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A Comprehensive Study of Fuzzy Logic

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Abstract -- Fuzzy logic is a developed field of Computer science, Artificial intelligent and Mathematic. In this survey paper, we are elaborating fuzzy logic, fuzzy sets, its properties and operations. In this paper we also show that "why fuzzy logic is so important" and "How fuzzy logic is future of artificial intelligent". Today, our computers operate only on a binary true or false value i.e. 0 and 1 but our world is not binary. The world where we live is full of ambiguities. Fuzzy logic is an approach where we compute on the bases of "degrees of truth" from (1 to 0) rather than the usual "true or false" (1 or 0). Here, we are going to focus on fuzzy set theory, its properties and how can we relate it with our world.

Keyword -- Fuzzy Logic, Introduction of Fuzzy logic, Fuzzy Set Mathematically, How crisp sets different from fuzzy logic, Fuzzy Set properties, Fuzzy Set Operation, Concentration of fuzzy set, Dilation of a fuzzy set, Fuzzy Set Applications.

I. Introduction

The idea of fuzzy logic was first advanced by Dr. Lotfi Zadeh of the University of California at Berkeley in the 1960s. He claimed that many sets in the world that surrounds us are defined by a non-distinct boundary [3]. In Fuzzy logic, there are not just two alternatives but a whole continuum of truth values for logical propositions. A proposition A can have truth value 0.7 and its complement A^c the truth value 0.3. [2]

In fuzzy logic 0 and 1 are the extreme cases of truth but also includes the various states of truth in between so that [3], for example, Milk {not sweet, little sweet, sweet and too sweeter} can be written as {0/not sweet; 0.2/little sweet; 0.4/sweet; 1/too sweeter;}

Fuzzy logic designed to solve problems in the same as our brain solve. We aggregate data and form a number of partial truths which we aggregate further into higher truths [3]. In real world, we use fuzzy knowledge, knowledge that is ambiguous, probabilistic, imprecise, uncertain or inexact in nature. There is fuzzy information involved in human thinking and reasoning like young, tall, good, high, cold [4].

- There is no single quantitative value which defines the term young.
- For some people, 22 ages is young and for others, 34 is young.
- Little bit cold is also cold and freeze is also cold.

However, our systems are not able to answer many questions. The reason is, most systems are designed based upon classical set theory and two valued logic (true, false) which unable to cope with unreliable and incomplete information and give expert opinions [4].

We want our systems also be able to cope with unreliable and incomplete information and give expert opinions. Fuzzy sets have been able provide solutions to many real world problems.

II. Crisp Sets

A crisp set is defined in such a way that all the individuals in a given universe can be partitioned into two classes: those who belong to the set, and those who do not belong to the set. Mathematically, we give the following definition.
If U is the universe, then the set of elements in U having property P (the property is such that each element of the universe either has the property or does not have the property) is denoted by B, and can be written as
B = {x : x ∈ U and x has property P}. If A and B are two sets of the universe U, then A is said to be a subset of (or contained in) B, denoted by $A \subset B$ or $B \supset A$, if and only if $x \in A \Longrightarrow x \in B$. Two sets A and B of the same universe U are said to be equal if $A \subset B$ and $B \supset A$.

A set consisting of no elements is said to be a null set; it is denoted by φ. Note that a null set is a subset of every set, and any set is contained in its universe. [13]

All crisp sets can be a Fuzzy Set:
U = {1, 2, 3, 4, 5}
Crisp Set: - Let A = {1, 3, 4}
Fuzzy Set A = {1/1, 0/2, 1/3, 1/4, 0/5}
But all fuzzy sets can't be Crisp Set:

Let a fuzzy set A = {0.2/1, 0.5/2, 0.7/3, 1/4}

Note: there is different degree from (0, 1) and in crisp set we only use (0, 1) so we can't written it in crisp set.

III. Fuzzy Set

Fuzzy set theory is an extension of classical set theory where elements have degrees of membership. It is a class of object with a continuum of degree of membership. Such a set is characterized by a membership function which assigns to each object a degree of membership ranging between 0 and 1. In mathematics a set, by definition, is a collection of things that belong to some definition. Any item either belongs to that set or does not belong to that set [5].

Let us look at simple example, the set of tall men. We shall say that people taller than or equal to 6 feet are tall. This set can be represented graphically as follows:

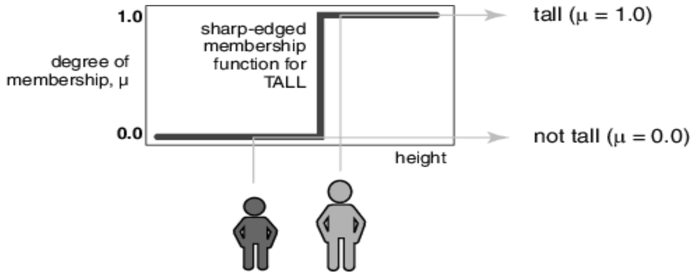

Fig 1 Classical Set theory representation [5]

The function shown above describes the membership of the "tall" set, you are either within it or not. This sharp edge binary function works nicely for binary operations and mathematics, but it doesn't as nicely in describing the real world. Their makes no distinction between somebody who is 6'3" and someone who is 7'3", they are both simply tall. Clearly there is a significant difference between the two heights. The other side of this lack of distinction is the difference between a 5'11" and 6'0" man. This is only a difference of one inch; however this membership function just says one is tall and the other is not tall. [5]. Fuzzy set approach provides a much better representation of the tallness of a person. The set shown below is defined by a continuously inclining function.

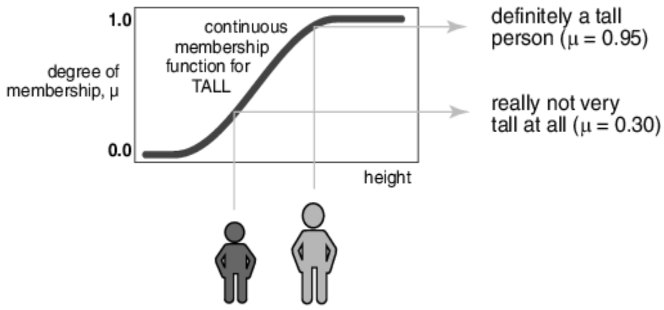

Fig 2 Fuzzy set theory representation [5]

The membership function defines the fuzzy set for the possible values underneath of it on the horizontal axis. The vertical axis, on a scale of 0 to 1, provides the membership value of the height in the fuzzy set. So for the two people shown above the first person has a membership of 0.3 and so is not very tall. The second person has a membership of 0.95 and so he is definitely tall. He does not, however, belong to the set of tall men in the way that bivalent sets work; he has a high degree of membership in the fuzzy set of tall men.

A. *Fuzzy Set Mathematically*

Let X be a space of points, with a generic element of X denoted by x. Thus X = {x}.

A fuzzy set A in X is characterized by a membership function $f_A(x)$ which associates with each point in X a real number in the interval [0,1], with the values of $f_A(x)$ at x representing the "grade of membership" of x in A. Thus, the nearer the value of $f_A(x)$ to unity, the higher the grade of membership of x in A. When A is classical set than its membership function can take on only two values 0 and 1, with $f_A(x)$ = 1 or 0 according as x does or does not belong to A. When there is need to differentiate between a set and fuzzy set, the set with two values (0, 1) characteristic functions will be referred to as *ordinary sets* or simple sets. [6, 4]

Example 1: Consider a set of numbers:

X = {1, 2,…...10}

a fuzzy set labeled 'large number' can be defined as

A = 0.2/6 + 0.5/7 + 0.8/8 + 1/9 + 1/10

Explanation:

‘Large Number’	‘Membership degree’	Comment
10	1	‘Surely’
9	1	‘Surely’
8	0.8	‘Quite poss.’
7	0.5	‘Maybe’
6	0.2	‘In some cases but not usually’
5	0	‘Surely’
4	{$\mu_A(0)$ usually omitted from A}	‘Definitely’
3		‘Not’
2		
1		

Usually a fuzzy set is denoted as:
$A = \mu_A(x_i)/x_i + \ldots\ldots\ldots\ldots + \mu_A(x_n)/x_n$ [11]

Example 2: Set **TALL** in set **X** consisting of Tall persons
Assume: TALL (4’5”) = 0, TALL (5’0) = 0.2, TALL (5’4”) = 0.4, TALL (5’7”) = 0.6, TALL (6’0”) = 1 for u>=6’1”
Note that a fuzzy set can be defined precisely by associating with each x, its grade of membership in TALL.

1) *Equality of Crisp Set or classical set:*

Thus a classical set theory µA (x) has only the value 0 (false) and 1(true). Such sets are called crisp sets. [4]

Let a set of prime numbers:
A= {3, 5, 11, 7}
B= {5, 11, 3, 7, 5}
C= {11, 3, 17, 5}
According to crisp set theory
A=B and A≠C

2) *Equality of Fuzzy Set:*

Two fuzzy sets A and B are equal, written as A = B, if and only if $f_A(x) = f_B(x)$ for all x in X. We can written it simply as $f_A = f_B$.

Let a set of Tall persons:
A= 0.2/Rocky, 1/Bill, 0.7/John, 0.4/Robert
B= 1/Bill, 0.2/Rocky, 0.4/Robert, 0.7/John
However if

C= 0.8/Bill, 0.3/Rocky, 0.7/John, 0.4/Robert
According to Fuzzy set theory
A=B and A≠C
In this example, “Rocky” is an element and “.2” is a degree.

3) *Empty Fuzzy Set:*

A fuzzy set is empty if and only if its membership function is identically Zero on X.

Example: Let EMPTY be a set of people, in India, older than 130. The Empty set is also called the Null Set.
$A \equiv \varnothing$ if $\mu_A(x) = 0, \forall x \in X$
$\forall x \in X$: “for any element x in X” [6, 8]

4) *Inclusion of one set into another set:*

Fuzzy set A□ X is included in (is a subset of) another fuzzy set, B□ X
IF AND ONLY IF (iff)
□A(x) □ □B(x) □ x□ X

Example: Consider X= {1, 2, 3} and
A = 0.2/1 + 0.7/2 + 1/3
B = 0.5/1 + 0.8/2 + 1/3
Then A is a subset of B

B. Fuzzy Set Operations

A fuzzy set operation is an operation on fuzzy sets. The fuzzy set operations are generalization of crisp set operations. There is more than one possible generalization. The most widely used operations are called standard fuzzy set operations. There are three operations: fuzzy unions, intersections, and complements.

Union (U): $(A \cup B)(x) = \max [A(x), B(x)]$
Intersection: $(A \cap B)(x) = \min [A(x), B(x)]$
Complement: $cA(x) = 1 - A(x)$

1) Union:

The union of two fuzzy sets A and B with respective membership function $f_A(x)$ and $f_B(x)$ is a fuzzy set C, written as C = A U B.
Abbreviated form $f_C = f_A \vee f_B$
We can define union in more appealing way as follows: Union is the smallest fuzzy set that contains both A and B. The union of A and B is denoted by A U B.
The following relation must be satisfied for the union operation: [10, 6]
For all x in the set X, $(A \cup B)(x) = \max [A(x), B(x)]$

Example 1: Union of Fuzzy **A** and **B**
A(x) = 0.85 and **B(x)** = 0.75
$(A \cup B)(x) = \max [0.85, 0.75] = 0.85$

Example 2: Union of **AGE1** and **AGE2**
AGE1(x) = 0.75/Rocky, 0.4/Bill, 0.6/John, 1/Robert
AGE2(x) = 0.85/Rocky, 0.6/Bill, 0/John, 0.8/Robert}
(AGE1 ∪ AGE2)(x) = max [AGE1(x), AGE2(x)]
(AGE1 ∪ AGE2)(x) = 0.85/Rocky, 0.6/Bill, 0.6/John, 1/Robert

2) Intersection:

Let **A** and **B** be fuzzy sets defined in the space X. Intersection is defined as the greatest fuzzy set that include both **A** and **B.** It can be denoted as (**A ∩ B**). The following relation must be satisfied for the intersection operation: [4]
For all x in the set X, $(A \cap B)(x) = \min [A(x), B(x)]$
Example 1: Intersection of Fuzzy **A** and **B**
A(x) = 0.85
B(x) = 0.75
$(A \cap B)(x) = \min [0.85, 0.75] = 0.75$

Example 2: Intersection of AGE1 and AGE2
AGE1(x) = 0.75/Rocky, 0.4/Bill, 0.6/John, 1/Robert
AGE2(x) = 0.85/Rocky, 0.6/Bill, 0/John, 0.8/Robert}
(AGE1 ∩ AGE2)(X) = min [AGE1(x), AGE2(x)]
(AGE1 ∩ AGE2)(X) = 0.75/Rocky, 0.4/Bill, 0/John, 0.8/Robert

3) Complement:

Let A be a fuzzy set defined in the space X. Then the fuzzy set B is a complement of the fuzzy set A, if and only if, for all x in the set X, B(x) = 1-A(x). Complement of Fuzzy set A can be denoted by A' or $\bar{A}$. [4]
Complement**:** $cA(x) = 1 - A(x)$
Example 1**:** Complement of AGE1
AGE1(x) **=** 0.75/Rocky, 0.4/Bill, 0.6/John, 1/Robert
AGE1(X)**' =** 0.25/Rocky, 0.6/Bill, 0.4/John, 0/Robert

4) Bonded Sum of two fuzzy sets:

If A and B are fuzzy subsets of X, then the bonded sum of A and B is denoted by: A□ B [11]
$\square_{A\square B}(x) = \min (1, \square_A(x)+\square_B(x))$ for all x□ X
Example 1: Let X = {1, 2, 3, 4}
A = 0.4/1 + 0.9/2 + 1/4 and
B = 0.6/1 + 0.5/2 then
= min (1, (0.4 + 0.6)/1) + min (1, (0.9 + 0.5)/2) + min (1, (1 + 0/4)
A□B = 1/1 + 12/ + 1/4

5) Bonded difference of two fuzzy sets:

If A and B are fuzzy subsets of X, then the bonded difference of A and B is denoted by: [11]
A Ө B
and

$\square_{A\Theta B}(x) = 0\square\ (\square_A(x) - \square_B(x)0/x$
$\square_{A\Theta B}(x) = \max(0,(\square_A(x) - \square_B(x)0/x$

Example 1: Let X = {1, 2, 3, 4}
A = 0.4/1 + 0.9/2 + 1/4
B = 0.6/1 + 0.5/2
AΘB = max(0, (0.4-0.6)/1) + max(0, (0.9-0.5)/2) + max(0,(1-0)/4)
AΘB =0/1 + 0.4/2 + 1/4

6) *Concentration of fuzzy set:*

This operation is unique in fuzzy set. It helps to increase the intensity of element like ‘Good’ to ‘Very Good’, ‘Hot’ to ‘Very Hot’.

The concentration of the fuzzy set A is denoted by A^2 and is defined by: [11]
$\square_A{}^2(x) = \square_A(x))^2/x$
for all x □ X
Example 1: Let X = {1, 2, 3, 4}
A = 0.5/1 + 0.9/2 +0/3 + 0.7/4
then A^2=0.25/1 + 0.81/2 + 0/3 + 0.49/4
$\square_A{}^2(x)$ is much more sharply defined membership function than, say $\square_A(x)$

7) *Dilation of a fuzzy set:*

This operation is also unique in fuzzy set. Dilation operation is opposite of concentration operation.

Dilation of a fuzzy subset A (of X) is denoted as $A^{1/2}$ and the membership function of this set is written as: [11]
$\square_{A1/2}(x)/x = (\square_A(x))^{1/2}/x$
Example 1: Let X = {1, 2, 3, 4}
A = 0.5/1 + 0.9/2 +0/3 + 0.7/4
Then
$A^{1/2}$ = 0.70/1 + 0.95/2 + 0/3 + 0.83/4

IV. Applications

There are various fields and application where we can use fuzzy logic. Every sector in this world wants a system which is itself intelligent to solve any problem according to the inputs and can processed fuzzy knowledge. Some applications are as follows: [14, 15]

1) Mobile Robot Navigation
2) Image Processing
3) Artificial Intelligence
4) Humanoid Robot
5) Automobiles
6) Defense and Security
7) Internet And Computer Security
8) Decision Making (Law, Business, Etc.)

There are more applications where we can use fuzzy logic.

V. Conclusion and Future Work

By studying fuzzy logic we had concluded that as per as technology is developing and our requirement increases in the field of artificial intelligence and others, the need of fuzzy logic is parallel increase. As computer scientists are trying to model human brain in Artificial Intelligence (AI). We look forward when a computer will be able to understand human behavior and act accordingly. In future we can combine it with Computational Automata and Artificial Neural Networks.

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