED METHOD FOR REAL POWER GENERATION INCREASED AT P-V BUS G2 BY 0.05PU

<i>Lines</i>	<i>Gen1</i>	<i>Gen 2</i>	<i>Load3</i>	<i>Load4</i>	<i>Total</i>
1-4	0.1740	0.0119	0.0119	0.1740	0.372
1-3	0.0668	-0.0040	0.0596	0.0032	0.126
3-4	0.0022	0.0352	-0.0101	0.0476	0.075
4-2	0.0006	0.336	0.0246	0.3120	0.674
Total	0.2436	0.3791	0.0860	0.5368	1.246

TABLE-15: CURRENT PROJECTIONS INCREASING Q4 DEMAND BY 0.25PU

<i>Lines</i>	<i>Gen1</i>	<i>Gen2</i>	<i>Load3</i>	<i>Load4</i>
Line 1-4	0.2482	0.0412	0.0447	0.2447
Line 1-3	0.1763	-0.0385	0.0997	0.0382
Line 3-4	0.0243	0.0948	-0.0755	0.1946
Line 4-2	0.0126	0.2740	0.0343	0.2523
Line 2-3	-0.0127	0.3426	0.1696	0.1604

TABLE-16: ALLOCATED LOSS IN MW WANG'S METHOD [15] FOR VAR DEMAND INCREASED AT LOAD BUS L4 BY 0.25PU.

Lines	Gen1	Gen 2	Load3	Load4	Total
1-4	0.2873	0.0477	0.0518	0.2833	0.670
1-3	0.1459	-0.0319	0.0825	0.0316	0.228
3-4	0.0145	0.0565	-0.0450	0.1159	0.142
4-2	0.0180	0.3926	0.0492	0.3616	0.821
Total	0.4657	0.4649	0.1385	0.7924	1.862

TABLE-17: ALLOCATED LOSS IN MW BY PROPOSED METHOD FOR VAR DEMAND INCREASED AT LOAD BUS L4 BY 0.25PU.

Lines	Gen1	Gen 2	Load3	Load4	Total
1-4	0.3164	0.0184	0.0215	0.3135	0.670
1-3	0.1197	-0.0057	0.0926	0.0214	0.228
3-4	0.0076	0.0633	-0.0126	0.0835	0.142
4-2	0.0023	0.4083	0.0162	0.3946	0.821
Total	0.4460	0.4843	0.1177	0.8130	1.862

### B. Four -Bus Cyclic System

Then we study a 4 bus cyclic transmission system whose single line diagram, line data and bus data are given in the fig 3 and also in [17] which was studied by Wang et al [15]. Here also we consider convergence tolerance =0.001 pu for load flow study. The load flow data (power flow through lines and bus power injections) are given in fig 4. From load flow result it is seen that generator bus 1 has lesser power injection (36.283MW, 11.179MVar), compared to generator bus 2 (50MW, 28.744MVar). Current injections by generator 1 and generator 2 are (0.3456-j0.1065)pu and (0.7420-j0.2603)pu respectively.

i) *Base Case*: From table 3 we see the current projection components due to individual generators and loads on net branch currents. Loss allocated to L4 is higher than L3 by proposed method (table-5) with comparison to Wang's method (table 4)[15] and it is acceptable as L4 has greater amount of load than L3. From table 4 we see loss allocations to generator 1 and 2 by Wang's method are 0.3255MW and 0.3195MW respectively and from table 5 we see using our proposed method the loss allocations are 0.3148MW and 0.3302 MW respectively. As G2 has higher level of generation than G1, so G2 should be allocated more loss than G1. Sharing the loss parcel due to mutual term of current projections in proportion to their magnitude the branch loss allocations are fairer compared to Wang's method and all other methods and as a result overall allocation to generators and loads are fairer. The line 2-3 for which orthogonal projection of current due to generator bus 1 and 2 are respectively -0.0047 pu and 0.3054 pu, the lowest and the highest one among all current projections, but the resistance is given as zero. The real power loss allocation to generator 2 is not high as it's power level, thus taking into consideration the topology of the network. If the resistance would be some value then allocation to generator 2 would increase greatly.

ii) *Case-1(Real power demand increased by 0.05pu at load bus L3)*: For the 4 bus cyclic system mentioned in fig 2 consider 5 MW load increment at bus 3 keeping all other specified quantities same as base case, let us study the allocation method's robustness. From load flow result we get (as generator bus 1 is slack bus) it's power increases to 41.402MW, 10.978MVar and reactive power of generator bus increases to 29.66MVar. From table 7 and 8 we see loss allocated to bus 3 is increased dominantly and loss allocated to other load (bus 2) increases by small amount 0.031 MW(table-7,8) by proposed method and Wang's method [15] respectively for line 1-3. Similarly loss allocated to bus 4 is increased by 0.0084MW and 0.0127 MW (table-7,8) by proposed method and Wang's method[15] for same the line. Here loss allocated to bus 3 changes by more amount(0.035 MW) as per proposed method than the amount (0.031 MW) (table-7,8) by Wang's method[15]. Similarly the changes occurs in bus 4 are 0.0084MW and 0.0127MW(table:7,8) as per proposed method and Wang's method[15] respectively. As we only change the load 3 retaining load 4 unchanged(that reflects on projection components prominently) it is expected that loss allocated to load 3 will be changed prominently than to load 4. This expectation is met by our proposed method only.

iii) *Case-2(Real power demand increased by 0.05pu at load bus L4)*: Now 5 MW load is increased at bus 4. So the projection component of bus 3 and 4 is incremented by 0.0055pu and 0.027pu respectively for line 1-4(table-9). As a result loss allocated to bus 4 is increased by 0.0602MW and 0.054 MW by proposed method compared to Wang's method [15] for line 1-4(table-10,11). Besides, Hence loss allocated to bus 3 is increased by 0.0053MW and 0.0113 MW by proposed method and Wang's method [15] respectively for line 1-4(table-10,11). This result indicate more loss increment of bus 4 and less loss increment of bus 3 comparatively by proposed method. Same result will be obtained for bus 1. By proposed method loss allocated to a line connected to the bus which is subjected to change in load or generation, is affected prominently.

iv) *Case-3(Generation increased by 0.05pu at P-V bus G2)*: As G2 is increased by 5 MW and demand is fixed, to supply the same demand generation of G1 will be decreased. Increased power of G2 should flow through either line 4-2 or 2-3. But resistance of line 2-3 is zero and hence we will focus on loss allocation of line 4-2. In this case total loss allocation of G2 is increased by 0.0486MW and 0.04887MW by Wang's method [15] and proposed method respectively. Whereas total loss allocation of G1 is increased by 0.071MW and 0.0713MW by Wang's method [15] and proposed method respectively (table-13,14). Moreover increase in total loss allocation across branch 4-2 is comparatively more by proposed method (0.08624MW) than Wang's method [15] (0.0841MW) (table-13,14). Another fact is that increased share of G2 for branch 4-2 is more by proposed method (0.0432MW) than Wang's method [15] (0.0431MW). All the above results accord with branch power flow and loss allocation. It is expected that G2 would share more loss than G1 for branch 4-2 and this is met by proposed method. Though difference between results are small but it can indicate that the proposed method allocate loss among participants in a more fair and accurate way.

v) *Case-4(Reactive power demand increased by 0.25pu at load bus L4)*: In power system network some loads may operate at poor power factor. These loads draw more reactive power and cause more active power losses and burden the system. To check effectiveness of the proposed method reactive power of L4 is increased by 25 MVar (deteriorate power factor). As a result, total branch loss of line 1-4 and line 2-4 as well as share of L4 for branch loss is increased. This scenario indicate that the proposed method takes into consideration the reactive power. So, loads operating at poor power factor (lagging) would be allocated more losses when using proposed method.

Proposed method may produce negative loss allocation to indicate counter flows and dominant flows. Acceptance of negative loss allocation depends on market and its participants. In this case negative loss may indicate positional strategy to help reduce system losses. Otherwise modifications could be made by unsubsidized strategy presented in paper [7].

## VI. CONCLUSIONS

This paper presents a new transmission loss allocation method applying orthogonal projection concept and having following characteristic

1. This paper propose the share of losses due to mutual term of current projection component to be in proportion to their magnitude, leading more fair way of loss allocation.
2. Results are independent of voltage reference bus
3. Like incremental loss allocation method and Z bus loss allocation, modified Ybus method, this method can yield negative loss allocation to give positional benefit to participants.
4. Takes into consideration network topology, nature of load (Power factor), location of participants etc.
5. It is extremely simple to formulate and implement numerically.
6. Branch loss allocation for each generators and loads is more reasonable than Wang's method [15] in case of varying loads or generation which has already discussed in the analysis.

From case studies it is seen that generators and loads contributed largely to branches directly connected to them, and slightly to ones far away from them. Compared with other methods this method gives more fair, transparent and reasonable loss allocation results and gives clear explanation of the obtained branch loss for each participants in competitive electricity markets.

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