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Fuzzy Theory Concept Applied in Analytic Network Process

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Abstract- The Analytic Network Process (ANP) is a multi-criteria decision making (MCDM) theory of measurement used to derive relative priority scales of absolute numbers from individual judgments (or from actual measurements normalized to a relative form) that also belong to a fundamental scale of absolute numbers. These judgments represent the relative influence, of one of two elements over the other in a pairwise comparison process on a third element in the system, with respect to an underlying control criterion. Through its supermatrix, whose entries are themselves matrices of column priorities, the ANP synthesizes the outcome of dependence and feedback within and between clusters of elements. There can be many situations where the decision makers cannot provide exact numerical value or the information is not complete. We can apply Fuzzy theory in such situation. In this paper we will integrate Fuzzy theory and ANP which can provides efficient result even in uncertain conditions.

Keywords- ANP, Multi-Criteria Decision Making (MCDM), Fuzzy theory, FANP, Supermatrix.

I. INTRODUCTION

The Analytic Network Process (ANP) is a generalization of the Analytic Hierarchy Process (AHP) [1]. The basic structure is an influence network of clusters and nodes contained within the clusters. Many decision problems cannot be structured hierarchically because they involve the interaction and dependence of higher-level elements in a hierarchy on lower-level elements. Not only does the importance of the criteria determine the importance of the alternatives as in a hierarchy, but also the importance of the alternatives themselves determines the importance of the criteria. Feedback enables us to factor the future into the present to determine what we have to do to attain a desired result. Many decision problems cannot be structured hierarchically because they involve the interaction and dependence of higher-level elements in a hierarchy on lower-level elements. Not only does the importance of the criteria determine the importance of the alternatives as in a hierarchy, but also the importance of the alternatives themselves determines the importance of the criteria. Feedback enables us to factor the future into the present to determine what we have to do to attain a desired future. In 1965 Zadeh proposed the fuzzy set theory to solve problems in which description of activities, observations and judgments are subjective, vague and imprecise [2]. The fuzzy set theory is useful in modeling of uncertain systems in industry, nature and humanity, and facilitators for common-sense reasoning in decision making in the absence of complete and precise information. The conventional AHP/ANP-based decision model seems to be ineffective in dealing with the inherent fuzziness or uncertainty in judgment during the pair wise comparison process. Although the use of the discrete scale of 1-9 to represent the verbal judgment in pair wise comparisons has the advantage of simplicity, it does not take into account the uncertainty associated with the mapping of one's perception or judgment to a number. In real-life decision-making situations, the decision makers or stakeholders could be uncertain about their own level of preference, due to incomplete information or knowledge, complexity and uncertainty within the decision environment. Such situations will occur when selecting and evaluating an optimal project. Therefore, it's better to make project selection and assessment under fuzzy conditions. Therefore, Fuzzy ANP method was taken to calculate the weights of factors and sub-factors of the model, which can solve problems where there is lack of complete knowledge or information effectively and will provide more accurate result.

II. RELATED WORK

Many researchers are doing research on how to improve software quality and using which technique or methodology the quality of software will improve and fulfil all the requirements of the customers or users. Like

Thomas L. Saaty [3] have proposed that ANP is a useful way to deal with complex decisions that involve dependence and feedback analyzed in the context of benefits, opportunities, costs and risks. It has been applied literally to hundreds of examples.

Thomas L. Saaty[4] have illustrates the basic concepts of the ANP and shows how informed intuitive judgments can lead to real life answers that are matched by actual measurements in the real world (for example, relative dollar values) as illustrated in market share examples that rely on judgments and not on numerical data. The Analytic Network Process (ANP) is a multicriteria theory of measurement used to derive relative priority scales of absolute numbers from individual judgments (or from actual measurements normalized to a relative form) that also belong to a fundamental

scale of absolute numbers. These judgments represent the relative influence, of one of two elements over the other in a pairwise comparison process on a third element in the system, with respect to an underlying control criterion.

Hua Gao [5] have proposed combined of ANP and Fuzzy theory approach to solve the problem of project management performance evaluation which was considered a multi-criteria decision problem.

Ran Bi and Jin-yu We [6] proposed that it is convenient for the decision makers to learn which factor or sub-factor needs more attention to select an appropriate production line which is evaluated as a critical factor in Enterprise Resource Planning (ERP).

III. ANALYTIC NETWORK PROCESS (ANP)

The Analytic Network Process (ANP) was proposed by T.L.Saaty in 1996. The Analytic Network Process (ANP) provides a general framework to deal with decisions without making assumptions about the independence of higher-level elements from lower level elements and about the independence of the elements within a level as in a hierarchy [4]. In fact the ANP uses a network without the need to specify levels. As in the AHP, dominance or the relative importance of influence is a central concept.

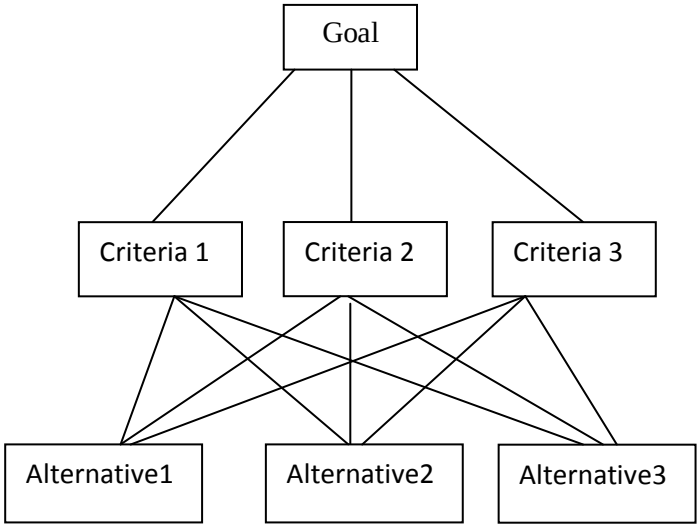

Fig 1: Network in ANP

In the ANP, one provides a judgment from the fundamental scale of the AHP by answering two kinds of questions with regard to strength of dominance:

1) Given a criterion which of two elements is more dominant with respect to that criterion,

2) Which of two elements influences a third element more with respect to a criterion? In order that all such influences be considered with respect to the same criterion so they would be meaningful to synthesize, it is

essential that the same criterion be used to make all the comparisons. Such a criterion is called a control criterion.

A control criterion is an important way to focus thinking to answer the question of dominan

TABLE1
FUNDAMENTAL SCALE FOR JUDGEMENT

1	Equal importance
3	Moderate importance of one over another
5	Strong or essential importance
7	Very strong or demonstrated importance
9	Extreme importance
2,4,6,8	Intermediate importance

Judgments are first given verbally as indicated in the scale and then a corresponding number is associated with that judgment.

A. Why Analytic Network Process?

Many decision making problem cannot be solved hierarchically because they involve interaction and dependence of higher level elements in a hierarchy on lower level elements. Analytic Network Process (ANP) allow consideration of

interdependence among factors and sub-factors. In order to overcome this limitation Analytic Network Process (ANP) was proposed by T.L.Saaty. Analytic Network Process (ANP) lies in its use of ratio scales to capture all kinds of interactions and make accurate prediction and even further to make better decision.

1) Steps of Analytic Network Process

The ANP modeling process can be divided into three steps which are described as follows:

Step 1: forming initial supermatrix

This is done through pairwise comparisons by asking "How much importance does a criterion have compared to another criterion with respect to our interests or preferences?"

The relative importance value can be determined using a scale of 1- 9 to represent equal importance to extreme importance.

Assume a network structure is composed of hierarchy C_h (h=1,2,…,m) . For each hierarchy C_h, assume there exist elements $e_{h1}, e_{h2}, ..., e_{hm}$, so the influence of C_h (h=1,2,…,m) can be denoted as below:

$$W = \begin{pmatrix} W_{11} & W_{12} & W_{13} & \dots & W_{1m} \\ W_{21} & W_{22} & W_{23} & \dots & W_{2m} \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ W_{m1} & W_{m2} & W_{m3} & \dots & W_{m} \end{pmatrix}$$

It is the general form of the supermatrix. W_{ij} shows the influence of each element of the i_{th} hierarchy on the j_{th} hierarchy, which is called a block of a supermatrix[7].

Step 2: obtaining weighted supermatrix

The priorities of elements in one hierarchy according to a certain criterion can be denoted with a supermatrix, which means each column of each hierarchy in the supermatrix is column stochastic. But the influence that other hierarchy according to this criterion is not concerned. As a result, each column of the supermatrix is not column stochastic. It is essential to consider the influence between each two hierarchy. The particular method is: regarding each hierarchy as an element, and pairwise comparing according a certain hierarchy, then computing corresponding priorities. Suppose a_{ij} is the influence weight of the i_{th} hierarchy on the j_{th} hierarchy, let

$$\overline{W}_{ij} = a_{ij} W_{ij}$$

W is a weighted supermatrix. In a weighted supermatrix, addition of elements in each column is 1. Matrix has this trait is called column stochastic [8].

Step 3: calculating limited supermatrix

Feedback and interdependency among the factors have made the computation of elements priorities complex. For example, W_{ij} can reflect direct comparison relationship between elements in i_{th} hierarchy and j_{th} hierarchy. Feedback and interdependency among the factors have made the computation of elements priorities complex. Therefore, limited supermatrix should be computed. The weighted supermatrix is raised to limiting powers such as to get the global priority vector or called weights, so the most important criterion or the best alternative are acquirable [9].

2) Pros and cons of Analytic Network Process (ANP)

Pros of Analytic Network Process

- Due to its more general approach to use a network instead of hierarchy ANP is the general approach for any kind of decision problem.
- It is an ideal tool to gain deeper understanding of a specific problem and how its different factors and sub-factors inter-relate.
- It requires precise definition of nodes and interconnections. Therefore, it requires thorough thinking about the problem to be solved. Hence, the chance of committing error is reduced.

- Some real life problems can only be solved using ANP because they may involve interdependence and feedback.

Cons of Analytic Network Process

- Require some specific software to calculate results.
- Too complex for an implementation as a standard tool for practical decision making in an organisation.

IV. FUZZY THEORY

In 1965 Zadeh proposed the fuzzy set theory to solve problems in which description of activities, observations and judgments are subjective, vague and imprecise. The fuzzy set theory is useful in modelling of uncertain systems in industry, nature and humanity, and facilitators for common-sense reasoning in decision making in the absence of complete and precise information [10]. 'Fuzzy' generally refers to the situation in which boundary for the activity, observation or judgment cannot be well defined. Instead of assigning definite comparison scale 1-9 to represent criteria assessments, fuzzy sets are expressed from 1 *M* ='approximately 1'to 9 *M* ='approximately 9' to consider people's uncertain or imprecise assessments. Generally, a fuzzy set is defined by a membership function, which represents the grade of any element *x* of *X* that have the partial membership to *M*. Zadeh proposed to use values ranging from 0 to 1 for showing the membership of the objects in a fuzzy set. If an element *X* belongs to *M* $\mu_m(x) = 1$ and $\mu_m(x)=0$ if not.

A. Triangular fuzzy number

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function, which assigns to each object a grade of membership ranging between zero and one. A TFN is denoted simply as (*l*, *m*, *u*). The parameters *l*, *m* and *u*, respectively, denote the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event. Each Triangular Fuzzy Number has linear representations on its left and right side such that its membership function can be defined as

$$Um(x)=\begin{cases}\frac{x-l}{m-l} & l \le x \le m \\ \frac{u-x}{u-m} & m \le x \le u \\ 0 & otherwise\end{cases}$$

A triangular fuzzy number M is shown in Fig. 2 [11].

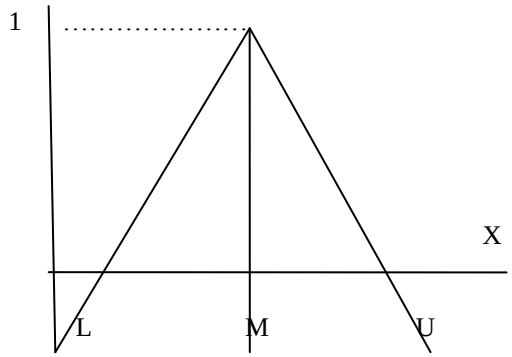

Fig 2: Triangular fuzzy numbers

B. FUZZY ANALYTIC NETWORK (FANP)

As we know, ANP has been proposed as a suitable multi-criteria decision analysis tool for project selection and evaluation. However, the conventional AHP/ANP-based decision model seems to be ineffective in dealing with the inherent fuzziness or uncertainty in judgment during the pairwise comparison process. Although the use of the discrete scale of 1-9 to represent the verbal judgment in pairwise comparisons has the advantage of simplicity, it does not take into account the uncertainty associated with the mapping of one's perception or judgment to a number. In real-life decision-making situations, the decision makers or stakeholders could be uncertain about their own level of preference, due to incomplete information or knowledge, complexity and uncertainty within the decision environment. Such situations will occur when selecting and evaluating an optimal project. Therefore, it's better to make project selection and assessment under fuzzy conditions.

1) Steps of Fuzzy Analytic Network Process (FANP)

The process of applying the group Fuzzy ANP under uncertainty that combines Fuzzy prioritization method, non-linear programming for group decision and ANP comprises of the following main steps:

Step 1: Identify alternatives, criteria and clusters to be used in the proposed model.

Step 2: Configure a network structure including clusters, criteria, alternatives and dependences among the factors and sub-factors.

Step 3: Construct pairwise matrices of the components with Fuzzy judgments. The Fuzzy scale regarding relative importance to measure the relative priorities. Pairwise comparison matrices are formed by the experts by applying fuzzy scale given in Table.

TABLE2
TABLE OF **LINGUISTIC SCALE**

Linguistic Scale for difficulty	**Linguistic Scale for importance**	**Triangular fuzzy scale**
Just equal	Just equal	(1,1,1)
Equally difficult(ED)	Equally important(EI)	(1/2,1,3/2)
Weakly more difficult(WMD)	Weakly more important(WMI)	(1,3/2,2)
Strongly more difficult(SMD)	Strongly more important(SMI)	(3/2,2,5/2)
Very strongly more difficult(VSMD)	Very strongly more important(VSMI)	(2,5/2,3)
Absolutely more difficult(AMD)	Absolutely more important(AMI)	(5/2,3,7/2)

Step 4: Constructing the fuzzy comparison matrix by using triangular fuzzy numbers:
the geometric mean represents the expert consensus and is the most widely used in practical applications. Here, the geometric mean models the triangular fuzzy numbers. Zadeh (1965) introduced the fuzzy set theory to deal with the uncertainty due to imprecision and vagueness. A major contribution of fuzzy set theory is its capability of representing vague data. A triangular fuzzy number is defined as (l,m,u) , where $l \le m \le u$. The parameters l,m and u respectively, denote the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event. (l,m,u) has the following triangular type membership function.

Step 5: Calculate fuzzy eigen value:
Saaty (1980) proposed a consistency index (C.I.) and consistency ratio (C.R.) to verify the consistency of the comparison matrix. Associated with the weights is an inconsistency. The consistency index of a matrix is given by

$C.I. = (\lambda_{max} - n)/(n-1)$

C.R = C.I/R.I

Where, R.I is Random Consistency Index. Eigen value is calculated to find whether the constructed matrix is consistent or not. It is recommended that should be less than or equal to 0.10. Inconsistency may be thought of as an adjustment needed to improve the consistency of the comparisons. But the adjustment should not be as large as the judgment itself, nor so small that it would have no consequence. Thus inconsistency should be just one order of magnitude smaller. On a scale from zero to one, the overall inconsistency should be around 10 %. The requirement of 10% cannot be made smaller such as 1% or 0.1% without trivializing the impact of inconsistency. But inconsistency itself is important because without it, new knowledge that changes preference cannot be admitted [12].

Step 6: Forming initial supermatrix:
The network of ANP is represented as a matrix. The matrix is composed by listing all nodes horizontally and vertically. This is done through pairwise comparisons by asking "How much importance does a criterion have compared to another criterion with respect to our interests or preferences?" Assume a network structure is composed of hierarchy C_h (h = 1,2,…,m) For each hierarchy C_h , assume there exist elements $e_{h1}, e_{h2}, \ldots, e_{hmk}$, so the influence of C_h (h= 1,2,…,m).

Step 7: obtaining weighted supermatrix
Obtain weighted supermatrix by multiplying the unweighted supermatrix by the corresponding cluster priorities, and then adjusting the resulting supermatrix to column stochastic.

Step 8: calculating limited supermatrix
Now, limit the weighted supermatrix by raising it to sufficiently large power so that it converges into a stable supermatrix (i.e, all columns being identical). Feedback and interdependency among the factors have made the computation of elements priorities complex. Therefore, limited supermatrix should be computed.

V. CONCLUSION

The ANP is a useful way to deal with complex decisions that involve dependence and feedback analyzed in the context of benefits, opportunities, costs and risks. It has been applied literally to hundreds of examples both real and hypothetical. Fuzzy theory is used when decision-makers cannot easily provide exact numerical values for factors. Hence, Fuzzy ANP is considered as an effective method in incomplete information situation and complex actual environment to weaken subjective factors from pairwise comparison. Therefore, Fuzzy Analytic Network Process (FANP) is an efficient method to solve many complex and real life problems.

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REFERENCES

[1]. Thomas L. Saaty, University of Pittsburgh, The Analytic Network Process.
[2]. Zadeh LA (1965) Fuzzy sets. Inf Control 8:338–353.
[3]. T. L. Saaty, and L. G. Vargas, "Decision making with the analytic network process," New York: Spring Science Publisher, 2006.
[4]. FUNDAMENTALS OF THE ANALYTIC NETWORK PROCESS – DEPENDENCE AND FEEDBACK IN DECISION-MAKING WITH A SINGLE NETWORK, JOURNAL OF SYSTEMS SCIENCE AND SYSTEMS ENGINEERING / Vol. 13, No. 2, June, 2004.
[5]. Hua Gao, A Fuzzy-ANP Approach to Project Management Performance Evaluation Indices System, 978-1-4244-7330-4/10/$26.00 ©2010 IEEE.
[6]. Ran Bi and Jin-yu Wei, Application of Fuzzy ANP in Production Line Selection Evaluation Indices System in ERP, Proceedings of the IEEE International Conference on Automation and Logistics Qingdao, China September 2008
[7]. Eddie W.L. Cheng, and Heng Li, "Application of ANP in process models: An example of strategic partnering", Building and Environment, vol 42, pp. 278-287, 2007
[8]. Xiaoli Tang, and Junwen Feng, "ANP Theory and Application expectation", *Statistics and Decision-making*, vol 12, No. 3, pp. 138-140, March. 2006.
[9]. Rachung Yu, and Gwo-Hshiung Tzeng, "A soft computing method for multi-criteria decision making with dependence and feedback",
[10]. *Applied Mathematics and Computation*, vol 180, No. 6, pp. 63-75, June 2006.
[11]. Deng H (1999) Multicriteria analysis with fuzzy pair-wise comparison. Int J Approx Reason 21:215–231.
[12]. Saaty, T. L. & Ozdemir, M. (2005). *The Encyclicon*. RWS Publications, 4922 Ellsworth Avenue, Pittsburgh.

