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Creation of Planar Graph Using Kuratowski's K5 and K3, 3 Non Planar Graph

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Abstract— ***If a geometric representation of a graph G can be drawn on a plane in such a way that no two edges of G intersect each other ,then G is said a planar graph. In this paper an approach to devise a planar graph using Kuratowski's two non-planar graph K5 (with minimum number of vertices) and K3,3(with minimum number of edges) is being proposed. A corollary for connected planar graph G having f regions, n vertices and e edges is also being proved as a witness of planarity.***

Keywords— ***Planar Graph, Non Planar Graph, Kuratowski's K5 and K3,3 Non Planar Graph, Tangle.***

I. INTRODUCTION

A Graph is said to be planar if there exists some geometric representation of G which can be drawn on a plane such that no two of its edge intersect. A graph that cannot be drawn on a plane without a crossover between its edges is called non planar. This document is a template. An electronic copy can be downloaded from the conference website. For questions on paper guidelines, please contact the conference publications committee as indicated on the conference website. Information about final paper submission is available from the conference website. Also by combining these two graphs (i.e K_5 and $K_{3,3}$) it also satisfies the inequality ($e \leq 3n-6$,where e represents the number of edges and n represents the number of vertices in a graph).

A. Planar Graph

A Graph G is plane if it is drawn in plane with no two edges crossing each other, and it is planar if it isomorphic to a plane graph. Otherwise, it is non-planar. A planar drawing of a graph is an injection of its vertices onto points in the plane, and a mapping of the edges into open curves between their endpoints. These curves are not allowed to touch each other, except in their common end points. Graph which admit such a planar drawing, are called planar graphs. Fig. below is the example of planar graph.

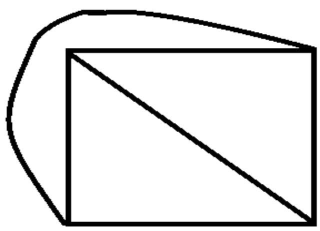

Figure 1.1 Planar Graph

In other words, the planar graph may also be defined with the help of considering the definition of a plane as below:

Definition

A graph G is planar if it is drawn in plane (or on the sphere) with no two edges crossing each other, and it is planar if it is isomorphic to a plane graph. Otherwise, it is non-planar. Stereographic projection caries plane embeddings to embeddings on a sphere and vice versa.

B. Non Planar Graph

An embedding of a graph G is a drawing of G on a certain surface in which the edges do not intersect. A non-planar graph can be always embedded on some surface, other than a plane (or sphere). For example, all graphs of polyhedra are planar, and the graphs *K*5 and *K*3,3 are non-planar.

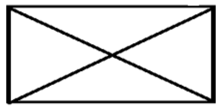

Figure 1.2 Non-Planar Graph

The most celebrated result about the planarity of graphs is Kuratowski's Theorem[1]. Two graphs G and G' are isomorphic modulo vertices of degree 2 if G is isomorphic to a graph G" obtained from G' by the addition or deletion of vertices with just two incident edges:

Theorem 1 **:** (Kuratowski's Theorem) Let G be a finite graph. G is planar iff it contains no subgraph isomorphic to K5 or K3,3 modulo vertices of degree 2 [1].
Short proof of the sufficiency part of this theorem is given by Makarychev [2],and the complete proof can be found, e.g., in the book [3].
The transformations described above are subdivision and contraction of a graph edge. A subdivision of a graph G is a graph obtained from G by a finite number of the following operations. Let v, w be the vertices of G which are connected by the edge vw. Introduce a new vertex x and replace the edge vw by two edges vx and xw, i.e., insert a vertex x in the middle of an existing edge vw.

Theorem 2: A graph G is planar iff it contains no subgraph which has K5 or K3,3 as a contraction. A special kind of contraction where edges forming a bigon are contracted simultaneously plays an important role in analyzing *KL*s. We call such contraction a *bigon collapse* [7].Planar embeddings of graphs can contain intersections of edges, called *crossings*,which are not the vertices of the graph. For planar graphs, there is always a plane diagram that avoids such (nugatory) crossings. But an important invariant of nonplanar graphs is their *graph crossing number* (usually denoted by *k*)—the minimal number of edge crossings among all possible planar diagrammatic representations of the graph[4,5].Since it is defined over all possible diagrammatic representations of a graph in a plane, the graph crossing number is an invariant very hard to compute.
For planar graphs it is always k(G) = 0, and if we are able to find a plane diagram of a non-planar graph G with one crossing, than k(G) = 1.The length of the shortest graph cycle (if any) in a graph is called the girth of a graph *and* denoted by c. For graphs without multiple edges and loops, $c \geq 3$. The girth of a graph G can be computed using *Mathematica* function Girth[G] [6]. If
e denotes the number of edges, and c the girth of G, the lower bound of the graph crossing number $k(G)$ is given [4] by the formula:
$k(G) \geq e - [(v-2)c/(c-2)]$ (1)
If the right side of the preceding formula is a negative integer**, 1** we conclude that $k \geq 0$.Since we are interested only in graphs (shadows) of virtual *KL*s, all parameters in their Conway symbols take only non-negative values. A tangle of the form $p1\ p2 \ldots pn$ where $p1 \geq 1$ and all other $pi \geq 0$ is called (positive) *rational tangle*.
A tangle of the form $p1, p2, \ldots, pn$ ($n \geq 3$), where pi are positive twists is called (positive) pretzel tangle, and a tangle of the form $r1, r2, \ldots, rn$ ($n \geq 3$),where ri are (positive) rational tangles not beginning with 1 is called (positive) *Montesinos tangle* [7].

C. Kuratowski's Two Non Planar Graphs

The Polish mathematician Kazimierz Kuratowski discovered two non-planar graphs K_5 and $K_{3,3}$. K_5 which is a non-planar graph of minimum number of vertices, which cannot be embedded in a plane, means we cannot draw this graph without intersecting its edges. This graph is a complete graph of 5 vertices (complete graph is a graph in which every vertex is connected to every other vertex in a graph). Second graph is $K_{3,3}$, which is also a non-planar graph with minimum number of edges ,this graph have 6 vertices and 9 edges and this is a regular graph(Regular graph is a graph in which all vertices are of equal degree).

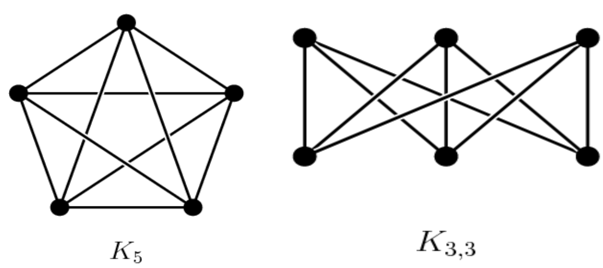

Figure 1.3 Kuratowski's Two Non Planar Graphs

D. Proposed Approach

After combining kuratowski's two non-planar graph ,a graph G(V,E) is appeared having a set of vertices V and edges E. The value of vertices and edges is 11 and 19 respectively which comes as the sum of vertices and edges of Kuratowski's two graph K_5 and $K_{3,3}$(Sum of total number of vertices is 5+6=11 from both graphs and also the sum of total number of edges is 10+9=19). The geometric representation of this graph G(V,E) is shown below.

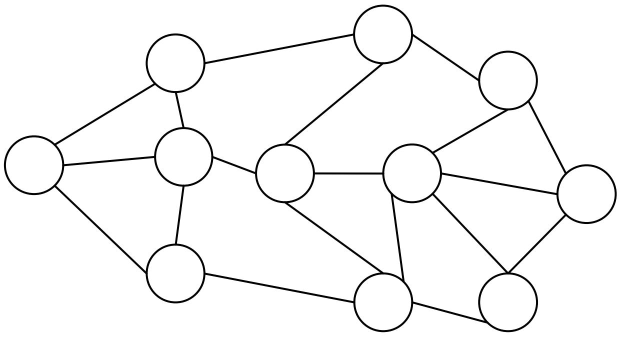

Figure 2.1 Geometric representation of this graph G(V,E)

E. Proof of Planarity for Graph G

From the geometrical representation of the graph G(V,E) as shown in figure 2.1,we can test that this graph can be drawn without intersecting any edges.So this graph is a planar graph. There is also another geometric representation of this which can also be embedded in the plane, so this graph is a planar graph , which is the combination of two nonplanar graphs.

F. Testing of Planarity using Euler's Formula

Applying euler's formula here , number of regions in this Graph G(V,E) can be calculated using the formula shown below[8]

R=e-n+2 **(2)**

Where e is the number of edges and n are the number of vertices.

Applying the formula,the number of regions,R,turns out to be 10.

R= 19-11+2=10

Testing the regions in graph G,it also contains the same number of regions i.e. 10.

The planarity test can also be verified by another corollary [8 narsinghdeo] as shown below.

COROLLARY : In any simple , connected planar graph with R regions, n vertices, and e edges (e>2), the following inequalities must hold :

e≥3/2 R **(3)**

e≤3n-6. **(4)**

Applying the above inequalities to resultant graph G(V,E),

Number of edges -: 19

Number of vertices -: 11

By putting these values in equation 3 & 4 :

19≥15

Or 19≤33-6

Or 19≤27

Which comes true.

On the basis of this analysis, it can be stated that the graph G(V,E) shown in figure 2.1 created by combining the Kuratowski's two non-planar graph K_5 and $K_{3,3}$ is a planar graph.

II. Conclusion and Future Scope

The approach used for creation of this planar graph G is not a generalized approach. This creation of a planar graph is a result of combining Kuratowski's two non-planar graph K_5 and $K_{3,3}$ and then planarity test is applied on each combination. A generalized approach can be develop for creating planar graph with the help of Kuratowski's two non-planar graph K_5 and $K_{3,3}$.

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