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Three-Dimensional High-Resolution Spectral Analysis: New method of estimating 3D frequencies

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Abstract*— *The present work is situated within the framework of the spectral analysis applied to the multidimensional signals, in particular the high-resolution methods known by their performances and their precision such as the 3D-ESPRIT method (Estimation of Signal Parameters via Rotationnal Invariance Techniques), the MEMP method (Matrix Enhancement and Matrix Pencil) and the ACMP method (Algebraically Coupled Matrix Pencil). In these methods, it generally poses the problem of the formation of the pairs or of the triplets of frequencies for the 2D or the 3D signals respectively. In this work, we are going to propose a new method of high-resolution spectral analysis type 3D-ESPRIT. The proposed method allows us to overcome the problem of the multiple triplets of the 3D-ESPRIT method. This new technique will be tested on a sum of 3D complex exponential (3DSCE) embedded in a white gaussian additive noise with various values of SNR.

Keywords*— *Spectral Analysis, High Resolution, 3D-ESPRIT, MEMP, ACMP, Autocorrelation matrix.

I. INTRODUCTION

Estimating the parameters of a model remains a problem which it is essential in the modeling of the signals by a sum of 3D complex exponential (3DSCE model) perturbed by an additive gaussian white noise. Indeed, this modeling is used in several applications such as telecommunications, antenna processing, image analysis by resonance or the seismic image or medical image processing. The passage from 1D to 2D or from 2D to 3D is not trivial and same problems which do not arise in the 1D case appear and they worth to be treated, we speak here about the pairing or the formation of frequential pairs or the frequential triplets. Note that the high resolution spectral analysis methods can be divided into two families: The first one group the methods which restore spectral information by the means of a functional depending of a frequency vector, these methods are known as of the pseudo-spectrum or scanning methods. In the second class, the methods exploit the matrix structure inherent to the 3DSCE model. It proposes a phase of estimate relating to the frequential triplets contained in the model, these methods are said analytics.

These later methods use the property of the invariance of the model. Indeed, MEMP [1] [2] uses the Teoplitz structure of the covariance matrix. The formation of triplets or pairing is necessary and the projection onto the signal sub-space allows us to free from this problem. The advantage of this method lies in the precision of the estimate (consistency). A second approach known as ACMP [3] is developed by considering the 3DSCE as a matrix form. ACMP offers the advantage to estimate simultaneously the frequency components of each dimension. The ACMP offers an automatic pairing but suffers from the bias of the estimated pairs. ESPRIT [4] extended to 2D dimensional case [5] [6] is a method which allows to combine the advantages of these two methods (MEMP and ACMP). In the case of the 3D signals, a new high-resolution method type 3D-ESPRIT is proposed to improve and overcome the problem of the formation of triplets if there are the multiples.

The rest of the paper is organized as follows: we are going to study in the second section the 3DSEC signal model then we will treat the autocorrelation matrix in the third section. In the fourth section, we will study MEMP and 3D-ESPRIT methods as well as the new proposed method. The fifth section will be devoted to make a simulation of these methods and to compare the results. Finally, one will complete our work by a conclusion.

II. FORMULATION OF PROBLEM

A. Model of signal3DSEC

Let us consider that every voxel (m, n, t) of block of observed image corresponds to the sum of two terms:

$$y(m,n,t) = x(m,n,t) + b(mn,n,t) \quad (1)$$

With $1 \le m \le M$, $1 \le n \le N$ and $1 \le t \le T$

The useful signal x (m, n, t) is modeled in the following way:

$$x(m,n,t) = \sum_{i=1}^{K} a_i \exp(j2\pi(f_{1i}m + f_{2i}n + f_{3i}m) + j\varphi_i) \quad (2)$$

-The K components of the signal are defined by the frequency triplets $\{f_{1i}, f_{2i}, f_{3i}\}$, the amplitudes $\{a_i\}$ and the phases $\{\varphi_i\}$.

- b(m, n, t) is a white, gaussian noise, centered whit variance σ_b^2.

Our objective is given a block $M \times N \times T$ of data, how to estimate the triplets $\{f_{1i}, f_{2i}, f_{3i}\}$ where i=1...K.

B. The autocorrelation matrix

Let us consider a $P \times Q \times L$ sub-block from the $M \times N \times T$ data block.

The purpose is to calculate the autocorrelation matrix of these $P \times Q \times L$ data. For that, we concatenate the data in a vector column $PQL \times 1$ noted Y_{PQL}. Thus to scan all the 3D block, we make a reading column by column in every layer of the volume as follows:

$$Y_{PQL}^T = [y_0^T, y_1^T \cdots y_{L-1}^T] \tag{3}$$

Where:

$$y_i^T = [y_{0,i}^T, y_{1,i}^T \cdots y_{Q-1,i}^T] \tag{4}$$

And

$$y_{i,j}^T = [y(m, n+j, t+i) \cdots y(m+P-1, n+j, t+i)] \tag{5}$$

For $i = 0,..., L-1$ and $j = 0,..., Q-1$.

Thus the autocorrelation matrix is given by:

$$R = E[Y_{PQL} Y_{PQL}^H] \tag{6}$$

With:

- E stands for the expectation operator.
- H is the conjugate transposition operator.

The eigen decomposition of the autocorrelation matrix is:

$$R = UDU^{-1} \tag{7}$$

Where D is the diagonal matrix composing of the eigenvalues, and the columns of U are the associated eigenvectors. The eigenvectors associated to the principal eigenvalues span the signal subspace noted U_s. The remainder eigenvectors span the noise subspace U_b.

R is the matrix $L \times L$ block Teoplitz given by:

$$R = \begin{bmatrix} R_0 & R_{-1} & \ldots & R_{-L+1} \\ R_1 & R_0 & \ldots & R_{-L+2} \\ . & . & . & . \\ R_{L-1} & R_{L-2} & \ldots & R_0 \end{bmatrix} \tag{8}$$

In which each block is a $Q \times Q$ block Teoplitz matrix:

$$R_i = \begin{bmatrix} r_{0,i} & r_{-1,i} & \ldots & r_{-Q+1,i} \\ r_{1,i} & r_{0,i} & \ldots & r_{-Q+2,i} \\ . & . & \ldots & . \\ r_{Q-1,i} & r_{Q-2,i} & \ldots & r_{0,i} \end{bmatrix} \tag{9}$$

And finally r_{ji} is a $P \times P$ Teoplitz matrix given by:

$$r_{j,i} = \begin{bmatrix} r_y(0,j,i) & r_y(-1,j,i) & \ldots & r_y(-P+1,j,i) \\ r_y(1,j,i) & r_y(0,j,i) & \ldots & r_y(-P+2,j,i) \\ . & . & \ldots & . \\ r_y(P-1,j,i) & r_y(P-2,j,i) & \ldots & r_y(0,j,i) \end{bmatrix} \tag{10}$$

Where $r_y(.,.,.)$ is the autocorrelation function defined by:

$$r_y(k,l,t) = \sum a_i \exp(j2\pi(f_{1i}k + f_{2i}l + f_{3i}t)) + \sigma_b^2 \delta(k,l,t) \tag{11}$$

The autocorrelation matrix can be also written in this way:

$$R = S^{1}_{[PQL,K]} \Psi S^{1H}_{[PQL,K]} + \sigma_b^2 I \tag{12}$$

With:

- $S^{1}_{[PQL,K]}$ the 3D Vandermonde matrix given by:

$$S^{1}_{[PQL,K]} = [s_{31L} \otimes s_{21Q} \otimes s_{11P}, \ldots, s_{3KL} \otimes s_{2KQ} \otimes s_{1KP}] \tag{13}$$

$$s_{mnA} = \begin{bmatrix} 1 & \exp(j2\pi f_{mn}) & \ldots & \exp(j2\pi f_{mn}(A-1)) \end{bmatrix} \tag{14}$$

Where:

- m = 1, 2, 3, n = 1, 2… K, and A=P, Q, L
- $\otimes$ stand for the Kronecker product.
- Ψ is the diagonal matrix containing the squares of the amplitudes.

$$\Psi = \underset{1\le i\le K}{diag}(a_i^2) \tag{15}$$

C. Recall high resolution estimation methods

1) *The MEMP method*

The development of the MEMP method is based on the study of the matrix structure of the autocorrelation matrix (12). Thus, for a reading column by column:

The subspaces spanned by the columns of the 3D Vandermonde matrix and by the eigenvectors of the signal subspace are the same. There exists thus an invertible matrix Θ_1 of rank K such as:

$$U_{s1} = S^{1}_{[PQL,K]} \Theta_1 \tag{16}$$

The 3D Vandermonde matrix $S^{1}_{[PQL,K]}$ can be written using the 2D Vandermonde matrix $S^{1}_{[PQ,K]}$ as follows:

$$S^{1}_{[PQL,K]} = \begin{bmatrix} S^{1}_{[PQ,K]} \\ S^{1}_{[PQ,K]}\Phi_3 \\ . \\ S^{1}_{[PQ,K]}\Phi_3^{L-1} \end{bmatrix} \tag{17}$$

With:

$$\Phi_3 = \underset{1\le i\le K}{diag} \exp(j2\pi f_{3i}) \tag{18}$$

In the same way, 2D Vandermonde matrix $S^{1}_{[PQ,K]}$ can be written using the 1D Vandermonde matrix $S^{1}_{[P,K]}$:

$$S^{1}_{[PQ,K]} = \begin{bmatrix} S^{1}_{[P,K]} \\ S^{1}_{[P,K]}\Phi_2 \\ . \\ S^{1}_{[P,K]}\Phi_2^{Q-1} \end{bmatrix} \tag{19}$$

With:

$$S^{1}_{[PK]} = [s_{11,P} \quad s_{12,P} \quad \ldots \quad s_{1K,P}] \tag{20}$$

$$\Phi_2 = \underset{1\le i\le K}{diag} \exp(j2\pi f_{2i}) \tag{21}$$

However the property of invariance is expressed at the level of the matrix $S^{1}_{[PQL,K]}$, thus we can clarify two distinct partitionings relating to a matrix EM1:

$$S^{1}_{[PQL,K]} = \begin{bmatrix} S^{1}_{[PQ,K]} \\ ---- \\ EM1 \end{bmatrix} \begin{matrix} \updownarrow PQ \\ \\ \\ \end{matrix} = \begin{bmatrix} EM1\Phi_3^{-1} \\ ---- \\ S^{1}_{[PQ,K]}\Phi_3^{L-1} \end{bmatrix} \begin{matrix} \\ \\ \updownarrow PQ \end{matrix} \tag{22}$$

By applying these last relations to the equation (15), we have:

$$U_{S1} = \begin{bmatrix} xxx \\ ---- \\ \overline{U}_{S1} \end{bmatrix} \updownarrow PQ \quad \Theta_1 = \begin{bmatrix} \underline{U}_{S1} \\ ---- \\ xxx \end{bmatrix} \updownarrow PQ \quad \Theta_1$$
$$= \begin{bmatrix} S^1_{[PQ,K]} \\ ---- \\ EM1 \end{bmatrix} \updownarrow PQ \quad \Theta_1 = \begin{bmatrix} EM1\Phi_3^{-1} \\ ---- \\ S^1_{[PQ,K]}\Phi_3^{-L} \end{bmatrix} \updownarrow PQ \quad \Theta_1 \tag{23}$$

Thus:

$$\begin{cases} EM1\Theta_1 = \overline{U}_{S1} \\ EM1\Phi_3^{-1}\Theta_1 = \underline{U}_{S1} \end{cases} \Rightarrow \begin{cases} EM1 = \overline{U}_{S1}\Theta_1^{-1} \\ \overline{U}_{S1}\Theta_1^{-1}\Phi_3^{-1}\Theta_1 = \underline{U}_{S1} \end{cases} \tag{24}$$

$$\Rightarrow \Theta_1^{-1}\Phi_3^{-1}\Theta_1 = \overline{U}_{S1}{}^{\dagger}\underline{U}_{S1}$$

Where † is the pseudo reverse.
We find finally the following equality:

$$F_3 = (\overline{U}_{S1})^{\dagger}\underline{U}_{S1} = \Theta_1^{-1}\Phi_3^{-1}\Theta_1 \tag{25}$$

In order to retrieval the frequencies of the first spatial axis, we have to make a reading line by line in the 3D Vandermonde matrix, by following the same stages as previously and commutate the Kronecker product contained in $S^1_{[PQL,K]}$, we have the following relations:

$$U_{s2} = S^2_{[PQL,K]}\Theta_2 \tag{26}$$

$$S^2_{[PQL,K]} = E_1^2 S^1_{[PQL,K]} \tag{27}$$

$$U_{s2} = E_1^2 U_{s1} \tag{28}$$

Where**:**

$$E_1^2 = \sum_{i=1}^{P}\sum_{j=1}^{Q}\sum_{k=1}^{L} E_{i,j}^{PQ} \otimes E_{j,k}^{QL} \otimes E_{k,i}^{LP} \tag{29}$$

And $E_{k,l}^{QP}$ is the $P \times Q$ elementary matrix having value 1 at the (k, l) coordinate and zeros elsewhere.
We find finally the following equality:

$$F_1 = (\overline{U}_{S2})^{\dagger}\underline{U}_{S2} = \Theta_2^{-1}\Phi_1^{-1}\Theta_2 \tag{30}$$

To extract the frequencies of the second special axis, we have to read data layer by layer for the 3D Vandermonde matrix, by following the same stages as previously and commutate the Kronecker product contained in $S^3_{[PQL,K]}$, we have the following relations:

$$U_{s3} = S^3_{[PQL,K]}\Theta_3 \tag{31}$$

$$S^3_{[PQL,K]} = E_2^3 S^2_{[PQL,K]} \tag{32}$$

And

$$U_{s3} = E_2^3 U_{s2} \tag{33}$$

$$E_2^3 = \sum_{i=1}^{Q}\sum_{j=1}^{L}\sum_{k=1}^{P} E_{i,j}^{QL} \otimes E_{j,k}^{LP} \otimes E_{k,i}^{PQ} \tag{34}$$

We find finally the following equality:

$$F_2 = (\overline{U}_{S3})^{\dagger}\underline{U}_{S3} = \Theta_3^{-1}\Phi_2^{-1}\Theta_3 \tag{35}$$

Until now, the components of the 3D frequencies are estimated, but they are not associated in the good form. For this end, we use the property of orthogonality between the noise subspace and the 3D Vandermonde matrix. The best triplets maximize the projection onto the signal subspace according to the following criterion:

$$C(i,j,k)=\sum_{l=1}^{K}\left\| U_{s1}{}^{H}(l)(s_{3k,L}\otimes s_{2j,Q}\otimes s_{1i,P})\right\| \quad (36)$$

2) *The 3D-ESPRIT method*

The 3D-ESPRIT method succeeds to combine the advantages of the MEMP method and those of the ACMP method. Indeed, 3D-ESPRIT offers a precision comparable to with that of MEMP and reforms the frequency pairs in an automatic way as that of ACMP.

3D ESPRIT uses the relations (25), (30) and (35):

$$\begin{cases} F_3=(\overline{U}_{S1})^{\dagger}\underline{U}_{S1}=\Theta_1^{-1}\Phi_3^{-1}\Theta_1 \\ F_1=(\overline{U}_{S2})^{\dagger}\underline{U}_{S2}=\Theta_2^{-1}\Phi_1^{-1}\Theta_2 \\ F_2=(\overline{U}_{S3})^{\dagger}\underline{U}_{S3}=\Theta_3^{-1}\Phi_2^{-1}\Theta_3 \end{cases} \quad (37)$$

3D-ESPRIT exploits a supplementary property consisting in that F_1, F_2 and F_3 are diagonalizable by the same transformation:

$$\Theta_1=\Theta_2=\Theta_3 \quad (38)$$

This shows that Θ_1 diagonalise F_1, F_2 and F_3 and it is calculated using the following relation:

$$F=\alpha_1 F_1+\alpha_2 F_2+(1-(\alpha_1+\alpha_2))F_3=\Theta^{-1}\Delta\Theta \quad (39)$$

Where α_1 and α_2 are scalars.

By applying the transformation Θ to matrices F_1, F_2 and F_3, the frequencies are extracted from the diagonals of Φ_1, Φ_2 and Φ_3:

$$\begin{cases} \Phi_1=\Theta F_1\Theta^{-1} \\ \Phi_2=\Theta F_2\Theta^{-1} \\ \Phi_3=\Theta F_3\Theta^{-1} \end{cases} \quad (40)$$

3) The proposed new 3D-ESPRIT method

The 3D-ESPRIT frequencies estimation method remains an effective one. Especially the triplets are combined in an automatic and with high accuracy. However, for certain cases, this method can form erroneous triplets; especially if they are some multiples. To overcome this problem, we propose a new method of high resolution estimation type 3D-ESPRIT. Indeed, after having determined the matrices F_1, F_2 and F_3, we pass in the stage of the diagonalisation but this time it is not with the matrix Θ . But this later matrix will be a tool to form three other permutation matrices P_1, P_2 and P_3 given by:

$$\begin{cases} P_1=\Theta^{-1}\Theta_1 \\ P_2=\Theta^{-1}\Theta_2 \\ P_3=\Theta^{-1}\Theta_3 \end{cases} \quad (41)$$

Finally the estimated frequencies are classified in a correct way by using the permutation matrices in the following way:

$$\begin{cases} \Phi_1^{'}=P_1^{-1}\Phi_1 P_1 \\ \Phi_2^{'}=P_2^{-1}\Phi_2 P_2 \\ \Phi_3^{'}=P_3^{-1}\Phi_3 P_3 \end{cases} \quad (42)$$

III. NUMERICALS EXAMPLES

In this section, we present some numerical examples; our approach is tested on a 3DSCE model. The data are generated according to the model of the equation (1). We consider three waves i.e. K=3 with the amplitude a_i=1, the 3D frequencies are given in the table 1. The data and the sizes of the autocorrelation matrix are respectively (M, N, T) = (16, 16, 16), and (P, Q, L) = (5, 5, 5). We consider two values of signal to noise ratio (SNR): SNR=0dB and SNR=20dB. The estimated frequencies using MEMP, 3D-ESPRIT and the proposed methods are given in Table II.

TABLE I
3D FREQUENCIES USED IN THE SIMULATION

	f_{1i}	f_{2i}	f_{3i}
1st wave	0.11	0.14	0.17
2nd wave	0.24	0.23	0.21
3rd wave	0.45	0.48	0.47

TABLE II
ESTIMATED FREQUENCIES USING MEMP, 3D ESPRIT AND THE NEW PROPOSED METHOD FOR SNR=0 DB AND SNR=20 DB

			1st wave	2nd wave	3rd wave
MEMP method	f_{1i}	0 dB	0.1090	0.2386	0.4495
		20 dB	0.1100	0.2400	0.4499
	f_{2i}	0 dB	0.1380	0.2284	0.4806
		20 dB	0.1400	0.2300	0.4800
	f_{3i}	0 dB	0.1706	0.2093	0.4703
		20 dB	0.1700	0.2101	0.4699
3D-ESPRIT method	f_{1i}	0 dB	0.1095	0.2402	0.4489
		20 dB	0.1098	0.2401	0.4501
	f_{2i}	0 dB	0.1406	0.2289	0.4800
		20 dB	0.1399	0.2299	0.4799
	f_{3i}	0 dB	0.1697	0.2091	0.4702
		20 dB	0.1700	0.2100	0.4699
the new 3D-ESPRIT method	f_{1i}	0 dB	0.1110	0.2409	0.4489
		20 dB	0.1100	0.2400	0.4500
	f_{2i}	0 dB	0.1404	0.2307	0.4801
		20 dB	0.1400	0.2299	0.4799
	f_{3i}	0 dB	0.1700	0.2101	0.4715
		20 dB	0.1699	0.2100	0.4700

In order to propose a quantitative measure of the 3D estimated frequency, we evaluate the variance of the estimation error versus the SNR value in the range -10 to 30dB. For each SNR, 100 trials are used. The result is plotted in Fig. 1.

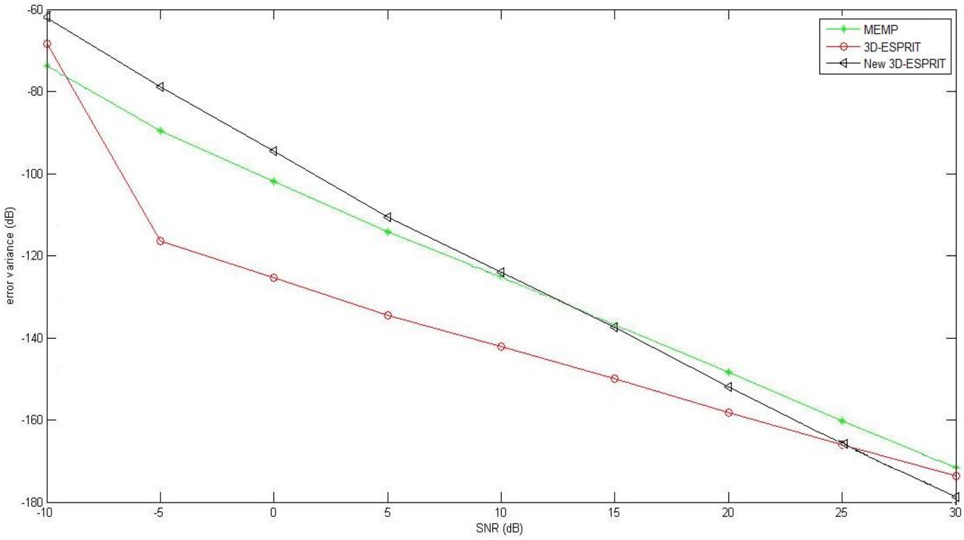

Fig. 1 Estimation-error variance versus the SNR.

Comment

The formation of triplet or the problem of pairing remains one of the major problems which are obvious during passage from 2D to 3D. Indeed, this problem is very concrete when there are the multiples, and the formed triplets are not correct. In the table II, the obtained results show it; we can see that the new proposed method estimate the frequencies with high accuracy compared to MEMP and the 3D-ESPRIT methods. The results given by the new method are similar to true real value. In that, this method is more efficient with good constancy in comparison with the MEMP and the 3D-ESPRIT methods.

IV. CONCLUSIONS

The methods of high resolution spectral analysis are the important methods and which allow resolving some problems of appariement. Indeed, the 3D-ESPRIT method allows extracting the triplet in an efficient and automatic way. But if there is the multiple, this method doesn't give a better triplet. The new proposed try to overcome this problem by using three appropriate permutation matrices. In perspectives, we propose to calculate the Cramer-Rao Bound for the estimated frequencies using the new proposed method; we will also try to develop this technology for signals embedded in a colored noise.

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