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Design Technique for an MP Driver Based on the Theory of Automata – A Case Study

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***Abstract*— This paper deals with the use of the theory of automata to design a Mathematical Pathway [MP] driver which lets a child progress through the Mathematical learning pathway in a systematic way. The MP driver is an integral part of the Learning model. The Concept flow graph constructed using graph theoretical approach is used to design the automata machines for the various concepts of Mathematics that a child has to learn between 0 and 12 years. The Learning Finite State machines not only ensure smooth learning transitions but also track the progress of the child. Using this, the learning curve of a child can be obtained at any instant of time. Based on this necessary and timely intervention can be provided and also learning trends inferred. The design techniques use Mathematical modeling and Computer based techniques so as to ensure high reliability and precision. Use of Mathematical concepts to optimise Mathematical Learning is a novel approach.**

***Keywords*— Mathematical Pathway driver, Automata Theory, Learning Finite State Machine, Concept Flow Graph, Optimisation**

I. INTRODUCTION

Mathematics with its interdisciplinary nature has a significant role in all the fields. Hence it is a core subject at the primary level of Learning. A good foundation is very essential. This will help children to not only understand higher level Mathematics better but also pursue careers in Mathematics. **Mathematical Pathway** is an optimal learning progression that can be computerised to track children's progress in Mathematics and provide them with timely assistance and guidance to make maths learning effective. This is implemented using a **Learning Model "Ganitha Vithika".[1]** The **Mathematical Pathway driver** [**MP driver]** is a tracking, assessment and guidance tool that tracks the progress of a child and can be used to diagnose any need for remedial learning or recognise extraordinarily bright children. The purpose of the MP Driver is to let a child make transitions from a level of learning to the other and also record the progress. The main **objective** is to design the Mathematical Pathway using design techniques that ensure effectiveness and efficiency of the Learning model. Learning of any subject involves understanding various concepts and being able to apply them appropriately. Due to the interdependencies between concepts the learning pathway has to ensure that a child has all the prerequisites before the child progresses to the next concept. This gives rise to a **Concept flow graph**. This Concept flow graph designed using graph theoretical approach [1][2] for each concept of Mathematics to be learnt is the basis for the design of the MP Driver. This approach can be easily extended to any other subject. The content of learning changes but the design approach remains the same. The paper is organised as follows: Section 2 deals with a brief review of the work pertaining to the subject of research. Section 3 deals with data set description. Section 4 deals with the methodology. Section 5 deals with the experimental results and discussion. Section 6 deals with the conclusion.

II. RELATED WORK

In this section a brief overview of the literature pertaining to the subject of research is presented. Child Developmentalists [3][4][5][6][7][8][9][10] [11] through research and interactions with children have understood the various aspects of Mathematical learning and Thinking. Significant findings have brought about novel aspects in the Math Pedagogy. Various Learning models for effective Math pedagogy have also been designed [12][13]. Researchers have also worked on Learning trajectories [14] for Mathematics for high quality teaching based on Mathematical thinking and learning. These attempts have been from a purely pedagogical perspective. Though Computer based teaching [15][16][17] has been used to make teaching learning effective, use of Mathematical or computer based approach has not been considered for design and development of these Learning Pathways. Though Automata has been used in Knowledge spaces and machine learning [18][19][20][21] no attempt to use it in education for the purpose of learning has been made. Hence this paper explores an innovative approach of use of Mathematics for enhancing the quality of learning of Mathematics.

III. DATA SET DESCRIPTION

The data used for the development of the automata machine for the MP driver is the Mathematical content to be learnt between 0 and 12 years at the primary level. The primary source of data was obtained from educators and experts in Mathematics through interviews and questionnaires. Data was collected across India and also the ICSE, CBSE and State boards. Observation method was also used to understand the learning progression in children. The secondary source was the syllabus and frameworks defined by the various boards followed by the different schools across the globe. The National Curriculum Framework of the Department for Education was the backbone for this work.

In total, eighteen concepts were identified and flow graphs for each of these concepts were constructed based on this data[1][2]. These flow graphs and Concept framework were used to design the Automata machine.

IV. METHODOLOGY

The **Concept Flow Graph and Concept Framework** [1][2] obtained from the data collected were used to build the automata machine for the MP driver. The Concept Framework obtained from the concept flow graph is a collection of Concept Framework Tables where each table represents a level of learning with respect to a concept.

On Studying the Concept Framework Tables it was observed that each Concept has several levels of learning. To complete a level of learning with respect to a concept, the child has to accomplish the mandatory learning objectives corresponding to that level. This is best represented through a **Finite State Automata** [FSA].

The Concept framework was represented using the Finite State automata Computational model. **One FSM was constructed per level of learning** of a concept. **Each learning objective pertaining to that level of learning was a state**. The final state of the FSM declares whether that level has been completed successfully. **Completion of a level is therefore equivalent to reaching the final accepting state of the FSM corresponding to that level of the concept.** Within a state an **input function**, ***test*** evaluates the accomplishment of the learning objective represented by the state. This generates the input for the next state. If the child succeeds then it produces a 1 or else it produces a 0. Whenever a 1 is produced the **transition function** takes the machine from the current state to the next state and on a 0 it continues in the current state. During transition if a **final state** is reached then it indicates that that level of learning has been completed. The input set of the FSM is {0,1}.

On analyzing the Concept Framework, it is observed that due to some learning objectives being optional sometimes transitions from one state to many states is possible on a single input hence the automata is **nondeterministic in nature.**

Hence an NFA is defined as a 5-tuple, (Q, Σ, T, q_0, F), consisting of

- a finite set of states, Q
- a finite set of input symbols, Σ
- a transition function, $T : Q \times \Sigma \rightarrow P(Q)$
- an initial state, $q_0 \in Q$
- a set of final states, $F \subseteq Q$.

to represent the Concept framework.

Every time a learning objective has been completed a record of the same has to be made for further analysis. So every state produces an output. This output is dependent on the current state. Hence the FSM also behaves like a Moore machine.

This is defined by a 6-tuple ($Q, \Sigma, T, q_{0,} \Lambda,$, □) consisting of the following:

- a finite set of states, Q
- a finite set of input symbols, Σ
- a transition function, $T : Q \times \Sigma \rightarrow Q$ mapping a state and the input alphabet to the next state
- an initial state, $q_0 \in Q$
- a finite set of output alphabet, Λ
- an output function, □ : $Q \rightarrow \Lambda$ mapping each state to the output alphabet

A Moore machine by definition is a transducer or an output producer and does not have accept states. So this FSM that represents the Mathematical Pathway is a **combination of an NFA which is an acceptor and a Moore machine which is a transducer.**

Formally, the **Learning Finite State Machine [LFSM]** for the Mathematical Pathway is represented by a 7-tuple, MP = ($Q, \sum, T, q_0, F, \lambda$,□ ,)

where

a. Q is a finite set of states.
b. $\sum$ is a finite set of input symbols.
c. $T : Q \text{ X } \sum \rightarrow P(Q)$ is the transition function where $P(Q)$ denotes the power set of Q.
d. $q_0 \in Q$ is the initial state.
e. $F \subseteq Q$ is a set of final states.
f. λ is a finite set of output symbols.
g. □ : $Q \rightarrow \lambda$ is the output function.

Based on this definition a Learning finite state automata machine LFSM is designed using the Concept Framework.

V. EXPERIMENTS AND RESULTS

The Concept flow graph for each of the eighteen concepts were developed using Graph Theoretical approach [1][2]. All the eighteen concepts were used as inputs to design the Learning Finite State Automata. Here the analysis is focused on one of the eighteen concepts i.e addition. Fig 1. represents the **Concept Flow graph** and Table 1 the **Concept Framework Table** [CFT] for the addition concept.

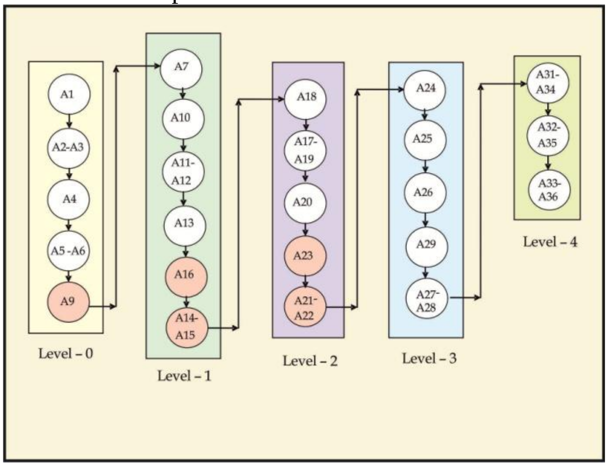

Fig. 1 Addition CFG

Table 1 Addition CFT

LEVEL 0

STEP	PREREQUISITE	LEARNING OBJECTIVE	MLL
A0-1	N0	**Increase**: Learns concept of One more	P-1
A0-2	A0-1	**Joining is adding**: Joining of collections to make bigger collections. Learns joining together or putting together is adding. . Learns the vocabulary 'add', 'addition', 'plus', 'sum', 'combine', 'altogether', 'put with'. Uses collections of real objects or picture representation to add two numbers.	P-1
A0-3	A0-2	**Addition**: Uses number line to Add two numbers whose sum does not exceed 9. Uses plus symbol '+'.	P-1
A0-4	A0-3	**Addition Table**: Makes single digit number pairs whose sum does not exceed 9. Mentally adds a pair of number whose sum does not exceed 9. Learns adding zero to a number does not change its value.	P-1
A0-5	A0-4	**Application:** Solves one step problems of real life on single digit addition presented through pictures and verbal description and writes number sentences using + and =	P-1

LEVEL 1

STEP	PREREQUISITE	LEARNING OBJECTIVE	MLL
A1-1	N1,A0	**Addition T able**: Builds the cayley addition table for sums up to 10. Mentally adds a pair whose sum does not exceed 10.	1-2
A1-2	A1-1	**Addition**: Adds two digit numbers by drawing representations of tens and ones without and with regrouping.	1-2
A1-3	A1-2	**Addition without carry:** Adds 2-3 two digit number without carry and the sum not exceeding 99. Adds both vertically and horizontally. Understands order /commutative property of addition. Learns vocabulary augends, addend and sum.	1-2
A1-4	A1-3	**Addition with carry:** Adds 2-3 two digit number with carry from ones to tens and the sum not exceeding 99.	1-2
A1-5	A1-4	**Estimation:** Estimates the result of addition and compares the result with another given number.	

A1-6	A1-5	**Application:** Solves 1-2 step problems of real life on two digit addition presented through pictures and verbal description and can write number sentence for the problem. Describes orally the situations that corresponds to the given addition facts.	1-2

LEVEL 2

STEP	PREREQUISITE	LEARNING OBJECTIVE	MLL
A2-1	N2,A1	**Addition Table**: Extends cayley table based on tens to hundreds. Adds two digit numbers and doubles two digit numbers mentally	2-3
A2-2	A2-1	**Addition without carry:** Adds 2-3 three digit numbers without carry and the sum not exceeding 999. Reads and writes the number name for the sum. Adds numbers by writing them vertically. Uses place value in standard algorithm of addition.	2-3
A2-3	A2-2	**Addition with carry:** Adds 2-3 three digit numbers with carry from a) ones to tens and b) tens to hundreds c) ones to tens and tens to hundreds provided the sum does not exceed 999.	2-3
A2-4	A2-3	**Estimation:** Estimates sum of two given numbers.	2-3
A2-5	A2-4	**Application:** Solves 1-2 step problems of real life on three digit addition presented through pictures and stories by writing number sentences using '+' and '='. Frames problems for addition facts.	2-3

LEVEL 3

STEP	PREREQUISITE	LEARNING OBJECTIVE	MLL
A3-1	N3,A2	**Addition Table**: Extends cayley addition table to thousand. Adds multiples of 10 and 100 mentally	3-4
A3-2	A3-1	**Addition without carry:** Adds 2-3 four digit number without carry and the sum not exceeding 9999. Adds both vertically and horizontally.	3-4
A3-3	A3-2	**Addition with carry:** Adds 2-3 four digit number with carry and the sum not exceeding 9999.	3-4
A3-4	A3-3	**Estimation:** Estimates sums of given numbers and verifies using approximation.	3-4
A3-5	Reading, Comprehension, A3-4	**Application:** Solves 1-2 step problems of real life on four digit addition by writing number sentences using '+' and '='. Frames word problems.	3-4

LEVEL 4

STEP	PREREQUISITE	LEARNING OBJECTIVE	MLL
A4-1	N4,A3	**Addition without carry:** Adds 2-3 three five to nine digit numbers without carry and the sum not exceeding 99 crores.	4-6
A4-2	A4-1	**Addition with carry:** Adds 2-3 five to nine digit numbers with carry and the sum not exceeding 99 crores.	4-6
A4-3	Reading, Comprehension, A4-2	**Application:** Solves 1-2 step problems of real life on five to nine digit addition by writing number sentences using '+' and '='.	4-6

Each level in Concept Framework Table is converted into an Finite State Machine [FSM]. The transitions for the FSM's are represented using **transition diagram**. The transition diagram includes the output for each state. Each state is labelled with a compound symbol $q_i|x$ to indicate that T ($q_{i,}$ a) = q_j and □ ($q_{i,}$ a) = x. Fig 2. represents the transition diagrams for Addition.

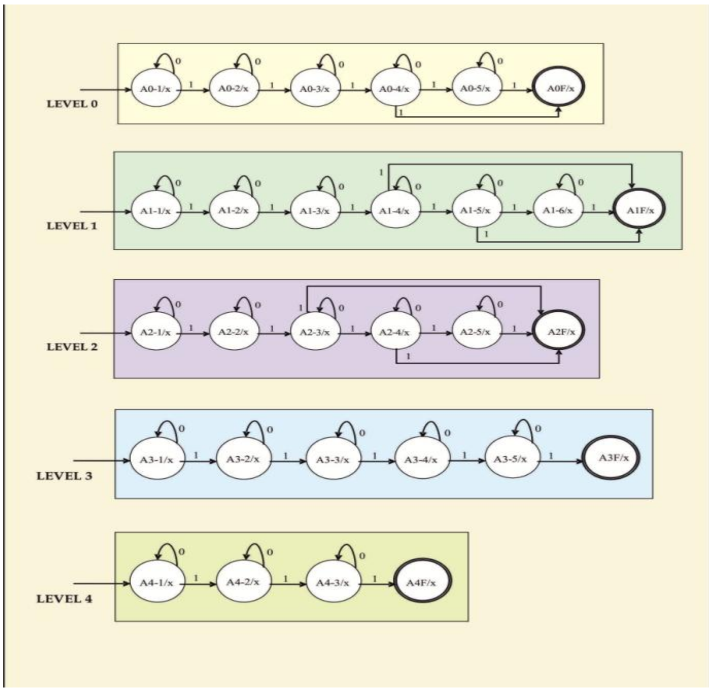

Fig 2. Finite State Machine for Addition

The FSM can also be represented as a **transition table** which is useful during programming. Here in a Moore machine as every state is associated with output, the transition table is called **transition and output table.** The rows of the table correspond to states and columns correspond to inputs and output. A **transition entry** defines the possible states to which the machine can transit on the given input. The output for a normal state is <concept code> - <level number> - <state number>. For a final state the output is <concept code> - <level number>. Table 2 is the transition table for addition.

Table 2 Addition Transition Table

	Input → States↓	0	1	λ Output
Level 0	A0-1	A0-1	A0-2	2-0-40
	A0-2	A0-2	A0-3	2-0-41
	A0-3	A0-3	A0-4	2-0-42
	A0-4	A0-4	A0-5, A0F	2-0-43
	A0-5	A0-5	A0F	2-0-44
	A0F	Ø	Ø	2-0
Level 1	A1-1	A1-1	A1-2	2-1-45
	A1-2	A1-2	A1-3	2-1-46
	A1-3	A1-3	A1-4	2-1-47
	A1-4	A1-4	A1-5, A1F	2-1-48
	A1-5	A1-5	A1-6, A1F	2-1-49
	A1-6	A1-6	A1F	2-150
	A1F	Ø	Ø	2-1
Level 2	A2-1	A2-1	A2-2	2-2-51
	A2-2	A2-2	A2-3	2-2-52
	A2-3	A2-3	A2-4, A2F	2-2-53
	A2-4	A2-4	A2-5,A2F	2-2-54
	A2-5	A2-5	A2F	2-2-55
	A2F	Ø	Ø	2-2
Level 3	A3-1	A3-1	A3-2	2-3-56
	A3-2	A3-2	A3-3	2-3-57
	A3-3	A3-3	A3-4	2-3-58
	A3-4	A3-4	A3-5	2-3-59
	A3-5	A3-5	A3F	2-3-60

	A3F	Ø	Ø	2-3
Level 4	A4-1	A4-1	A4-2	2-4-61
	A4-2	A4-2	A4-3	2-4-62
	A4-3	A4-3	A4-4	2-4-63
	A4F	Ø	Ø	2-4

Here we have discussed in detail the application of the automata technique on the addition concept only. Using the same approach FSM for all eighteen concepts are constructed. The MP driver is a network of these FSM. The transitions through these FSM takes a child through the entire Mathematical pathway.

6. Conclusion

The finite automata machine constructed for the MP driver is a **Learning Finite State Machine**[LFSM] which is a combination of a transducer (Moore machine) and acceptor (NFA) represented formally by a 7 tuple, MP = $(Q, \sum, T, q_0, F, \lambda, \square,)$. Here the**input function** checks for the fulfillment of a learning objective pertaining to a specific level of learning of a concept. The **transition function** moves the LFSM on accomplishment of a learning objective to the next state. The **final state** of each LFSM declares completion of a level of learning with respect to a concept. The **transition diagrams** show transitions between states. The **transition tables** obtained from the LFSM is used to develop the MP Driver automata machine. The **MP Network** links all the LFSM's and helps a child traverse through the entire Mathematical Pathway. The MP Driver with the **Learning Finite State Machine [LFSM]** and the **Mathematical Pathway Network [MP Network]** works as a system that helps drive the Mathematical pathway. It is a model which is adopted for developing the automated **Progress tracking module.** This methodology is novel and unique and can be readily computerised. Also the same technique can be applied to any other subject learning which makes it general and versatile.

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