



Performance Evaluation of Extended AODV for Integrated MANET with Internet

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Abstract— *The integration of wired and wireless networks is gaining popularity due to its usefulness and practical use. This paper considers the problem of searching for a destination node in a large network, with the ability to conduct cost effective and fast searches. For this purpose, our attention is to the class of TTL based approaches that minimize the cost of such searches associated with the packet transmission, with the intention that they can be applied to wired cum wireless networks. The extended ad hoc on demand distance vector (AODV) routing protocol is used for interconnection of MANET with internet. The simulation is carried out for two different situations: when nodes are static and uniformly distributed along the square lattice and when nodes are mobile and randomly distributed. Simulation results and mathematical derivations used in this paper shows that the expected search cost is reduced in terms of route discovery delay, routing overhead and improved packet delivery ratio; with varying traffic sources. Thus, this is proved that the network wide search is the modification in TTL sequence values in Extended AODV routing protocol, provides the required outcome and more appropriate for integrated MANET with internet.*

Keywords— *Extended AODV, Integrated network, MANET, Search cost.*

I. INTRODUCTION

This paper selects that network which is the integration of MANET and Internet [13], [14]. Since the integration of wired and wireless networks is gaining popularity due to its usefulness and practical use.

The Internet Engineering Task Force (IETF) has proposed several routing protocols for MANETs [17], [19], [20], [24] such as Ad hoc On-Demand Distance Vector (AODV) [4], [5] Dynamic Source Routing (DSR) [22] Optimized Link State Routing Protocol (OLSR) [23]. However, these protocols were designed for communication within an autonomous MANET [18]. The ad hoc routing protocol AODV [1], [20] is one of the promising routing protocols and can be used to route packets between mobile nodes. However, it cannot provide Internet access to the mobile nodes because it does not support routing between a fixed network like the Internet and a mobile ad hoc Internet access to the mobile nodes. One solution is “MIPMANET - Mobile IP for Mobile Ad Hoc Networks” [14], [15]. This solution provides Internet access by using tunnelling and Mobile IP with foreign agent care-of addresses. This paper considers the problem of searching route to the destination node (located at the wired network domain), by a source node (located at MANET domain) in an integrated wired cum wireless MANET networks [25]. The solution of this problem is presented by the use of Internet draft “Global Connectivity for IPv6 Mobile Ad Hoc Networks” [1], [13] where the AODV protocol [4], [5] is modified in such a way that it can route packets not only within a mobile ad hoc network, but also to a fixed, wired network. This paper used the modified version of AODV which is known as extended AODV [2]. The interconnection of internet with MANET is achieved by introducing an Internet Gateway [21], [26] that provides the link to external hosts. Thus, a gateway acts as a bridge between a MANET and the Internet and all communication between the two networks must pass through gateway.

With the use of extended AODV for the interconnection of fixed wired and wireless network, the author worked on two types of network model. First, when nodes in several MANET domains are uniformly distributed along the square lattice. Second, the considered network model is that when nodes are randomly distributed [3] in MANET domain. The simulations and mathematical derivations are incorporated for both type of network model. Finally it is found that for integrated MANET with internet, the network wide search mechanism is the modification in TTL sequence values, provides the optimization in route discovery delay time and routing overhead with enhanced packet delivery ratio value compared to the conventional expanding ring search mechanism [16].

The remainder of this paper is organized as follows: Section II gives an overview of Extended AODV. Section III describes Performance metrics. Section IV presents Analytical Network Models: (a) Uniformly Distributed nodes, (b) Randomly Distributed nodes. Section V presents network models used for simulation. Section VI provides Simulation results and discussion. Section VII concludes this paper.

II. EXTENDED AODV

When a destination is a fixed wired node and a mobile node wish to communicate with it; firstly mobile node searches its routing table for route towards destination. If a route is found, the mobile node starts sending packets. Otherwise, the

mobile node needs to begin route discovery process by broadcasting RREQ message as in conventional AODV routing protocol [6].

When a RREQ message is received by an intermediate mobile node, an intermediate mobile node searches its routing table for a route towards the wired destination. If a route is found, the intermediate node would normally send a RREP back to the originator of the RREQ. But in that case, the source would think that the destination is a mobile node that can be reached via the intermediate node. It is important that the source knows that the destination is a fixed node and not a mobile node, because these are sometimes processed differently. But in extended AODV [10] this problem has been solved by preventing the intermediate node to send a RREP back to the originator of the RREQ if the destination is a wired node. Instead, the intermediate node updates its routing table and rebroadcasts the received RREQ message. To determine whether the destination is a wired node or not, an intermediate node consults its routing table. If the next hop address of the destination is a default route (see Table I), the destination is a wired node. Otherwise, the destination is a mobile node or a gateway. Since neither the fixed node nor the mobile nodes in the MANET can reply to the RREQ, it is rebroadcasted until its TTL value reaches zero. When the timer of the RREQ expires, a new RREQ message is broadcasted with a larger TTL value. However, since the fixed node cannot receive the RREQ message (no matter how large the TTL value is) the source will never receive the RREP message it is waiting for. This problem has been solved by letting the source assume the destination is a fixed node if a network wide search has been done without receiving any corresponding RREP. In that case, the source must find a route to a gateway [11], [21] (if it does not have one already) and send its data packets towards the gateway, which will forward them towards the fixed node [2]. It should be mentioned that when using the expanding ring search [16] a considerable route discovery delay will occur if the destination is a fixed node. Modifying the parameters involved in the expanding ring search technique (such as TTL_START and TTL_THRESHOLD) can decrease the route discovery delay if the destination is a fixed node.

TABLE I
ROUTING TABLE OF MOBILE NODE

Sr.	Destination Address	Next Hop Address
1	Fixed Node	Default
2	Default Node	Gateway
3	Gateway Node	IMN

III. PERFORMANCE METRICS

The route discovery delay, routing overhead and packet delivery ratio metrics are used in the performance evaluation of Extended AODV for Integrated MANET with Internet. The route discovery delay is defined as the time required receiving the first data packet by the destination. The routing overhead is defined as the total number of routing packets transmitted during simulation. For packets sent over multiple hops, each transmission of packet (each hop) count as one transmission. The packet delivery ratio is defined as ratio of data packets delivered to the destination to those generated by CBR sources.

IV. ANALYTICAL NETWORK MODEL

This paper incorporates two types of network models of MANET domains for analysis, as follows:

A. Uniformly Distributed Network Model

Within the context of TTL based controlled flooding search [7], [3] in integrated wired cum wireless mobile ad hoc network [8] the nodes in a wireless MANET domains are uniformly distributed in a square lattice. For analysis, assume that a search with TTL value of k will reach all neighbours that are k hops away from originating node, and the cost associated with this search is a function of k , denoted by C_k .

Let L denotes the minimum TTL value required to search every node within the network, and will also refer to L as the dimension or size of the network. Also assumes that the destination exists; using a TTL value of L will locate the destination with probability 1.

Consider, X to denote the minimum TTL value required to locate the destination. Note that X is an integer-valued random variable taking values between 1 and L such that $P_r(X \in \{1, 2, \dots, L-1, L\}) = 1$. We denote the cumulative distribution of X by $F(\cdot)$. By definition $F(k) = P_r(X \leq k)$.

Similarly, the tail distribution of X is denoted by $\bar{F}(\cdot)$, so that $\bar{F}(k) = 1 - F(k) = P_r(X > k)$. Note that $\bar{F}(L) = 0$ and $\bar{F}(0) = 1$ for any X . For a given search strategy, denote u_i as the TTL value used during the i -th round, and let $\mathbf{u} = [u_1, u_2, \dots, u_N]$ be a vector denoting the increasing sequence of N TTL (integer) values. The N -tuple \mathbf{u} represents a specific search strategy. For any sensible strategy must have $u_i < u_{i+1}$, for all $1 \leq i \leq N-1$. Note that it is not essential to use the entire sequence in a specific search experiment. However, in order to guarantee that the strategy \mathbf{u} will locate the destination with probability 1, it must be true that $u_N = L$. Also note that the value of N can vary between different policies. The total expected search cost using strategy \mathbf{u} is given by

$$J_X^{\mathbf{u}} = \sum_{i=1}^{N_{\mathbf{u}}} C_{u_i} P_r(X > u_{i-1}) = \sum_{i=1}^{N_{\mathbf{u}}} C_{u_i} \bar{F}(u_{i-1}) \quad (1)$$

where n is number of elements in the vector \mathbf{u} , c is the cost of searching with TTL value u and $c_L = 0$. The search policy that minimizes this cost, denoted by \mathbf{u}^* , is thus

$$\mathbf{u}^* = \underset{\mathbf{u} \in U}{\operatorname{argmin}} J_X^{\mathbf{u}} = \underset{\mathbf{u} \in U}{\operatorname{argmin}} \sum_{i=1}^{N_u} C_{u_i} \bar{F}(u_{i-1}) \quad (2)$$

where U denotes the set of all admissible search strategies (TTL sequences), i.e., all vectors \mathbf{u} such that $u_i < u_{i+1}$ for all $1 \leq i \leq N - 1$ and $u_N = L$.

This minimization can be solved backward in time using standard dynamic programming techniques [3]. The most recently used TTL value is denoted by n , as the information state. For convenience, conditional tail distribution of the destination is denoted by $\bar{F}(j|n)$, that the most recently used TTL value n did not locate the destination, i.e.,

$$\begin{aligned} \bar{F}(j|n) &= P_r(X > j | X > n) \\ &= \begin{cases} 1 & 1 \leq j \leq n \\ \bar{F}(j) / \bar{F}(n) & n + 1 \leq j \leq L \end{cases} \end{aligned}$$

The following dynamic programming equations can be solved recursively for $1 \leq n \leq L - 1$:

$$V(L) = 0$$

$$V(n) = \min_{n+1 \leq l \leq L} \{C_l + \bar{F}(l|n)V(l)\}$$

where the value function $V(n)$ is the minimum expected cost-to-go (over all choices of TTL values), given that the most recently used TTL value n did not locate the destination.

The initial condition $V(0)$ reflects the fact that using a TTL value of L ensures finding the destination and thus there would be no more remaining cost. Equation for $V(n)$, follows from the fact that after unsuccessfully searching with a TTL value of n , the remaining choices for TTL values are the integers from $n + 1$ to L . Any such choice l incurs an immediate search cost C_l plus an expected future cost if the destination is not located using l . Note that because $\bar{F}(j|n) = \bar{F}(j) / \bar{F}(n)$ for any value of n , $V(L-1) = C_L$ for any search strategy. This agrees with the fact that if searching with a TTL value of $L-1$ is unsuccessful, then the only remaining option is to search with a TTL value of L . Solving this set of equations backward we can obtain $V(n)$ for all n and determine the optimal TTL sequence \mathbf{u}^* . Finally $V(0)$ is the optimal (minimum) total expected search cost $\min_{\mathbf{u} \in U} J_X^{\mathbf{u}}$.

Consequently the minimum total expected cost is

$$\begin{aligned} V(0) &= \min_{1 \leq l \leq L} \{C_l + \bar{F}(l)V(l)\} \\ &= \min_{1 \leq l \leq L} \left\{ C_l + \frac{L-1}{L} C_L \right\} \\ &= C_L \min_{1 \leq l \leq L} \left\{ \frac{l}{L} + \frac{L-1}{L}, 1 \right\} = C_L \end{aligned}$$

Therefore, the optimal search cost when X is uniformly distributed with linear search cost is C_L . The minimum can be obtained by either using an initial TTL value of L so that $\mathbf{u} = [L]$.

B. Randomly Distributed Network Model

The previous section was presented the search strategies that minimize the expected search cost for a given destination location distribution. But now the problem of finding good search strategies when this probability distribution is not known a priori. In this case, a natural performance criterion is the worst-case performance. Thus, the goal is to find a search strategy that has the lowest worst-case cost. It is in fact always possible to find a random TTL sequence for any given non random sequence that performs better in the worst case. Therefore, under this criterion the best search strategies are randomized strategies [8].

A randomized search strategy \mathbf{u} is a TTL sequence that consists of random variables of certain probability distributions (discrete), i.e., $\mathbf{u} = [u_1, u_2, \dots, u_N]$ where u_i is a random variable, $1 \leq i \leq L$, and their distribution can be independently or jointly defined [3].

The set of all possible search strategies U to include both non random and random search strategies. With this inclusion, a much bigger set of strategies can be used.

The notation $J_X^{\mathbf{u}}$ to denote the expected search cost of using strategy \mathbf{u} for destination location X , noting that when \mathbf{u} is a deterministic vector \mathbf{u} is the average over all realizations of X as defined in (1), while \mathbf{u} being a random vector means $J_X^{\mathbf{u}}$ is the average over all realizations of X as well as all realizations of \mathbf{u} . Consider, the case where the random vector \mathbf{u} and X are mutually independent since the distribution of X is not known a priori.

On the other hand, the expected search cost using an ideal observer who knows precisely the location (realization of X) is $E[C_X]$, where $C_X = C(X)$ is the cost of searching with TTL value X . This is also the best (minimum cost) obtainable.

The ratio between these two costs $\frac{J_X^{\mathbf{u}}}{E[C_X]}$, also referred to as the cost ratio, to evaluate the performance of strategy \mathbf{u} . The worst-case performance (cost ratio) for any strategy \mathbf{u} is therefore

$$\rho^{\mathbf{u}} = \sup_{\{p_X^{(\mathbf{u})}\}} \frac{J_X^{\mathbf{u}}}{E[C_X]}$$

where the supremum is taken over all possible probability distributions $\{p_x\}$ for the destination location. The quantity J is also known as the competitive ratio with respect to an oblivious adversary [12] who knows the search strategy u in advance.

The best worst-case strategy, denoted by u^* , is one that achieves the minimum over all admissible search strategies, denoted by u ;

$$\rho^* = \inf_{u \in U} \rho^u = \inf_{u \in U} \sup_{\{p_x^{(u)}\}} \frac{J_X^u}{E[C_X]}$$

It is required that any probability distribution within the set of $\{p\}$ must satisfy $E[C_X] < \infty$, so that the destination can be located with finite average cost.

V. SIMULATION NETWORK MODEL

The mathematical model [3] mentioned above for uniformly and randomly distributed nodes in MANET domain are analysed by the simulation. The performance of Extended AODV routing protocol is evaluated by considering two different scenarios. Simulation was carried out using network simulator NS2 (2.33) [9], [10].

A. Uniformly Distributed Network Model

This is the scenario, where the wireless domain nodes are uniformly distributed along the square lattice. The locations of all nodes in wireless domain are shown in Fig 1. The positions of MANET domain nodes are static. The simulation scenario consists of ten MANET domains and one wired domain. Each MANET domain consists of total six nodes. Out of six nodes, one of the nodes is gateway node in each MANET domain. The wired domain consists of two fixed nodes. The topology is rectangular area with 2200 meter length and 1000 meter width. The varying traffic sources (10, 20, 30) were used to send data from the shown MANET domains to the wired domain.

Each MANET domain nodes are located in such a way that the two consecutive nodes are in direct communication range. The communication range is taken for simulation is 250 meters.

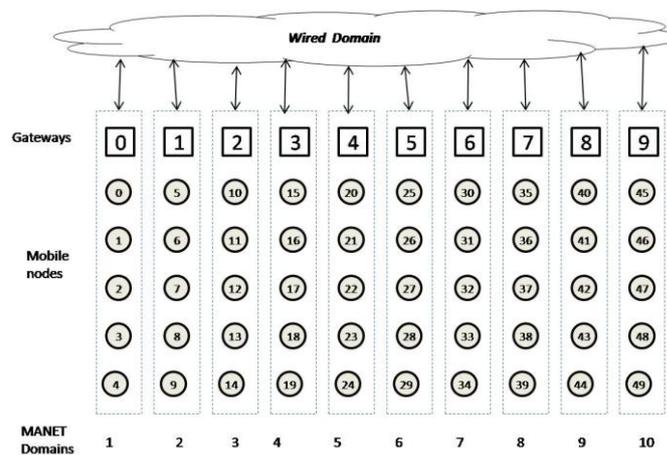


Fig. 1 Uniformly distributed network model

B. Randomly Distributed Network Model

In contrast to previous network model, the wireless domain nodes are randomly distributed. This random location distribution is obtained by the use of ns-2.33 [9], [10] utility for mobility scenario generation i.e. setdest. Otherwise, all the parameters and their values are same as the first scenario. Table II shows the simulation parameters for both scenarios.

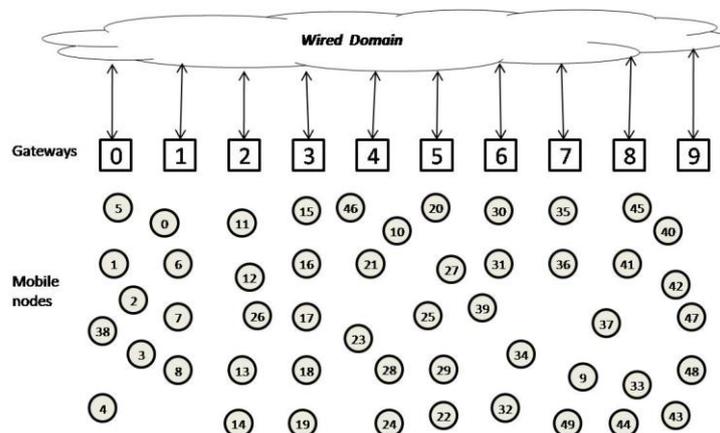


Fig. 2 Randomly distributed network model

TABLE II
SIMULATION PARAMETERS

Sr	Parameter	Value
1	Mobile node Transmission range	250 m
2	Topology size	2200 m x 1000 m
3	Domain	1 Wired, 10 MANET
4	Number of gateways in each MANET domain	1
5	Mobile Nodes in each MANET domain	5
6	Traffic type	Constant bit rate
7	Packet size	512 bytes
8	Packet interval	0.25 s

VI. SIMULATION RESULTS AND DISCUSSION

Fig. 3, 4, 5 are the results obtained for the scenario1, when wireless domain nodes are uniformly distributed in a square lattice. Fig. 3 shows the plot of route discovery delay versus varying sources. It is clear that when expanding ring search is used with TTL_START = 1, 5; TTL_INCREMENT = 2; TTL_TRESHOLD = 7; the route discovery delay is high. This is because; the source nodes in wireless MANET domain is required to send multiple times RREQ across the network diameter; until the route to the destination is not found. But when network wide search used with TTL_START equal to network diameter while keeping other parameters same as previous; the route discovery delay is reduced. This is because only one time the RREQ is broadcasted to entire network, without receiving any corresponding RREP (as mentioned in extended AODV) for the destination node which is located in wired domain. Fig. 5 is a plot of routing overhead versus number of sources. Because of similar reasons, the routing overhead is reduced for network wide search. Fig. 4 shows that the packet delivery ratio is increased for network wide search with varying number of sources. This is desirable and obvious, as once the route is established in between source and destination, then all the transmitted successfully received by the destination node which is located at wired domain. Since the route discovery delay time is reduced in case of network wide search compare to the expanding ring search, so that, the probability of lost packet is less in network wide search. Hence, the packet delivery ratio is enhanced.

Fig. 6, 7, 8 are the results obtained for the scenario 2, i.e. when wireless domain nodes are randomly distributed, which is shown in Fig. 2. The plots of route discovery delay, routing overheads and packet delivery ratio versus varying sources (10, 20, 30) shows the same results as obtained from scenario 1. This means that the network wide search performs better for integrated MANET with wired network regardless of distribution of nodes in wireless network.

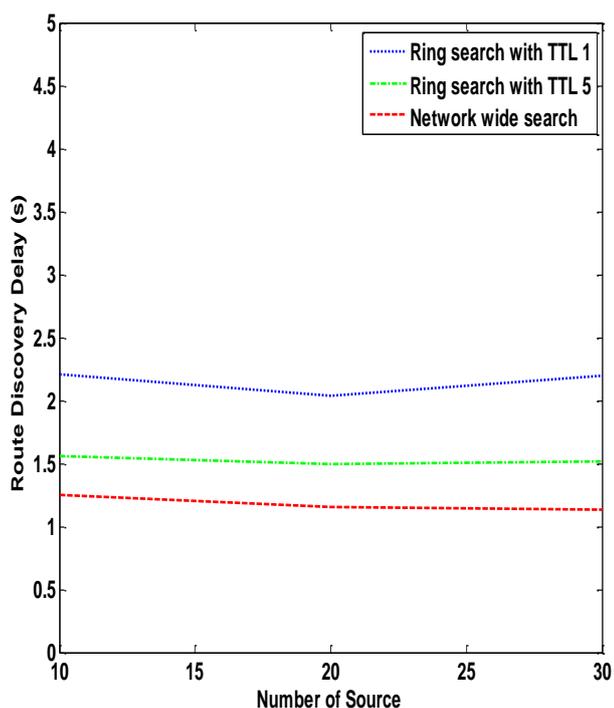


Fig. 3 Scenario-1 Route discovery delay

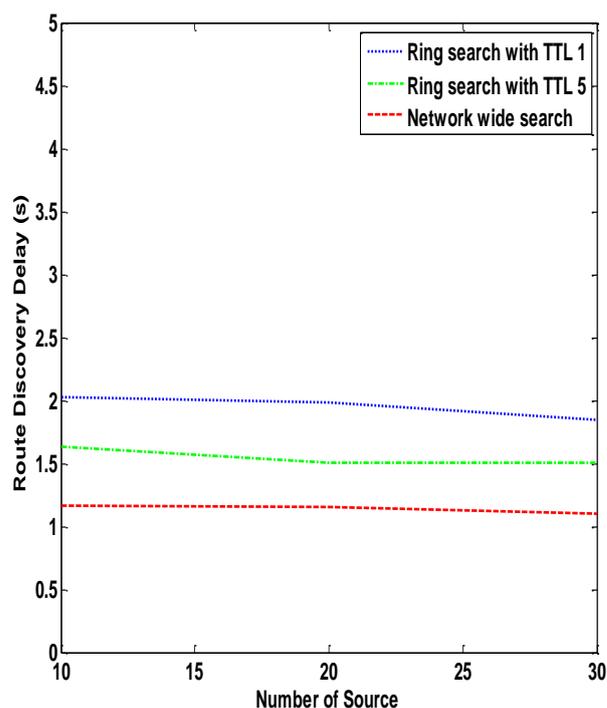


Fig. 6 Scenario-2 Route discovery delay

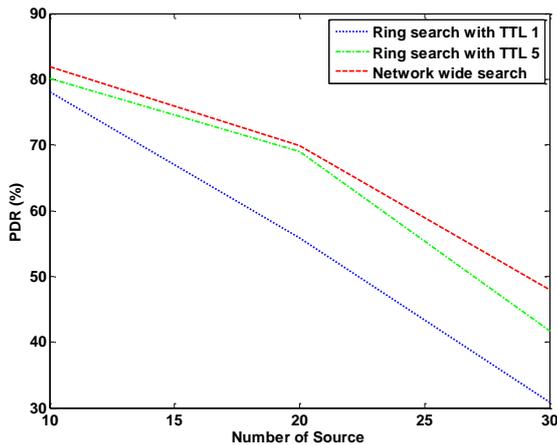


Fig. 4 Scenario-1 Packet delivery ratio

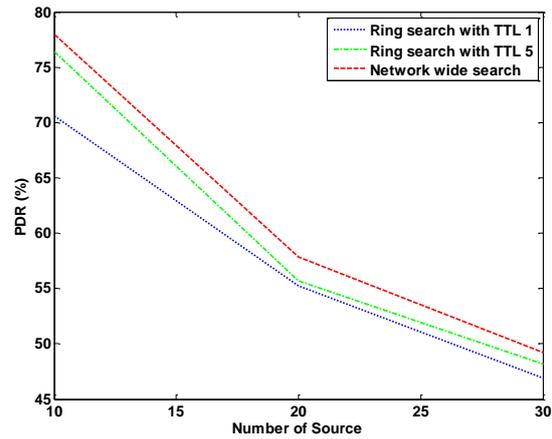


Fig. 7 Scenario-2 Packet delivery ratio

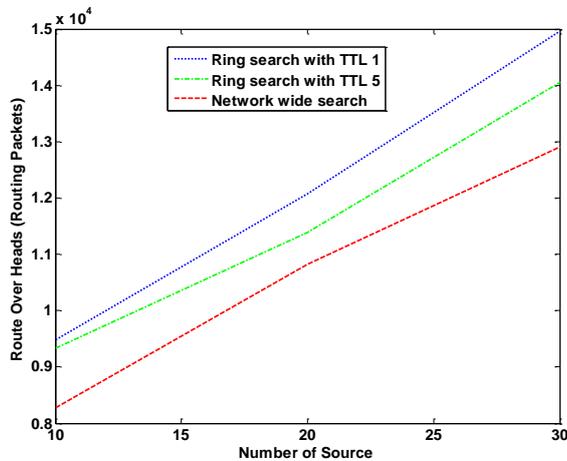


Fig. 5 Scenario-1 Routing overheads

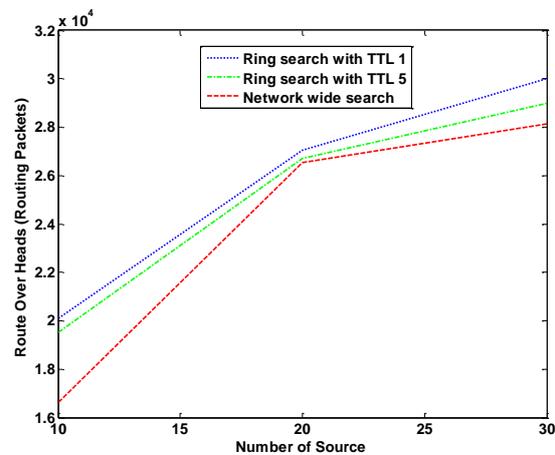


Fig. 8 Scenario-2 Routing overheads

VII. CONCLUSIONS

The use of extended AODV ad hoc routing protocol provides the accessibility of mobile node in MANET domain to the fixed wired internet domain. The modification in TTL value of the proposed approach which is known as network wide search, performs better than conventional expanding ring search for Integrated MANET with Internet. Simulation results shows that the network wide search provides the reduced route discovery delay time, low routing overhead and high value of packet delivery ratio as desired, for both uniformly and randomly distributed network models. The mathematical model used for the class of TTL based search method also justifies the obtained simulation results for integrated MANET with internet. The mathematical derivation proves that the network wide search minimizes the expected search cost.

This paper also concludes that there is no effect on the performance of extended AODV, whether the network model of MANET domain is uniformly or randomly distributed.

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