



## Calculation of Peak to Average Power Ratio of Chu Sequence Using Oversampling

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**Abstract**—Highly spectral efficient formats like orthogonal frequency division multiple access suffers from large fluctuations in signal envelope, hence resulting in high peak to average power ratio. Exact measurement of peak to average power ratio is of main concern. And it comes out to be less for discrete orthogonal frequency division multiplexing signal than for continuous one. This paper presents a measurement strategy of peak to average power ratio for the same taking example of a Chu sequence.

**Keywords**— Chu sequence, Interpolation, Orthogonal Frequency Division Multiplexing, Oversampling, Peak to Average Power Ratio.

### I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) systems have high Peak to Average Power Ratio (PAPR) because in OFDM system many subcarrier components are added using an IFFT operation. PAPR is the ratio between the maximum power and the average power of the complex pass band signal  $s(t)$  [1], that is,

$$\text{PAPR}\{\tilde{s}(t)\} = \frac{\max |s(t)|^2}{E\{|s(t)|^2\}} \quad \dots (1)$$

The PAPR defined in the Equation (1) deals with the pass band signal with a carrier frequency of  $f_c$  in the continuous time domain. As  $f_c$  generally is much higher than  $1/T_s$ , a continuous time baseband OFDM signal  $x(t)$  with the symbol period  $T_s$  and the corresponding pass band signal  $\tilde{x}(t)$  with the carrier frequency  $f_c$  have almost the same PAPR [2]. But the PAPR for the discrete-time baseband signal  $x[n]$  may not be the same as that for the continuous-time baseband signal  $x(t)$ . Indeed, as  $x[n]$  may not have all the peaks of  $x(t)$ , the PAPR for  $x[n]$  is lower than that for  $x(t)$  [3, 4]. Practically, the PAPR for the continuous-time baseband signal can be measured only after implementing the actual hardware, including digital-to-analog convertor (DAC). It implies that measurement of the PAPR for the continuous-time baseband signal is not straightforward. For that reason, there must be some means of estimating the PAPR from the discrete-time signal  $x[n]$ . Oversampling is one of the methods using which actual value of PAPR for the discrete time signal  $x[n]$  can be calculated [3].

The remainder of this paper has been organized as follows: Section II describes the technique of oversampling for calculating PAPR, followed by discussing Chu sequence. Section III presents the implementation of oversampling technique and simulation results which are obtained using MATLAB. Finally, conclusions are drawn in Section IV followed by references.

### II. OVERSAMPLING TECHNIQUE FOR MEASUREMENT OF PAPR

Oversampling technique can measure the true value of PAPR of discrete time OFDM signal. The discrete-time baseband signal  $x[n]$  can show almost the same PAPR as continuous time baseband OFDM signal  $x(t)$ , if it is  $L$ -times interpolated (oversampled) where  $L \geq 4$  [3].

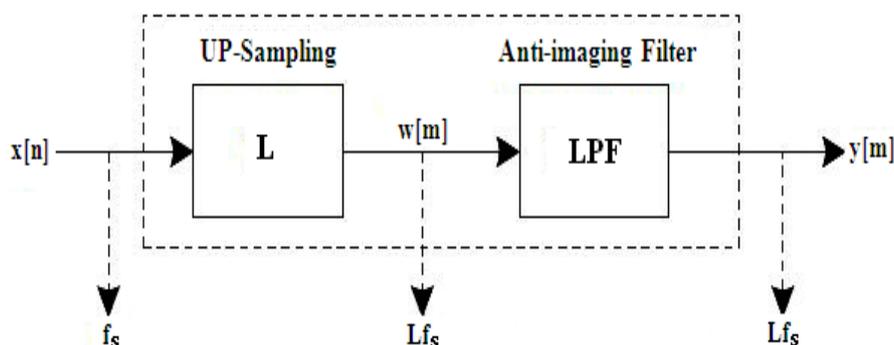


Fig. 1 Block Diagram of Interpolator

Fig. 1 shows the block diagram of interpolator with a factor of L [5]. This is done by inserting (L-1) zeros between the samples of  $x[n]$  to yield  $w[m]$  as follows

$$w[m] = \begin{cases} x[m/L], & \text{for } m = 0, \pm L, \pm 2L \dots \\ 0, & \text{elsewhere} \end{cases} \dots (2)$$

To construct the L-times-interpolated version of  $x[n]$  from  $w[m]$ , a Low Pass Filter (LPF) is used. For the LPF with an impulse response of  $h[m]$ , the L-times-interpolated output  $y[m]$  can be represented as

$$y[m] = \sum_{k=-\infty}^{\infty} h[k]w[m-k] \dots (3)$$

Fig. 2 and 3 illustrate the signals and their spectra appearing in the oversampling process with a sampling frequency of 2 kHz to yield a result of interpolation with  $L = 4$ . As per these figures, the IFFT output signal  $x[n]$  can be expressed in terms of the L-times interpolated version as

$$x'[m] = \frac{1}{\sqrt{L.N}} \sum_{k=0}^{LN-1} X'[k]. e^{j2\pi m \Delta f k / L.N}, m = 0, 1, \dots, NL - 1 \dots (4)$$

With

$$X'[k] = \begin{cases} X[k], & \text{for } 0 \leq k \leq N/2 \text{ and } NL - N/2 < k < NL \\ 0, & \text{elsewhere} \end{cases} \dots (5)$$

Where  $N$ ,  $\Delta f$  and  $X[k]$  represents FFT size, subcarrier spacing and complex symbol carried over a subcarrier  $k$  respectively. For such interpolated signal, PAPR can be redefined as

$$\text{PAPR} = \frac{\max_{m=0,1,\dots,NL} |x'[m]|^2}{E\{|x'[m]|^2\}} \dots (6)$$

To see the effect of interpolation on PAPR, a Chu sequence has been considered, which is defined as

$$X_i(k) = \begin{cases} e^{j\frac{\pi}{N}i^2k} & \text{if } N \text{ is even} \\ e^{j\frac{\pi}{N}(i+1)k} & \text{if } N \text{ is odd} \end{cases} \quad i = 0, 1, 2, \dots, N - 1 \dots (7)$$

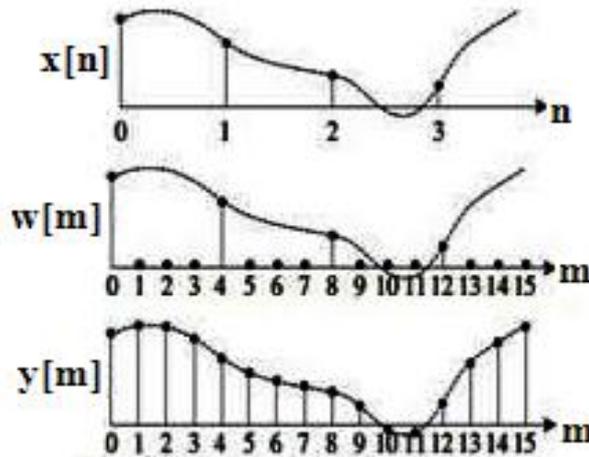


Fig. 2 Time Domain Representation of Interpolation with L=4

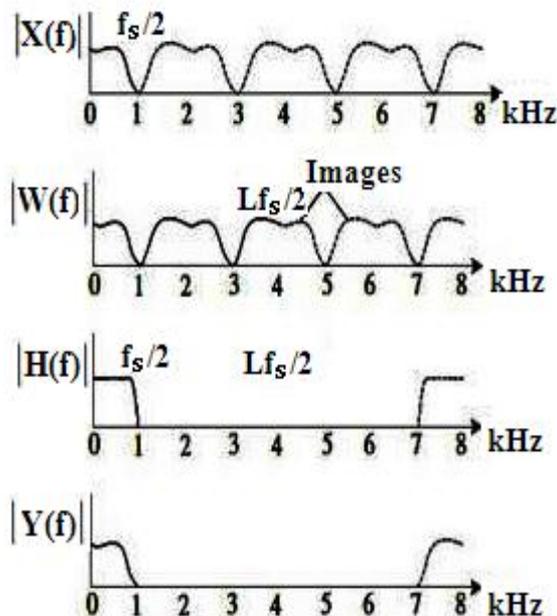


Fig. 3 Frequency Domain Representation of Interpolation with L=4

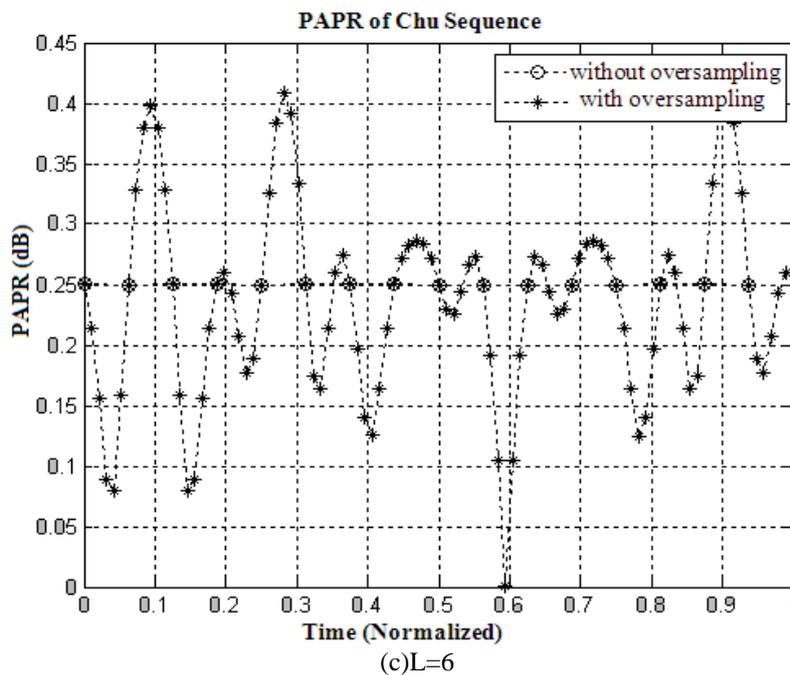
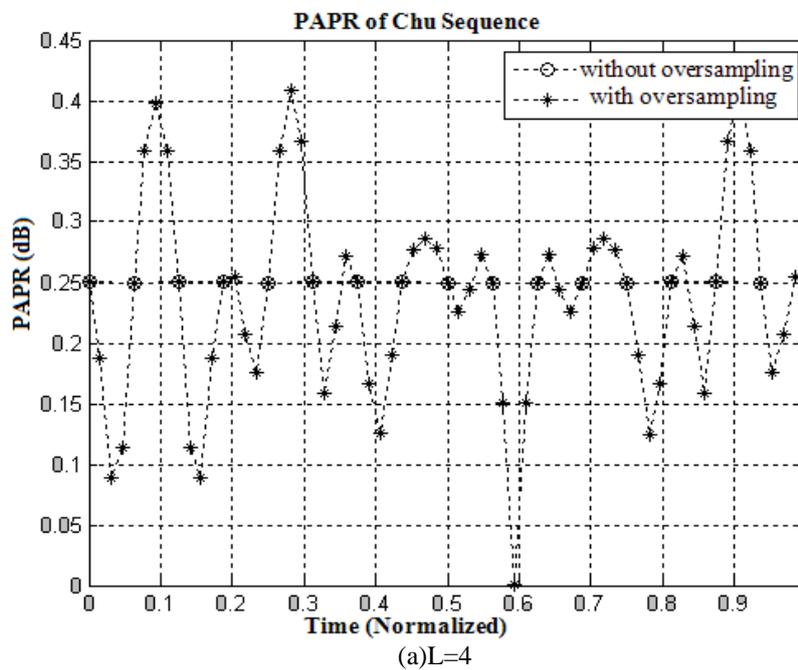
III. IMPLEMENTATION OF OVERSAMPLING TECHNIQUE FOR CHU SEQUENCE

Simulation results for the effect of oversampling on the measurement of PAPR are obtained by using MATLAB. Table 1 shows the value of PAPR with and without oversampling with N=16 and 32 point IFFT of Chu sequence and varying number of L.

TABLE I  
PAPR FOR N=16 AND 32 FOR DIFFERENT VALUES OF L

Value of L	Without Oversampling	With Oversampling	
		N=16	N=32
L=4	0.0000	4.2714	4.8104
L=5	0.0000	4.1551	4.6828
L=6	0.0000	4.2714	4.8104
L=7	0.0000	4.2227	4.7549
L=8	0.0000	4.2714	4.8104

By using oversampling technique the value of PAPR has been calculated as 4.2714, which is very close to the actual value, but it happens only when oversampling factor is an even number and N=16.



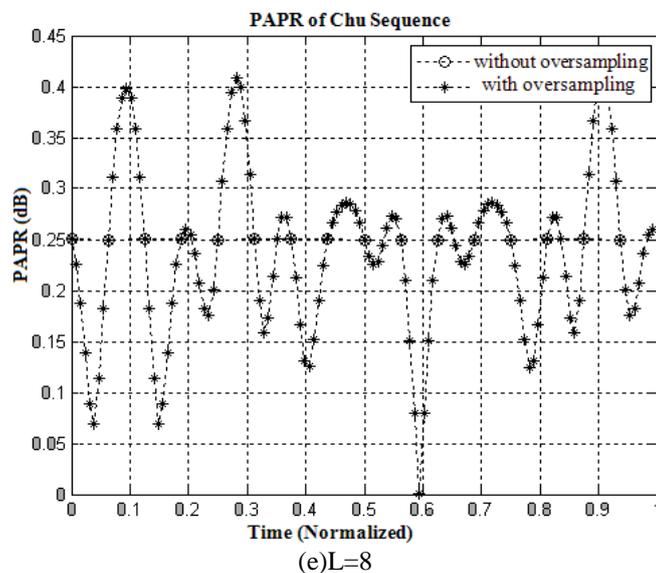
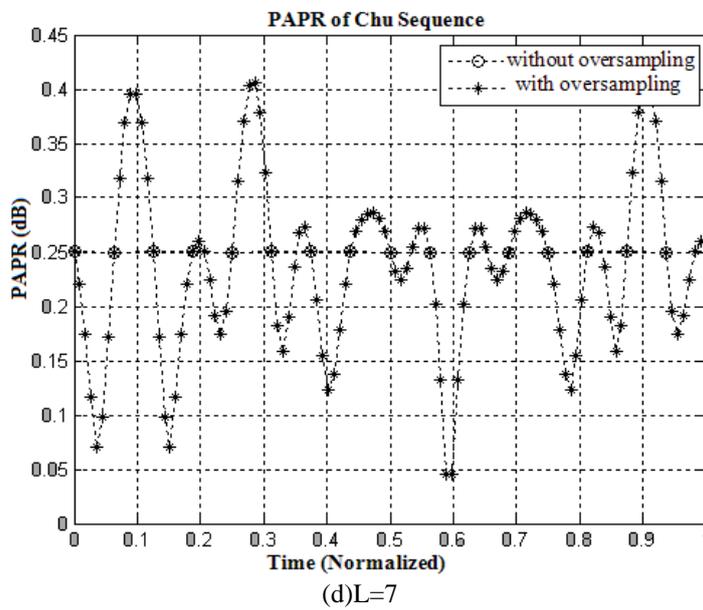
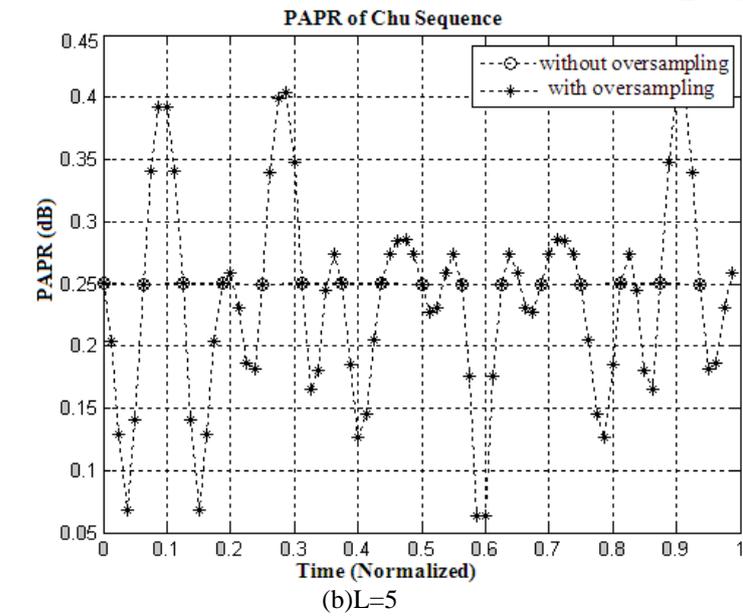


Fig. 4 (a)-(e) PAPR Characteristics of Chu Sequence for Different Interpolation Factors When N=16

Fig. 4(a)-(e) shows the variation of PAPR of Chu sequence with time for different values of interpolation factor for N=16.

#### IV. CONCLUSIONS

The measurement of PAPR for the discrete time baseband signal cannot be made by using the techniques used for measurement of PAPR of continuous time signal as they will not give accurate value of PAPR. Oversampling can be used by which true value of PAPR for the discrete time signal can be calculated. And it is shown in the results that when discrete time signal is oversampled by a factor greater or equal to 4 and is even, the measured value of PAPR of discrete OFDM signal comes out to be same as that of an analog OFDM signal.

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