



Realization of High Order IIR Digital Bandstop Filter Using Domain Transfer Algorithm

Subhadeep Chakraborty

Department of Electronics and Communication Engineering
West Bengal University of Technology, India.

Abstract— Filter is one of the most important device to perform the filtering operation for reduction of unwanted components from the actual signal. Bandstop filter is one of the important filter in Signal Processing. In Digital Signal Processing, there are mainly two type of filters, one is Infinite Impulse Response(IIR) Filter and another is the Finite Impulse Response(FIR) Filte. The first one is of recursive type meaning that the present output depends upon the past input, past outputs and the present input but the FIR filter is of nonrecursive type. Domain Transfer Algorithm (DTA) is applied to the predesigned analog filter to design its equivalent digital filter with less error and with increased computational speed. The Magnitude response shows the accuracy and the Pole-Zero plot shows the stability of the filter.

Keywords— Bandstop filter, Domain Transfer Algorithm,coefficient,realization,digital filter,high order filter

I. INTRODUCTION

Digital signal processing(DSP) deals with only the digital filter. The digital filter cannot be achieved or design directly. The digital filter can be designed from a predesigned analog filter with the application of domain transfer technique. The transfer function of the analog filter can be calculated in analog domain or s-domain on s-plane only. The Domain Transfer Algorithm (DTA) converts or maps the transfer function from s-domain to digital domain or z-domain[1][2][3]. Only after this operation, the Digital filter can be designed. The digital filter can performs its operation of filtration of signal in time domain. There are various types of filter available for the filtering operation. Bandstop filter is one of the category that eliminates the undesired frequency band. The Notch filter is a variation of Bandstop filter that is capable of eliminating a single frequency[4][6][9]. The analog filter can be designed from active or passive elements. Depending upon the designing elements, the analog filter can be categorized into two types, one is the active filter that is designed with the active elements such as the voltage source or current source and the other is the passive filter that is designed with the passive elements such as the resistor, capacitor and inductors[4][5][6][9][9][10]. In practical use, the IIR filter is designed with the OpAmp as it can show the recursive effect which is essential for the IIR filter. A number of methods are available to convert the domain from analog to digital for successful design of a digital filter, such as the Analog to Digital Mapping Technique, Modified Analog to Digital Mapping Technique, Bilinear Transform etc[18][26][27][31]. The Domain transfer technique requires less time to transfer the domain successfully and efficiently. The digital filter have several advantages and features like its small physical size, sensitivity, high accuracy, drift[1][2][3][12][9][10][11][12]. In this paper, a new algorithm, DTA is proposed to reduce the computational time and successful design of the High order Digital Filter.

II. ANALOG FILTER DESIGN

The analog filter can be designed from active components as well as the passive components also. The corresponding design of the Active filter and the passive filter are shown below.

A. Passive Filter

The passive filter is designed by using the resistance, capacitor and inductor. If the passive filter is designed with only inductor and capacitor, it is called LC-Filter and if the design involves the resistor and capacitor, it is called RC-Filter. The design of LC-Filter and the RC-Filter are shown below[12][28][29].

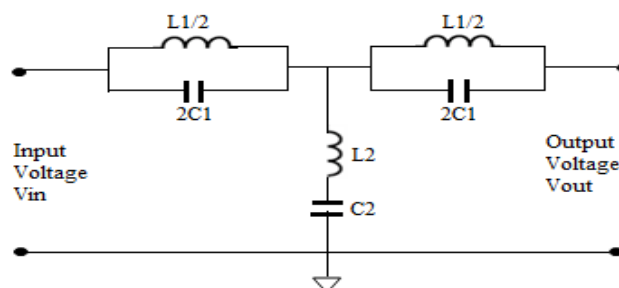


Fig.1 T-section LC Bandstop Filter

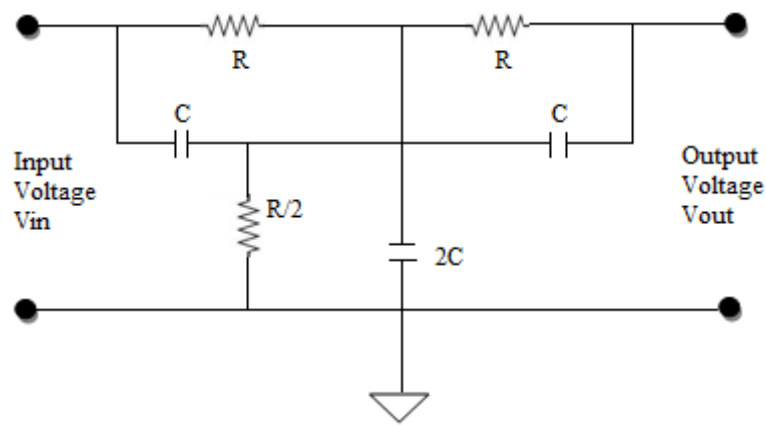


Fig.2 T-section RC Bandstop filter

The rejected frequency band can be determined by the proper selection of the value of R, L and C. Now, the transfer function of the above mentioned filters are in analog domain. The introduction of the analog to digital mapping technique maps the transfer function from analog domain or s-domain to digital domain or z-domain [2][7][12][27][31].

B. Active Filter

Active filter is designed with the active elements along with the required passive elements. The OpAmp chip is required to design the IIR filter because of its recursive effect. The active IIR bandstop filter is shown in the following figure.

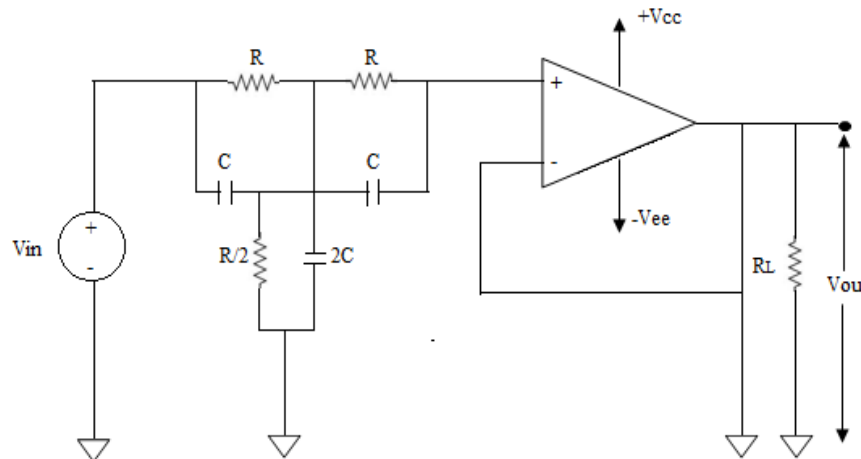


Fig.3 Active IIR Band stop filter

III. ANALOG TO DIGITAL MAPPING

ANALOG TO DIGITAL MAPPING IS VERY ESSENTIAL FOR THE DESIGN OF A DIGITAL FILTER. THIS IS ESSENTIALLY REQUIRED FOR THE CALCULATION OF THE TRANSFER FUNCTION OF THE DIGITAL FILTER IN Z-PLANE[2][7][12][18][21][31].

A. Transfer function in digital plane

The transfer function of an analog filter in s-plane can be mapped into z-plane with the application of bilinear transform. Bilinear transform operates on the analog plane to convert it to z-plane using the following equation[12][27],

$$z = \frac{1 + s}{1 - s} \tag{1}$$

where,

s = complex variable
 $= s = \sigma + j\Omega$

Now, putting the value of s into equation (1), we get,

$$z = \frac{1 + (\sigma + j\Omega)}{1 - (\sigma + j\Omega)} \tag{2}$$

$$\Rightarrow z = \frac{(1 + \sigma) + j\Omega}{(1 - \sigma) - j\Omega}$$

$$\Rightarrow |z|^2 = \frac{(1 + \sigma)^2 + \Omega^2}{(1 - \sigma)^2 + \Omega^2} \dots\dots\dots(3)$$

The transfer function of an IIR Digital filter can be described by the following equation[1][2][4][7][10][14][27][31],

$$H(z) = \frac{\sum_{n=0}^M b(n)z^{-n}}{1 + \sum_{n=1}^N a(n)z^{-n}} \dots\dots\dots(4)$$

$$\Rightarrow H(z) = \frac{B(z)}{A(z)}$$

$$\Rightarrow H(z) = \frac{b(0) + b(1)z^{-1} + b(2)z^{-2} + \dots\dots\dots + b(M)z^{-M}}{1 + a(1)z^{-1} + a(2)z^{-2} + \dots\dots\dots + a(N)z^{-N}} \dots\dots\dots(5)$$

Where,

H(z) = Transfer function and Z-transform of impulse response h(n)

b(n) = Numerator coefficient

a(n) = Denominator coefficient

Now, for a realizable filter, h(n) and H(z) can be described by[4][5][12][19][27][28][29],

$$h(n) = 0 \quad \text{for } n \leq 0 \dots\dots\dots(6)$$

$$\sum_{n=0}^{\infty} |h(n)| < \infty \dots\dots\dots(7)$$

$$H(z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n} \dots\dots\dots(8)$$

Equation (7) shows the satisfactory condition for a stable filter. Now, it is necessary to determine the transfer function of a digital filter i.e. of z-domain from the transfer function of s-domain. Next subsection shows the necessary derivation of the transfer function in z-domain.

B. Determination of Transfer Function in z-plane

Let we take the impulse response of a realizable filter in time domain is h(t). If the Laplace Transform is applied on h(t), the transfer function in s-plane can be achieved[3][17][20][27][31] by,

$$H(s) = L\{h(t)\} = \int_0^{\infty} h(t).e^{-st} dt$$

.....(9)

Now, for continuous time signal, the t of h(t) can be replaced by nT, that is,

$$h(t) = h(nT) \dots\dots\dots(10)$$

Where, T=Sampling time

If, T=1, then equation (10) becomes,

$$h(t) = h(n) \dots\dots\dots(11)$$

Now, h(n) is the impulse response in z-plane. So, in this process, the impulse response of z-plane can be obtained from the impulse response in s-plane. If the Z-Transform is applied on h(n), the transfer function in z-plane can be obtained[2][3][5][7][12][15].

$$H(z) = Z\{h(n)\} = \sum_{n=-\infty}^{\infty} h(n)z^{-n}$$

.....(12)

IV. DOMAIN TRANSFER ALGORITHM

This algorithm maps the transfer function from s-plane to z-plane efficiently. This is also a time consuming algorithm that perform faster with accuracy in the mapping operation. The algorithm is described below.

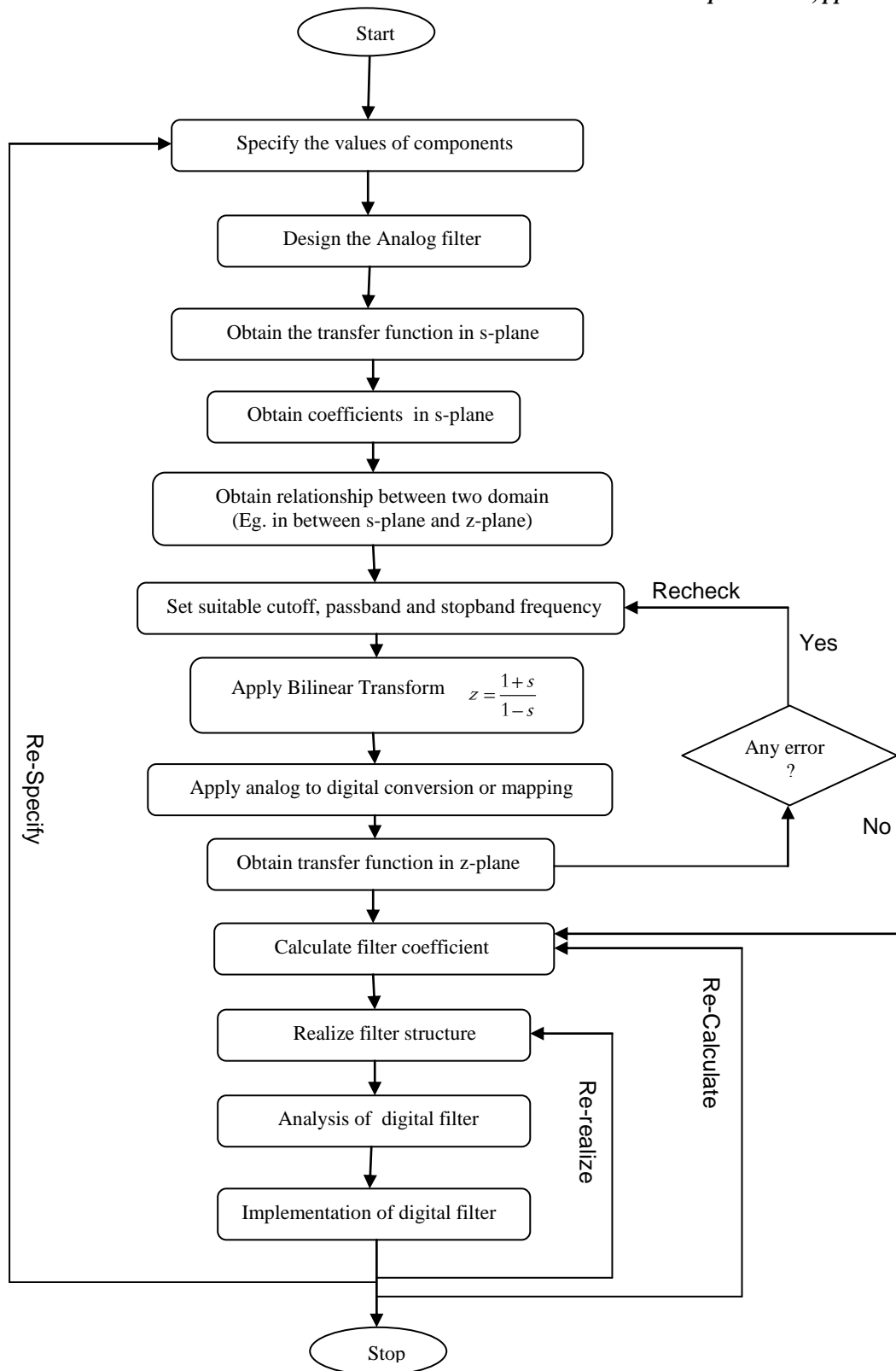


Fig.4 Domain Transfer Algorithm (DTA)

So, by applying the DTA, the preferred IIR Digital filter can be constructed. The stability of the generated digital filter can be found its pole-zero plot.

V. REALIZATION OF IIR DIGITAL BANDSTOP FILTER

IIR filter can be realized in various as there are various methods are available for the realization. The methods are listed below[12][14][21][22][25][27],

1. Direct form - I realization

2. Direct form - II realization
3. Transposed direct form realization
4. Cascade form realization
5. Parallel form realization
6. Lattice-Ladder structure realization

Let we consider the filter to be an Linear Time-Invariant (LTI) recursive system where $x(n)$ is the input sequence and $y(n)$ is the output sequence and can be described by difference equation[18][19][23][24][27],

$$y(n) = -\sum_{k=1}^N a_k y(n-k) + \sum_{k=0}^M b_k x(n-k)$$

.....(13)

Taking Z-Transform at both sides, we get,

$$Z\{y(n)\} = Z\{-\sum_{k=1}^N a_k y(n-k)\} + Z\{\sum_{k=0}^M b_k x(n-k)\} \quad \dots\dots$$

(14)

Arranging both sides we get,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=0}^N a_k z^{-k}} \quad \dots\dots$$

.....(15)

Let,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \bullet \frac{W(z)}{X(z)} \quad \dots\dots(16)$$

Where,

$$W(z) = \frac{1}{1 + \sum_{k=0}^N a_k z^{-k}}$$

.....(17)

Rearranging equation (17), we obtain that,

$$W(z) = X(z) - a_1 z^{-1}W(z) - a_2 z^{-2}W(z) \dots - a_N z^{-N}W(z) \quad \dots\dots(18)$$

Again,

$$\frac{Y(z)}{W(z)} = \sum_{k=0}^M b_k z^{-k} \quad \dots\dots$$

.....(19)

Rearranging equation (19), we obtain that,

$$Y(z) = b_0 W(z) + b_1 z^{-1}W(z) + b_2 z^{-2}W(z) \dots + b_M z^{-M}W(z) \quad \dots\dots(20)$$

Now equation (18) and equation (20) can be combined and can be expressed in difference equations that are,

$$w(n) = x(n) - a_1 w(n-1) - a_2 w(n-2) - \dots - a_N w(n-N) \quad \dots\dots(21)$$

$$y(n) = b_0w(n) + b_1w(n-1) + b_2w(n-2) - \dots - b_Mw(n-M) \quad \dots\dots(22)$$

We can see from the above difference equations that for an IIR filter, its present output is dependent upon the past inputs, past outputs and present input. This can be graphically represented in figure by applying Direct Form-II realization. So, the Direct Form-II realization of IIR Bandstop filter is shown below[1][2][3][6][8][14][20][23][26][27],

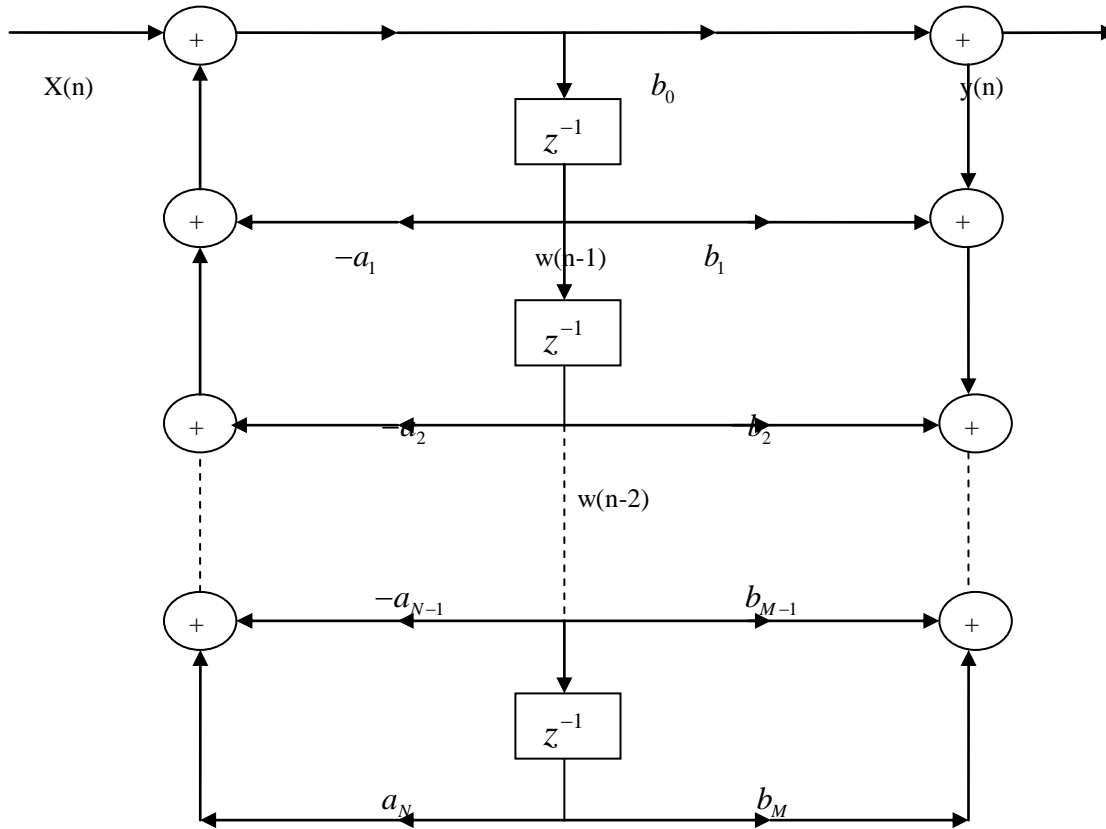


Fig. 5 Direct Form-II realization of IIR Bandstop filter

VI. SIMULATION RESULT

In the previous sections, the design and realization of IIR Bandstop Filter are shown. After successful realization and derivation of the required designing equations, the simulations are done. The required program for designing the Digital IIR Bandstop filter are simulated in Matlab7 (R2008a). After successful simulation without errors, the magnitude response, phaseresponse, impulse response and pole-zero plotting are obtained. Magnitude response actually shows the steady response at the output and pole-zero plot determines the stability of the filter by placing all the poles inside an unit circle. If the some or all of the poles are outside the unit circle, the system is called as an unstable system. Fig. 6 and Fig.7 shows a stable and unstable system respectively.

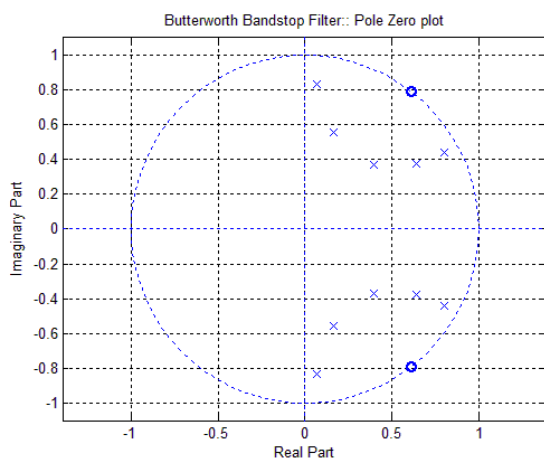


Fig.6 Stable IIR Bandstop filter (Order=5)

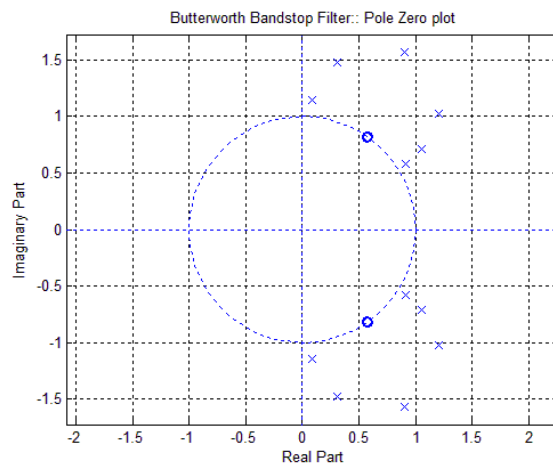


Fig.6 Unstable IIR Bandstop filter (Order=5)

Coefficient is the essential component for designing the digital filter. Coefficient of a filter is divided into two: one is Numerator coefficient and another is the Denominator coefficient. If the coefficients are calculated correctly, the design of the digital filter will be without error and shows the perfect result and satisfy with the stability. The Numerator coefficients and Denominator coefficients of the designed IIR Bandstop filter of order 18 and order 24 are shown in Table 1. The the magnitude response, phaseresponse, impulse response and pole-zero plotting are shown for the Bandstop IIR filetr of order 18 and order 24 from Fig. 7 to Fig. 10 and Fig.11 to Fig. 14. respectively.

Table 1

Filter Name	Order of Filter	Numerator Coefficients	Denominator Coefficients
Butterworth Bandstop Filter	18	1.399 , 24.78 ,26.41 ,237.1 , 217.4 , 1312 ,1020 , 4635 ,2984 , 1.083e004 , 5572 , 1.675e004 , 6485 ,1.653e004 , 4302 ,9435, 1245 , 2375	5.267 , 35.94 , 127.5 , 483.2 - 1262 , 3331 , - 6660 , 1.317e004 , 2.042e004,3.113e004 , - 3.719e004 , 4.391e004 , - 3.933e004 , 3.546e004 , - 2.211e004 , 1.463e004,- 5068 z , 2243
	24	1.664 , 28.28 , 36.97 , 320.5 , 372.4 , 2184 ,2245 , 9974 , 9000 , 3.216e004 , 2.52e004 7.511e004,5.026e004, 1.28e005, 7.144e004 ,1.579e005 , 7.091e004, 1.376e005 ,4.68e004 , 8.035e004, .849e004 , 2.822e004 , 3311 ,4505	- 4.717 , 35.88,- 124.5,529.4 , - 1441 , 4371 , - 9659 , 2.28e004 - 4.165e004,7.966e004 , - 1.213e005 , 1.918e005 , - 2.436e005 ,+ 3.21e005 , - 3.371e005 , 3.702e005 , - 3.15e005 , 2.858e005 , - 1.893e005 , 1.392e005 , - 6.579e004 , 3.789e004 , - 1.002e004 4228

The simulation results of IIR Butterworth filter of order 18 and 24 are shown below,

A. Butterworth Filter order = 18

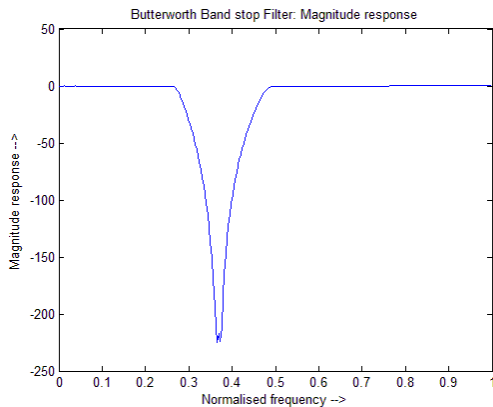


Fig.7 Magnitude response

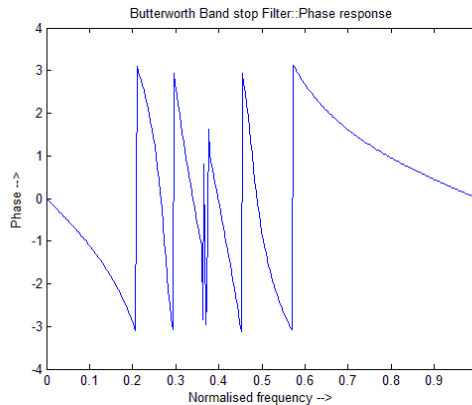


Fig.8 Phase response

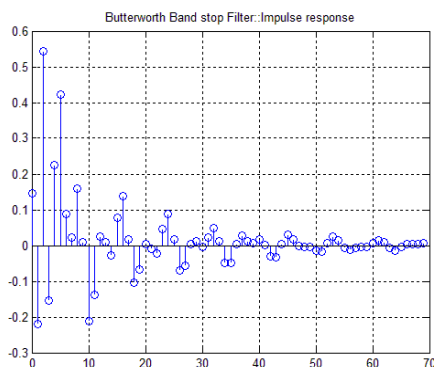


Fig.9 Impulse response

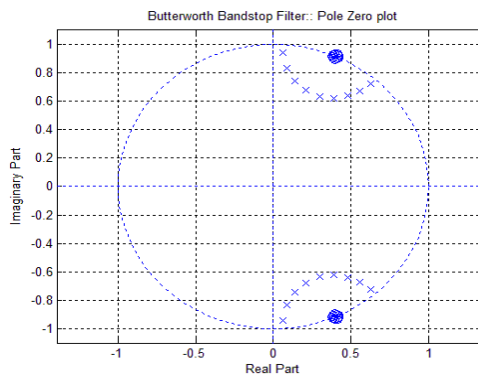


Fig.10 Pole-zero plot

Butterworth Filter order = 24

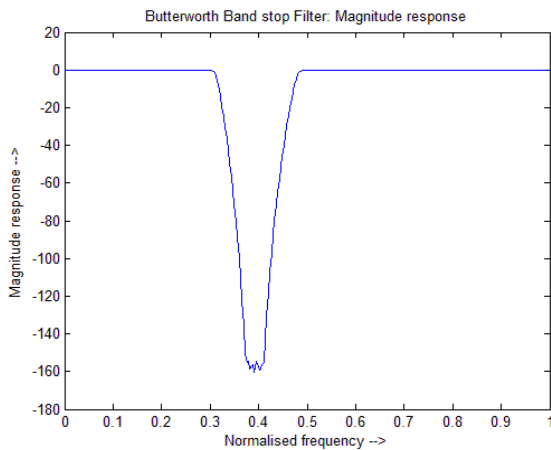


Fig.11 Magnitude response

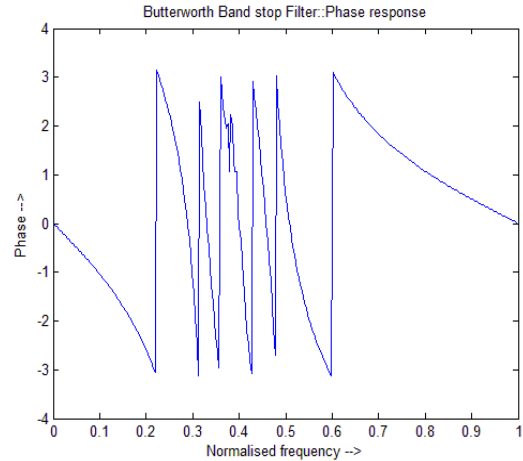


Fig.10 Phase response

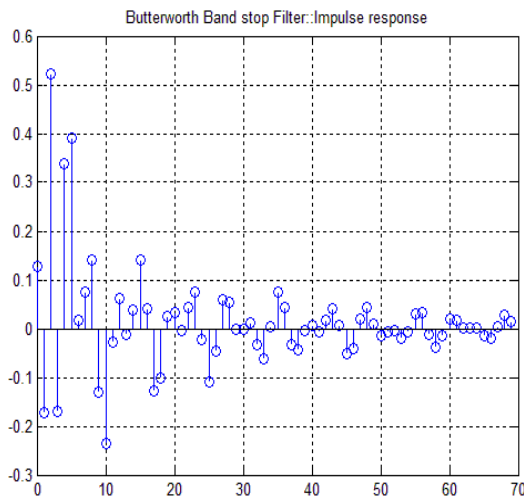


Fig.9 Impulse response

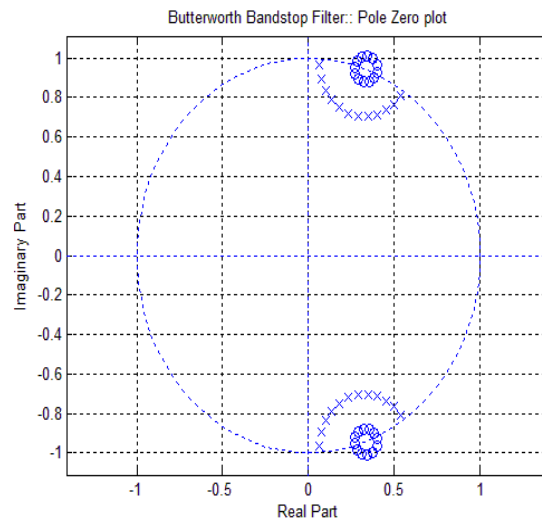


Fig.10 Pole-zero plot

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