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Principle Component Analysis in Image Processing

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Abstract— In this paper we address the Principle Component Analysis (PCA) and their application has been done in the area of image processing. Exploring basic theory of face recognition and mathematical procedure to transform a number of correlated variables in a number of uncorrelated variables and image processing toolbox written in open MATLAB have been studied. The Principle Components Analysis (PCA) ultimately reduces the number of effective variables used for classification which are compared with some statistical method.

Keywords— Principle Component Analysis, Recognition, Eigen Face.

I. Introduction

The Principal Component Analysis (PCA) is one of the most successful techniques that have been used in image recognition and compression. PCA is statistical method under the broad title of factor analysis. The purpose of PCA is to reduce the large dimensionality of the data space (observed variables) to the smaller intrinsic dimensionality of feature space (independent variables), which are needed to describe the data economically.[1] This is the case when there is a strong correlation between observed variables and in case of Image processing Toolbox provides a comprehensive set of reference standard algorithms and graphical tools for image processing, analysis, visualization, and algorithm development and most toolbox functions are written in the open MATLAB® language, giving we the ability to inspect the algorithms, modify the source code, and create our own custom functions. Image Processing Toolbox supports engineer and scientists in areas such as biometrics, remote sensing, surveillance, gene expression, microscopy, semiconductor testing, image sensor design, color science, and materials science. It also facilitates the learning and teaching of image processing techniques. We can restore noisy or degraded images, enhance images for improved intelligibility, extract features, analyze shapes and textures, and register two images. Face recognition has many applicable areas. Moreover, it can be categorized into face identification, face classification, or sex determination.[2] The most useful applications contain crowd surveillance, video content indexing, personal identification (ex. driver's licence), mug shots matching, entrance security, etc. The main idea of using PCA for face recognition is to express the large 1-D vector of pixels constructed from 2-D facial image into the compact principal components of the feature space. This can be called eigenspace projection. Eigenspace is calculated by identifying the eigenvectors of the covariance matrix derived from a set of facial images (vectors). PCA is a statistical method under the broad title of factor analysis. The purpose of PCA is to reduce the large dimensionality of the data space (observed variables) to the smaller intrinsic dimensionality of feature space (independent variables), which are needed to describe the data economically.[3] This is the case when there is a strong correlation between observed variables.

II. LITITERATURE SURVEY

1. Principle Component Analysis (PCA)

Principal component analysis (PCA) was invented in 1901 by Karl Pearson. PCA is a variable reduction procedure and useful when obtained data have some redundancy. This will result into reduction of variables into smaller number of variables which are called Principal Components which will account for the most of the variance in the observed variable. Problems arise when we wish to perform recognition in a high-dimensional space. Goal of PCA is to reduce the dimensionality of the data by retaining as much as variation possible in our original data set. On the other hand dimensionality reduction implies information loss. The best low-dimensional space can be determined by best principal components. The major advantage of PCA is using it in eigenface approach which helps in reducing the size of the database for recognition of a test images. The images are stored as their feature vectors in the database which are found out projecting each and every trained image to the set of Eigen faces obtained.[4] PCA is applied on Eigen face approach to reduce the dimensionality of a large data set.

2. Eigen Face Approach

It is adequate and efficient method to be used in face recognition due to its simplicity, speed and learning capability. Eigen faces are a set of Eigen vectors used in the Computer Vision problem of human face recognition. They refer to an appearance based approach to face recognition that seeks to capture the variation in a collection of face images and use this information to encode and compare images of individual faces in a holistic manner. The Eigen faces are Principal Components of a distribution of faces, or equivalently, the Eigen vectors of the covariance matrix of the set of the face images, where an image with N by N pixels is considered a point in N 2 dimensional space. Previous work on face recognition ignored the issue of face stimulus, assuming that predefined measurement were relevant and sufficient. This suggests that coding and decoding of face images may give information of face images emphasizing the significance of features. These features may or may not be related to facial features such as eyes, nose, lips and hairs. We want to extract the relevant information in a face image, encode it efficiently and compare one face encoding with a database of faces encoded similarly. A simple approach to extracting the information content in an image of a face is to somehow capture the variation in a collection of face images. We wish to find Principal Components of the distribution of faces, or the Eigen vectors of the covariance matrix of the set of face images. Each image location contributes to each Eigen vector, so that we can display the Eigen vector as a sort of face. Each face image can be represented exactly in terms of linear combination of the Eigen faces. The number of possible Eigen faces is equal to the number of face image in the training set. The faces can also be approximated by using best Eigen face, those that have the largest Eigen values, and which therefore account for most variance between the set of face images. The primary reason for using fewer Eigen faces is computational efficiency.[5]

2.1. Eigen Values and Eigen Vector

In linear algebra when operated by the operator, result in a scalar multiple of them. Scalar is then called Eigen value (λ) associated with the eigenvector (X). Eigen vector is a vector that is scaled by linear transformation. It is a property of matrix. When a matrix acts on it, only the vector magnitude is changed not the direction.

$$AX = \lambda X, \quad \text{where A is a vector function.}$$

$$(A - \lambda I)X = 0, \text{ where I is the identity matrix.}$$

This is a homogeneous system of equations and form fundamental linear algebra. We know a non-trivial solution exists if and only if-

$$Det(A - \lambda I) = 0, \text{ where det denotes determinant.}$$

When evaluated becomes a polynomial of degree n. This is called characteristic polynomial of A. If A is N by N then there are n solutions or n roots of the characteristic polynomial. Thus there are n Eigen values of A satisfying the equation.

$$AXi = \lambda\, iXi \text{ , where i = 1,2,3,.....n}$$

If the Eigen values are all distinct, there are n associated linearly independent eigenvectors, whose directions are unique, which span an n dimensional Euclidean space.

2.2. Face Image Representation

Training set of m images of size N×N are represented by vectors of size N2.

Each face is represented by $\Gamma1$, $\Gamma2$, $\Gamma3$, $\Gamma4$, ,Γm.

Feature vector of a face is stored in a N×N matrix. Now, this two dimensional vector is changed to one dimensional vector.

2.3. Eigen Face Space

The Eigen vectors of the covariance matrix AAT are AXi which is denoted by Ui. Ui resembles facial images which look ghostly and are called Eigen faces. Eigen vectors correspond to each Eigen face in the face space and discard the faces for which Eigen values are zero thus reducing the Eigen face space to an extent. The Eigen faces are ranked according to their usefulness in characterizing the variation among the images.

A face image can be projected into this face space by

$\Omega k = UT (\Gamma k - \Psi)$; k=1,....,M, where ($\Gamma k\ \Psi$) is the mean centered image.

Hence projection of each image can be obtained as $\Omega1$ for projection of image1 and $\Omega2$ for projection of image2.

III. FACE RECOGNITION

Once the eigenfaces have been computed, several types of decision can be made depending on the application. What we call face recognition is a broad term which may be further specified to one of following tasks:

- identification where the labels of individuals must be obtained,
- recognition of a person, where it must be decided if the individual has already been seen,
- categorization where the face must be assigned to a certain class.

PCA computes the basis of a space which is represented by its training vectors. These basis vectors, actually eigenvectors, computed by PCA are in the direction of the largest variance of the training vectors. As it has been said earlier, we call them eigenfaces. Each eigenface can be viewed a feature. When a particular face is projected onto the face space, its vector into the face space describes the importance of each of those features in the face. The face is expressed in the face space by its eigenface coefficients (or weights). We can handle a large input vector, facial image, only by taking its small weight vector in the face space. This means that we can reconstruct the original face with some error, since the dimensionality of the image space is much larger than that of face space. In this report, let's consider face identification only. Each face in the training set is transformed into the face space and its components are stored in memory. The face space has to be populated with these known faces. An input face is given to the system, and then it is projected onto the face space. The system computes its distance from all the stored faces.[6],[7]

However, two issues should be carefully considered:

1. What if the image presented to the system is not a face?
2. What if the face presented to the system has not already learned, i.e., not stored as a known face?

The first defect is easily avoided since the first eigenface is a good face filter which can test whether each image is highly correlated with itself. The images with a low correlation can be rejected. Or these two issues are altogether addressed by categorizing following four different regions:

1. Near face space and near stored face =*)* known faces
2. Near face space but not near a known face =*)* unknown faces
3. Distant from face space and near a face class =*)* non-faces
4. Distant from face space and not near a known class =*)* non-faces

Since a face is well represented by the face space, its reconstruction should be similar to the original, hence the reconstruction error will be small. Non-face images will have a large reconstruction error which is larger than some threshold *µr*. The distance ²*k* determines whether the input face is near a known face.

IV. Pca Algorithm

A number of algorithms have been proposed to extract the first p principal components from d dimensional (d>p) stochastic process with the help of a neural network. These algorithms are advantageous in adapting online to the data and thus no explicit computation of the covariance matrix and its eigenvalue decomposition is necessary. In local Hebbian type learning algorithms, the modification of the Ith row of the Weight matrix W between input and output layer depends only on the Ith output unit and the input. Due to this locality it has been argued that these algorithms are "biologically plausible". In such local algorithms have been analyzed and it was shown that only one part of these algorithms, the asymmetric algorithms, where there is a hierarchical connection between the output units, can lead to asymptotically stable equilibrium, whereas algorithms with a symmetric connection in the output layer is not asymptotically stable. Thus, it was concluded that asymmetric algorithms should be preferred, although they disallow "competition" and lack symmetry.

1. PCA Mathematical Modelling

A 2-D facial image can be represented as 1-D vector by concatenating each row (or column) into a long thin vector. Let's suppose we have M vectors of size N (= rows of image×columns of image) representing a set of sampled images. pj's represent the pixel values.

xi = [p1············ pn]T , *i* = 1··········M

The images are mean centered by subtracting the mean image from each image vector. Let m represent the mean image.

$$\mathrm{m} = \frac{1}{m}\sum_{k=1}^{m} \mathrm{Xi}$$

And let *wi* be defined as mean centered image

$$wi = xi - m$$

Our goal is to find a set of ei's which have the largest possible projection onto each of the wi's. We wish to find a set of M orthonormal vectors ei for which the quantity is maximized with the orthonormality constraint

$$\lambda \mathrm{i} = \frac{1}{m} + \sum_{n=1}^{m}(\mathrm{ei}^{\mathrm{T}}\mathrm{Wn})^2$$

$$e_l^T e_k = \delta lk$$

It has been shown that the ei's and ¸i's are given by the eigenvectors and eigenvalues of the covariance matrix

$$C = WW^T$$

Where W is a matrix composed of the column vectors wi placed side by side. The size of C is N × N which could be enormous. For example, images of size 64×64 create the covariance matrix of size 4096× 4096. It is not practical to solve for the eigenvectors of C directly. A common theorem in linear algebra states that the vectors ei and scalars ʎ i can be obtained by solving for the eigenvectors and eigenvalues of the M ×M matrix WTW.

Let di and µi be the eigenvectors and eigenvalues of WTW respectively.

$$W^T W.di = \mu i di$$

By multiplying left to both sides by *W*

$$WW^T(Wdi) = \mu i(Wdi)$$

Which means that the first M -1 eigenvectors ei and eigenvalues ʎ i of WWT are given by Wdi and µi respectively. Wdi needs to be normalized in order to be equal to ei. Since we only sum up a finite number of image vectors, M, the rank of the covariance matrix cannot exceed M - 1 (The -1 come from the subtraction of the mean vector m). The eigenvectors corresponding to nonzero eigenvalues of the covariance matrix produce an orthonormal basis for the subspace within which most image data can be represented with a small amount of error. The eigenvectors are sorted from high to low according to their corresponding eigenvalues. The eigenvector associated with the largest eigenvalue is one that reflects the greatest variance in the image. That is, the smallest eigenvalue is associated with the eigenvector that finds the least variance. They decrease in exponential fashion, meaning that the roughly 90% of the total variance is contained in the first 5% to 10% of the dimensions.

A facial image can be projected onto M'' (<< M) dimensions by computing

$$\Omega= [v1v2............v^{m'}]\ T$$

Where vi = eTiwi, vi is the Ith coordinate of the facial image in the new space, which came to be the principal component. The vectors ei are also images, so called, eigenimages, or eigenfaces in our case. They can be viewed as images and indeed look like faces. So Ω describes the contribution of each eigenfaces in representing the facial image by treating the eigenfaces as a basis set for facial images. The simplest method for determining which face class provides the best description of an input facial image is to find the face class k that minimizes the Euclidean distance

$$Ek =\|((\Omega - \Omega K))\|$$

Where ΩK is a vector describing the kTh face class? If Ek is less than some predefined threshold θK a face is classified as belonging to the class k.

V. Conclusion

We can conclude that Principal component analysis has been used in a number of fields. Case studies show that using formultivariate analysis, PCA analysis performs much better than in reducing the number of independent variables than those by Statistical analysis. In future, some of small principle component may have better performance of classification than the selected large principle components. So we should add some smaller principle component while removing some large principle component.

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