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Effects of Variable Fluid Properties on Free Convection Flows and Heat Transfer at an Isothermal Vertical plate Embedded in a Porous Medium in the presence of Magnetic field and Radiation

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Abstract*— Effects of magnetic field, radiation, variable viscosity and variable thermal conductivity on similarity solutions of free convection at an isothermal vertical plate in a porous medium are studied numerically. A similarity transformation is used to reduce partial differential equations governing the problem into ordinary differential equations and the equations are solved numerically subject to appropriate boundary conditions by the use of Runge-Kutta-Gill method together with shooting technique. The cases of hot and cold plate are considered and interesting features of the solutions are presented and discussed.

***Keywords*— Free Convection, Darcy Model, Variable fluid property, Magnetic field, Radiation.**

I. INTRODUCTION

Over the last five decades much insightful work has been done on convection boundary-layer flows in porous media. The analogous problems have important applications in fields such as geothermal energy extraction, oil reservoir modelling, analysis of insulating systems, food processing, casting and welding in manufacturing processes and the dispersion of chemical contaminants in different industrial processes in the environment. Books on Porous media by [8] and [11] stand evident to the fact that convective flows in porous media are of vital importance to these processes. Studies of radiation effects on free convection flow are important in the context of space technology and also in processes involving high temperature. Investigations both theoretical and experimental have been conducted on flow and heat transfer through Porous media covering a broad range of fields. The effect of radiation on free convection flow of fluids with variable viscosity from a porous plate is discussed in [1]. The fluid considered in that paper is an optically dense viscous incompressible fluid of linearly varying temperature dependent viscosity. Reference [10] discussed radiative convective flow past a semi-infinite vertical plate. Reference [3] discussed radiation effect on mixed convection along a vertical plate with uniform surface temperature. Reference [9] discussed coupled heat and mass transfer in Darcy-Forchheimer Mixed convection from a vertical flat plate embedded in a fluid saturated porous medium under the effects of radiation and viscous dissipation. Mixed convection boundary layer flow on a vertical surface in a saturated porous medium is studied in [7]. In that paper the flow of a uniform stream past an impermeable vertical surface embedded in a saturated porous medium and which is supplying heat to the porous medium at a constant rate is considered. In this paper, variable fluid properties and applied magnetic field, transfer of heat energy from the fluid due to radiation is also taken in to consideration. Rosseland approximation is used to describe radiative heat flux from the fluid and a numerical study is made of the effect of variable viscosity, variable thermal conductivity, radiation and magnetic field on convection flows at a vertical plate embedded in a porous medium.

Based on variations of viscosity (μ) and thermal conductivity (k_m) with temperature, fluids can be classified into four categories for which (i) both μ and k_m increase with increase in temperature- Type-I, (ii) both μ and k_m decrease with increase in temperature- Type-II, (iii) μ increases while k_m decreases with increase in temperature- Type-III and (iv) μ decreases while k_m increases with increase in temperature- Type-IV. Examples of fluids and appropriate ranges of temperature in which those fluids fall under the above categories are- Type-I: air (between 100^0 C and 800^0 C), steam (between 100^0 C and 1000^0 C) , Nitrogen (between 0^0 C and 1000^0 C); Type-II: saturated water (between 100^0 C and 200^0 C), unused engine oil (between 0^0 C and 160^0 C), Transformer oil (between -50^0C and -40^0 C): Type-III: Methyl Chloride (between -50^0C and 50^0C); Type-IV: Dichloro- Difluro Methane (between -50^0C and 20^0C) is observed in [4] & [5]. As such this data helps in interpreting the results of analysis as applicable to certain fluids in specific ranges of temperatures. In the present work authors made their attention towards two categories Type-III and Type- IV.

II. FORMULATION AND SOLUTION

Let an isothermal flat plate be embedded vertically in a porous medium, saturated with viscous incompressible homogeneous quiescent fluid. The porous medium is assumed to be homogeneous and is in thermal equilibrium with the surrounding fluid. Let a magnetic field of uniform strength is assumed to be acting a direction normal to the plate. The fluid is assumed to be a gray medium that emits and absorbs but do not scatter thermal radiation. It is also assumed that radiation from the fluid is only taken into consideration which is present in the form of a unidirectional flux, transverse to the vertical plate. Let *X*-axis be taken along the plate and *Y*-axis perpendicular to it and T_0 is assumed as the temperature of the plate. Orientation of the plate for both the cases (Hot and Cold) for free convection is presented in the Fig.1.

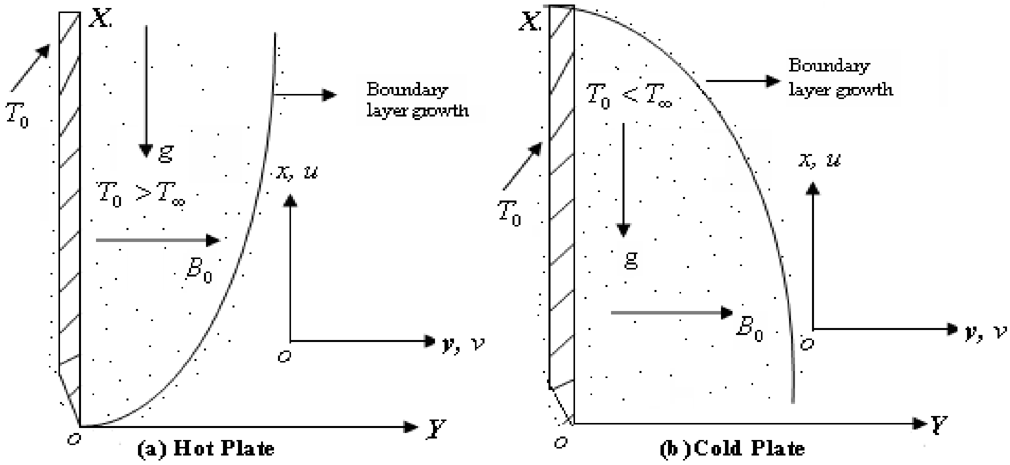

Fig .1 Physical model and coordinate system For Free convection

The equations governing free convection boundary- layer flow are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial p}{\partial x}+(\rho-\rho_\infty)g+\sigma B_0^2 u+\frac{\mu}{K}u=0 \tag{2}$$

$$\frac{\partial p}{\partial y}+\frac{\mu}{K}v=0 \tag{3}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k_m\frac{\partial T}{\partial y}\right)-\frac{\partial q_r}{\partial y} \tag{4}$$

where u, v are fluid velocity components, *T* is fluid temperature, K is Permeability, k_m is effective thermal conductivity of the porous medium, B_0 is the magnetic flux, σ is the electric conductivity, q_r is radiative heat flux. The Rosseland approximation is used in the energy equation to describe the thermal radiative heat transfer.

In fact the radiative heat flux q_r is given by $q_r=-\frac{4\sigma_s}{3k_e}\frac{\partial T^4}{\partial y}$, where σ_s the Stefan-Boltzmann is's constant and k_e is the mean absorption coefficient. Assuming temperature differences within the flow to be sufficiently small, the term $\frac{\partial q_r}{\partial y}$ of equation (4) is simplified by expanding T^4 into the Taylor series about T_∞, and neglecting higher order terms. It may be noted that because of the use of Rosseland approximation, the present analysis gets limited to optically thick fluids.

Taking $\rho=\rho_\infty[1-\beta(T-T_\infty)]$ in the body force term, introducing a stream function ψ and eliminating fluid pressure from equations (2) and (3), the governing equations are obtained as

$$\left(\mu+K\sigma B_0^2\right)\left(\frac{\partial^2\psi}{\partial y^2}\right)+\left(\frac{\partial\psi}{\partial y}\right)\left(\frac{\partial\mu}{\partial y}\right)=K g\rho_\infty\beta\left(\frac{\partial T}{\partial y}\right) \tag{5}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\left(k_m\frac{\partial^2 T}{\partial y^2}+\frac{\partial k_m}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{16\sigma_s T_\infty^3}{3\rho c_p k_e}\frac{\partial^2 T}{\partial y^2} \tag{6}$$

The boundary conditions on T and ψ are
$$\left.\begin{array}{l} at\ \ y=0,\ T_0=T_\infty+A,\ \dfrac{\partial\psi}{\partial x}=0, \\ as\ \ y\to\infty,\ T\to T_\infty,\ \dfrac{\partial\psi}{\partial y}\to 0 \end{array}\right\} \quad (7)$$

Introducing ***Rayleigh*** number (Ra_x), ***Hartman*** number (M^2), a magnetic interaction parameter ***C,*** Radiation parameter ***Rd*** and the non dimensional functions $f,\ \theta$ together with a similarity variable η through the relations

$$\left.\begin{array}{l} Ra_x=\dfrac{\rho_\infty K\, g\beta|T_0-T_\infty|\,x}{\alpha_m\mu_f} \\ M^2=\dfrac{B_0^2L^2\sigma}{\mu_f} \\ K^*=\dfrac{L^2}{K} \\ C=\dfrac{K^*}{K^*+M^2} \\ F=\dfrac{k_f\,k_c}{4\sigma_1T_\infty^3} \\ f(\eta)=\dfrac{\psi}{\alpha Ra_x^{\frac{1}{2}}} \\ \theta(\eta)=\dfrac{T-T_\infty}{T_0-T_\infty} \\ \eta=\dfrac{y}{x}Ra_x^{\frac{1}{2}} \end{array}\right\} \quad (8)$$

Equations (5), (6) are rewritten as

$$\left[1+C\gamma_\mu\left(\theta-\frac{1}{2}\right)\right]f''+C\gamma_\mu\ f'\theta'=C\theta' \quad (9)$$

$$\left[1+\gamma_k\left(\theta-\frac{1}{2}\right)+\frac{4(Rd)}{3}\right]\theta''+\frac{1}{2}f\,\theta'+\gamma_k\theta'^2=0 \quad (10)$$

Here $C=\dfrac{K^*}{K^*+M^2}$ is the ***magnetic field parameter***, $M^2=\dfrac{B_0^2L^2\sigma}{\mu_f}$ & $K^*=\dfrac{L^2}{K}$, $Rd=\dfrac{4\sigma_sT_\infty^3}{k\,k_e}$ is the radiation parameter, $Ra_x=\dfrac{\rho_\infty K\,g\,\beta(T_0-T_\infty)x}{\mu_f\ \alpha}$ is the *Rayleigh* number,

$\mu=\mu_f\left[1+\gamma_\mu\left(\theta-\frac{1}{2}\right)\right]$ and $k_m=k_f\left[1+\gamma_k\left(\theta-\frac{1}{2}\right)\right]$ (by [2]&[4]).

The boundary conditions (7) become

$$\left.\begin{array}{l} at\ \ \eta=0,\quad \theta=1,\quad f=0, \\ as\ \eta\to\infty,\ \theta\to 0,\ f'\to 0 \end{array}\right\} \quad (11)$$

Equation (9) can be integrated once using the condition on f' at infinity to get

$$f'=\frac{C\theta}{\left[C\gamma_\mu\left(\theta-\frac{1}{2}\right)+1\right]} \quad (12)$$

Evaluating this expression at $\eta=0$ which gives the slip velocity $f'(0)$ as

$$f'(0)=\frac{2C}{C\gamma_{\mu}+2} \qquad (13)$$

III. SOLUTION OF THE PROBLEM

A. **PARAMETERS OF THE PROBLEM-** In the free convection case, the flow and heat transfer depend on the parameters $C, \gamma_{\mu}, \gamma_{k}$ and *Rd* where *C* is magnetic field parameter, γ_{μ} is viscosity variation coefficient, γ_{k} is thermal conductivity variation coefficient and *Rd* is the radiation parameter.

The parameter *C* takes smaller values (less than unity) when either the porous parameter takes smaller values or the Hartmann number takes larger values, that is, when porosity of the medium is high or the intensity of the medium is high or the intensity of magnetic field is high. When there is no applied magnetic field M^2 takes zero value as a result *C* takes value of unity. Solutions are found for the values 0.1, 0.5 and 1 of *C*. Reduced flow can be expected for smaller values of *C* or for increased intensity of the magnetic field as the magnetic field lines obstruct the flow.

In the present work the attention is made for $T_0 > T_\infty$ (Hot Plate), fluids like (Dichloro Fluro Methane) $\gamma_{\mu} < 0, \gamma_{k} > 0$ and for $T_0 < T_\infty$ (Cold Plate) fluids like (Methyl Chloride) $\gamma_{\mu} > 0, \gamma_{k} < 0$.

The parameter γ_{μ} and γ_{k} takes positive as well as negative values, the limiting values being '-2' and '+2'. Zero value of γ_{μ} and γ_{k} corresponds to variation of constant viscosity and constant thermal conductivity. In this paper solutions are found for the values of -1, 0, 0.5 and 1 of both γ_{μ} and γ_{k}. Zero value for the parameter *Rd* corresponds to the case when transfer of heat energy through radiation from the fluid is neglected. Positive numerical values of *Rd* correspond to intensity of thermal radiation from the fluid. Solutions are found for the values 0, 0.5, 10 and 100 of the parameter *Rd*. Thermal radiation causes thickening of the thermal boundary layer and hence increasing values of the parameter *Rd* can increase thermal boundary layer thickness.

B. **NUMERICAL SOLUTION-** The equations for *f* and θ are integrated numerically subject to appropriate boundary conditions by **Runge-Kutta-Gill** method with a **Shooting technique** using **FORTRAN code**. The accuracy of the method is tested by comparing appropriate results of the present analysis with available results. Results of present work for *C =1*, *Rd* = 0, $\gamma_{\mu}=0$ $\gamma_{k}=0$ (i.e., no magnetic field, no radiation, constant viscosity, constant thermal conductivity) are in very good agreement with those in [3]. Also our results for *C* = 0.5, 0.1, *Rd* = 0, $\gamma_{\mu}=0,-1,1$ $\gamma_{k}=0,-1,1$ and $\lambda=0$ (i.e., magnetic field, no radiation, constant viscosity, constant thermal conductivity and isothermal plate) agree very well with those of [3].

IV. DISCUSSION OF THE RESULTS

In this paper, VFP is used as an abbreviation for variable fluid properties and CFP as an abbreviation for constant fluid properties.

Variations in skin friction with different parameters are shown in figures 2, 3 and 4. Skin friction $f''(0)$ can be observed to take negative values for all the values of the parameters under consideration. From figs. 2 and 3 absolute values of $f''(0)$ can be observed to increase with diminishing intensity of magnetic field (i.e., *C* takes increasing values) and with diminishing effect of radiation (i.e., as *Rd* takes diminishing values). The maximum absolute value corresponds to *C*= 1 and *Rd=0* i.e., when there is no magnetic field and there is no transfer of heat energy through radiation.

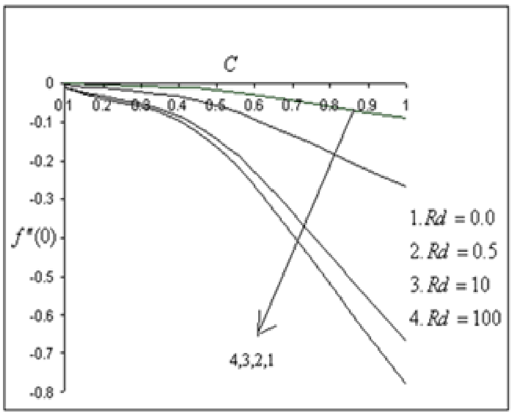

Fig 2 Variations of $f''(0)$ with C for $\gamma_{\mu}=-1$ & $\gamma_{k}=1$

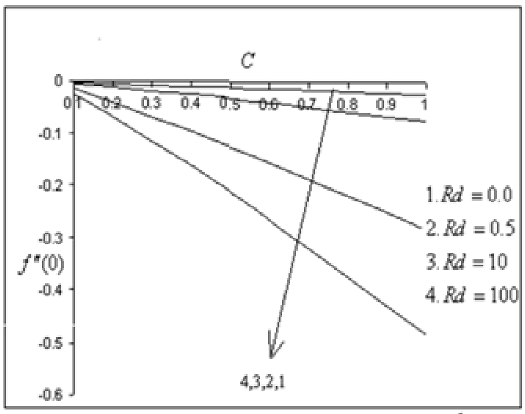

Fig 3 Variations of $f''(0)$ with C for $\gamma_{\mu}=1$ & $\gamma_{k}=-1$

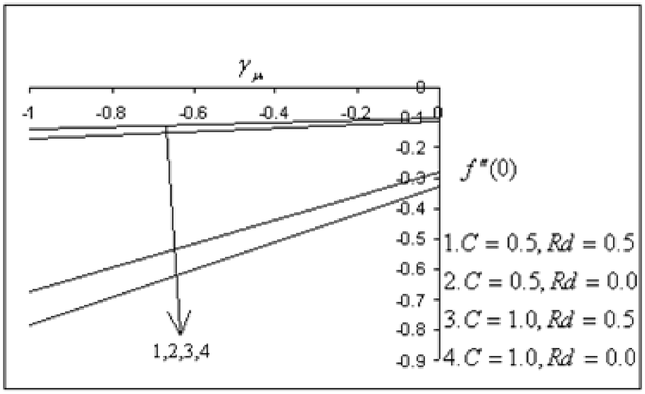

Fig. 4 Variations of $f''(0)$ with γ_μ for $\gamma_k = 1$

This behaviour of skin friction is true for liquids both for hot plate and cold plate. However, in the case of hot plate or upward flow the absolute values of $f''(0)$ are slightly larger than those of the other case. From fig. 4, absolute values of $f''(0)$ can be observed to be larger for negative values of γ_μ and increase as γ_μ approaches zero.

Variations in shear stress with different parameters in the presence of magnetic field are shown in fig. 5. The shear stress assumes a small negative value at the plate increases sharply and approaches zero away from the plate. Hydrodynamic boundary layer thickness can be seen to increase with increasing values of the Radiation parameter ***Rd***.

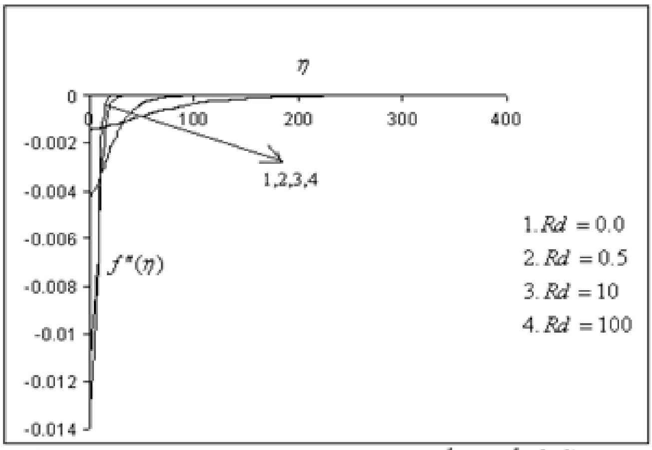

Fig. 5 Variations of $f''(\eta)$ with η for $\gamma_\mu = -1, \gamma_k = 1$ & $C = 0.1$

Variations in the heat transfer coefficient are presented in figs. 6, 7, and 8. In VFP case, heat transfer coefficient can be seen to take increasing values as the intensity of the magnetic field diminishes (as ***C*** takes increasing values) and also as the Radiation parameter ***Rd*** takes diminishing values. Heat transfer coefficient is also seen to take relatively larger values for $\gamma_\mu = 1, \gamma_k = -1$ then for $\gamma_\mu = -1$ and $\gamma_k = 1$. When γ_k is taken to be constant, heat transfer coefficient decreases as γ_μ changes from -1 to +1(it can be observe fig. 8). However with increasing intensity of magnetic field, heat transfer coefficient remains almost unaffected as γ_μ changes from -1 to +1.

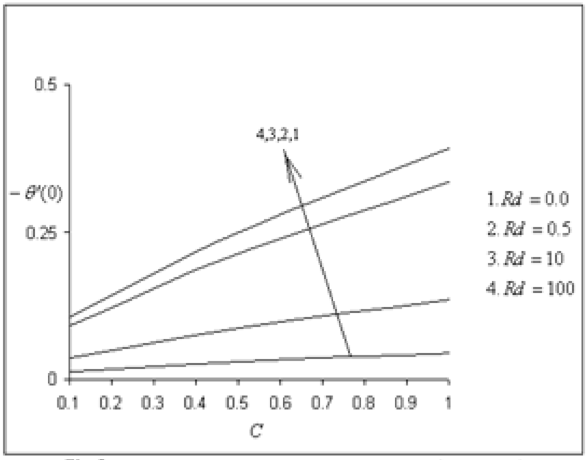

Fig. 6 Variations of '$-\theta'(0)$' with C for $\gamma_\mu = -1$ & $\gamma_k = 1$

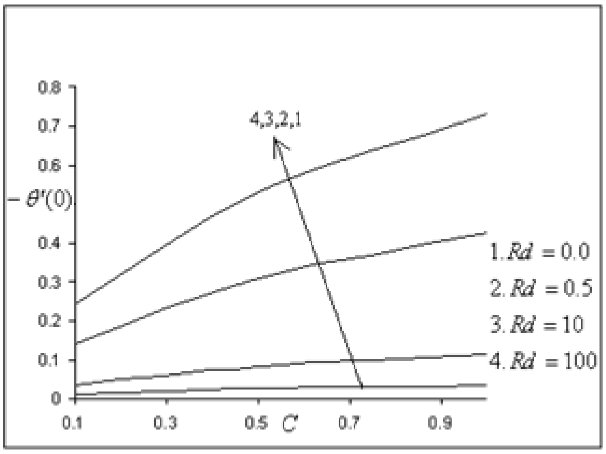

Fig. 7 Variations of '$-\theta'(0)$' with C for $\gamma_\mu = 1$ & $\gamma_k = -1$

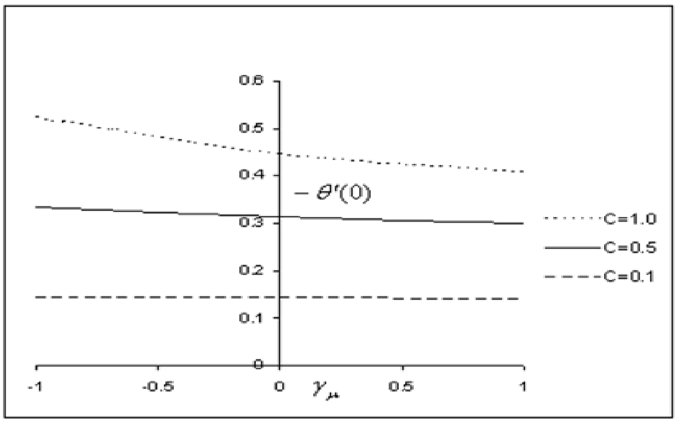

Fig. 8 Variations of '$-\theta'(0)$' with γ_μ for $\gamma_k = 0$ & $Rd = 0$

Fluid velocity profiles are presented in figs. 9, 10, 11 and 12 for different values of the parameters. Fluid velocity is seen to take a non-zero value at the plate diminish and approach zero away from the plate (as η takes large values). Hydrodynamic boundary layer thickness can be seen to very small for smaller values of *Rd* and increases considerably as *Rd* takes increasing values. Magnitudes of velocity diminish considerably and boundary layer thickness increases considerably as the intensity of the magnetic field increase (or as *C* takes diminishing values). (It can be observed from figs. 9 and fig. 10). Fluid velocity is also seen to take diminishing values for $\gamma_\mu = 1, \gamma_k = -1$ than for $\gamma_\mu = -1, \gamma_k = 1$ (compare figs-9, 11, and figs. 10, 12).

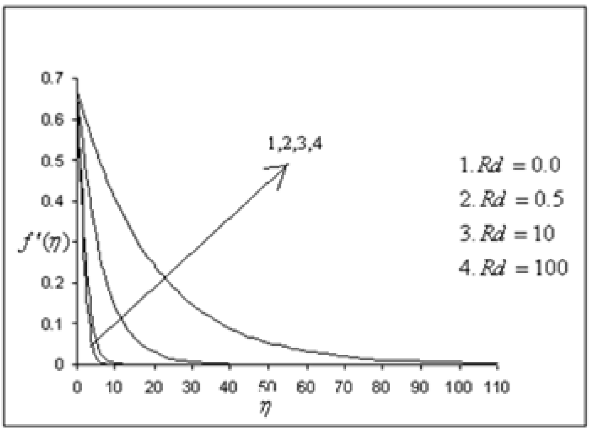

Fig .9 Variations of $f'(\eta)$ with η for $\gamma_\mu = -1$, $\gamma_k = 1$ & $C = 0.5$

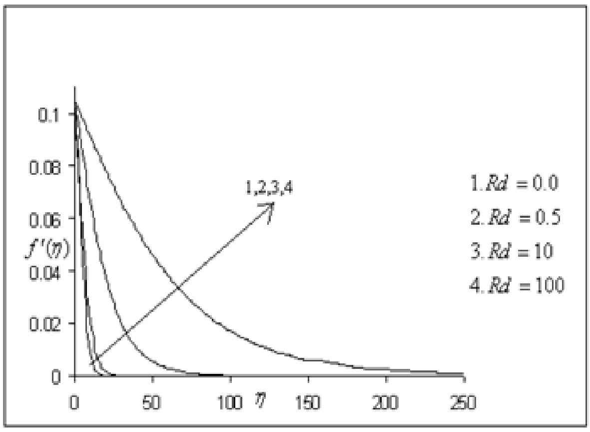

Fig 10 Variations of $f'(\eta)$ with η for $\gamma_\mu = -1, \gamma_k = 1$ & $C = 0.1$

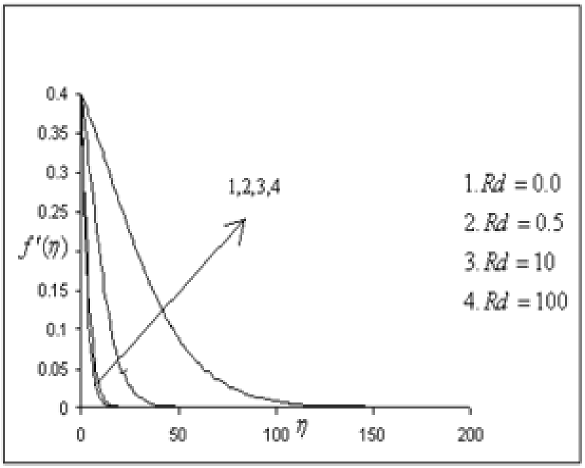

Fig 11 Variations of $f'(\eta)$ with η for $\gamma_\mu = 1$, $\gamma_k = -1$ & $C = 0.5$

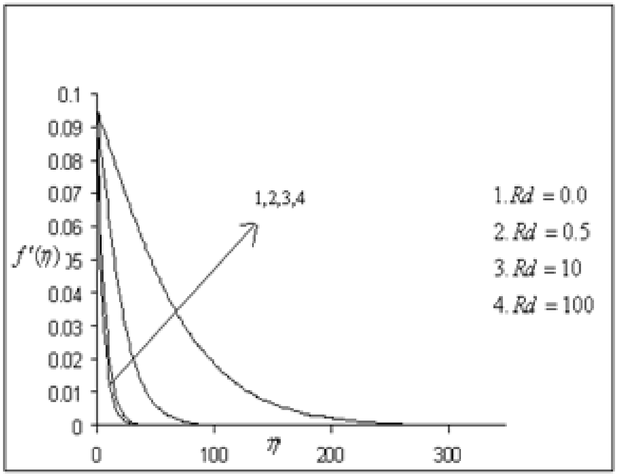

Fig .12 Variations of $f'(\eta)$ with η for $\gamma_\mu = 1$, $\gamma_k = -1$ & $C = 0.1$

Fluid temperature profiles are shown in figs. 13 and 14. Fluid temperature is seen to diminish from unity (the value at the plate) and approach zero far away from the plate. Thermal boundary layer thickness can be observed to increase with increasing intensity of radiation (or as *Rd* takes increasing values).Thermal boundary layer thickness is also seen to increase for $\gamma_\mu = 1, \gamma_k = -1$ than for $\gamma_\mu = -1, \gamma_k = 1$.

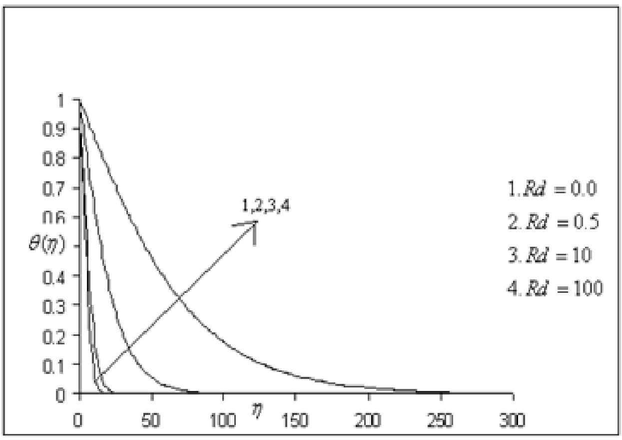

Fig.13 Variations of $\theta(\eta)$ **with** η **for** $\gamma_\mu = -1, \gamma_k = 1$ & $C = 0.1$

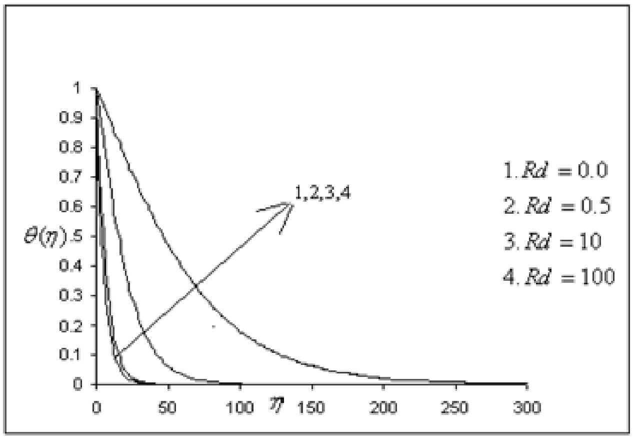

Fig 14 Variations of $\theta(\eta)$ **with** η **for** $\gamma_\mu = 1, \gamma_k = -1$ & $C = 0.1$

Slip velocity profile is shown in the fig. 15. Slip velocity also assumes larger values when γ_μ is negative. Since, when γ_μ is negative, the plate will be at higher temperature than the ambient fluid and hence enhanced flow can be expected . Also, it is clear from fig. 15, slip velocity diminish with increasing intensity of the magnetic field (i.e., as *C* diminishes). Reduced flow can be expected for smaller values of *C* or for increased intensity of the magnetic field as the magnetic field lines obstruct the flow discussed in (III. A). From Equation (13) slip velocity values can be calculated directly and also slip velocity will not varies as *Rd* increases from 0 to 100.

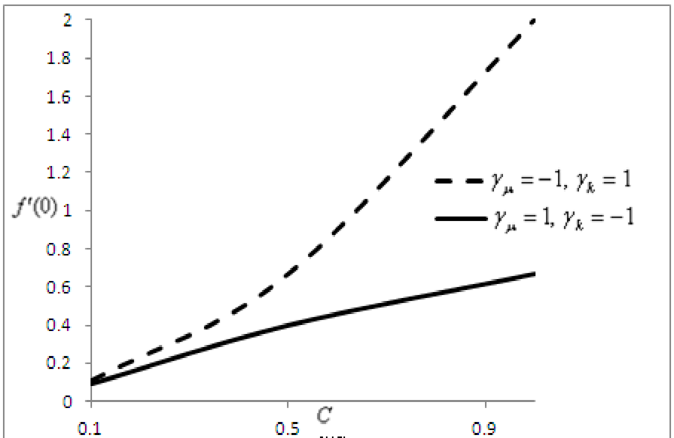

Fig 15 Variations of $f'(0)$ **with** C

V. CONCLUSIONS

1. Hydrodynamic boundary layer as well as thermal boundary layer increases with increasing intensity of radiation (***Rd***).
2. Thermal boundary layer thickness is also seen increase for fluid like Methyl Chloride (for $\gamma_\mu > 0, \gamma_k < 0$) (cold plate) than for fluid like Dichloro Fluro Methane (for $\gamma_\mu < 0, \gamma_k > 0$) (hot plate).
3. Heat transfer coefficient $(-\theta'(0))$ takes larger values with diminishing values of *C* (intensity of the magnetic field increases) and also the radiation parameter (***Rd***) takes diminishing values (the intensity of the radiation decreases).
4. Heat transfer coefficient is relatively larger values for fluid like Methyl Chloride (for $\gamma_\mu > 0, \gamma_k < 0$) (cold plate) than for fluid like Dichloro Fluro Methane (for $\gamma_\mu < 0, \gamma_k > 0$) (hot plate).
5. The absolute value of the skin friction $(f''(0))$ increases with diminishing intensity of the magnetic field and with diminishing effect of radiation.
6. The absolute value of the skin friction $(f''(0))$larger for fluid like Dichloro Fluro Methane (for $\gamma_\mu < 0, \gamma_k > 0$) (hot plate) than for fluid like Methyl Chloride (for $\gamma_\mu > 0, \gamma_k < 0$) (cold plate).
7. The value of the slip velocity $(f'(0))$ is larger for fluid like Dichloro Fluro Methane (for $\gamma_\mu < 0, \gamma_k > 0$) (hot plate) than for fluid like Methyl Chloride (for $\gamma_\mu > 0, \gamma_k < 0$) (cold plate).

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