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Total Bondage Number of an Interval Graph

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Abstract- ***Dominating sets play predominant role in the theory of graphs. Among the various applications of the theory of domination the most often discussed is a communication network. This network consists of communication links between a fixed set of sites. By constructing a family of minimum dominating sets, we compute the total bondage number. Suppose, communication network fails due to link failure. Then the problem is to find a fewest number of communication links such that the communication with all sites in possible. This leads to the introducing of the concept of total bondage number of graph. In this paper we consider the total bondage number $b_t(G)$ for an interval family corresponding to an interval graph G, which is defined as the minimum number of edges whose removal results in a new graph with larger total domination number.***

Keywords: Interval graph, Dominating set, Domination number, Total dominating set, Total bondage number.

1. INTRODUCTION

It is well known that the topological structure of an interconnection network can be modeled by a connected graph whose vertices represent sites of the network and whose edges represent physical communication links. A minimum dominating set in the graph corresponding to an interval family I, where each I_i is an interval on the real line $I_i = [a_i, b_i]$ for $i = 1, 2, ..., n$. Here a_i is called the left end point and b_i is the right end point of I_i, without loss of generality we may assume that all end points of the intervals I are distinct numbers between 1 and $2n$. Two intervals i and j are said to intersect each other if they have non-empty intersection. A subset D of V is said to be a dominating set of G if every vertex in $V \setminus D$ is adjacent to a vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of dominating set [8]. Also a minimum dominating set in the graph corresponds to a smallest set of sites selected in the network for some particular uses, such as placing transmitters. Such a set may not work when some communication links happen fault.

The bondage number $b(G)$ of a non-empty graph G is the minimum cardinality among all sets of edges E_1, for which $\gamma(G - E_1) > \gamma(G)$ [2]. Thus, the bondage number of G is the smallest number of edges whose removal will render every minimum dominating set in G a non-dominating set in the resultant spanning sub graph [5]. Since the domination number of every spanning sub graph of a non-empty graph G is at least as great as $\gamma(G)$, the bondage number of a non-empty graph is well defined [3,4,6,7].

A subset S of V is called a total dominating set if every vertex in V is adjacent to some vertex in S. The total domination number $\gamma_t(G)$ of G is the minimum cardinality taken over all total dominating sets of G [1]. The total bondage number $b_t(G)$ of a non-empty graph G is the minimum cardinality among all sets of edges E_1, for which $\gamma_t(G - e) > \gamma_t(G)$.

2. MAIN THEOREMS

THEOREM 2.1: Let G be an interval graph corresponding to the interval family I = {$i_1, i_2, ……, i_n$}. Let $i_1, i_2 \in I$ and suppose i_2 is contained in i_1, $i_1 \neq 1$ and there is no other interval that intersects i_2, other than i_1, then $b_t(G) = 1$.

PROOF: Let G be an interval graph corresponding to the interval family I = {$i_1, i_2, ……, i_n$}. Let $i_1, i_2 \in I$ and suppose i_2 is contained in i_1, $i_1 \neq 1$ and there is no other interval that intersects i_2. Then clearly $i_1 \in$TDS, the total dominating set of the interval graph G. Since there is no other interval in I, other than i_1 that totally dominates i_2.

Consider the edge (i_2, i_1) in the interval graph G. If we remove this edge from G, then i_2 becomes an isolated vertex in G-e, as there is no other vertex that intersects i_2 other than i_1 as given in the hypothesis. Hence TDS_1 = TDS ∪ {i_2} becomes the total dominating set of G-e and since TDS is the minimum total dominating set of G, it follows that TDS_1 is also a minimum total dominating set of G-e.

Therefore $|TDS_1| = \gamma(G-e) = |TDS| + 1 > |TDS|$. Thus b (G) = 1.

This statement is proved in the following illustration clearly,

2.2: ILLUSTRATION

Consider the following interval family I,

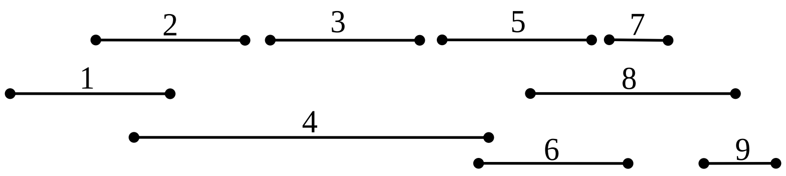

Fig. 1: Interval family I

The corresponding interval graph G is as follows,

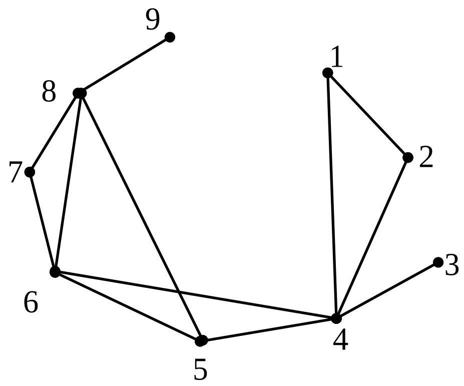

Fig. 2 : Interval graph G

Dominating set D = {4, 8}

Total dominating set TDS = {4, 5, 8} and γ_t(G)=3.

Remove the edge e = (3, 4) from G, then the corresponding interval graph G-e is as follows,

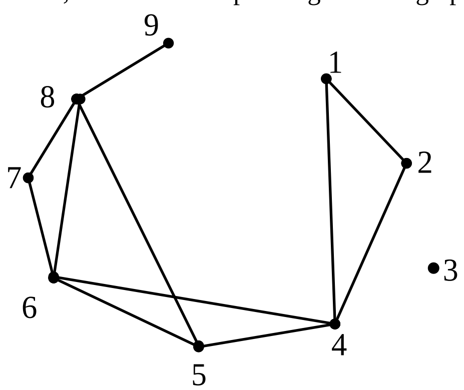

Fig. 3: Interval graph G-e

Total dominating set of G-e = TDS_1= {3, 4, 5, 8} and $\gamma_t(G-e) = 4$.

Therefore $\gamma_t(G-e) > \gamma_t(G)$ and hence b_t(G)=1.

Hence the proof of the theorem.

THEOREM 2.3: Let G be an interval graph corresponding to an interval family I = {i_1, i_2, ……, i_n}. Let the total dominating set TDS of G consists of four vertices only, say p, q, r and s. Suppose p and q vertices totally dominates the vertex set S_1 = {1,2,…..,i} and r and s vertices totally dominates the vertex set S_2 = { i+1,…..,n}.Suppose there is no vertex in S_1 other than the vertices p and q that totally dominates S_1 and no vertex in S_2 other than the vertices r and s that dominates S_2. Then b_t(G)=1.

PROOF: Let G be an interval graph corresponding to an interval family I = {i_1, i_2, ……, i_n} and the total dominating set TDS of G consists of four vertices only, say p, q, r and s.

Suppose p and q vertices totally dominates the vertex set S_1 = {1,2,……,i} and r and s vertices totally dominates the vertex set S_2 = { i+1,…..,n} and also consider that there is no vertex in S_1 other than the vertices p and q that totally dominates S_1 and no vertex in S_2 other than the vertices r and s that dominates S_2.

Since p and q are the only two vertices that are totally dominates S_1, there is no vertex in $S_3 = \{1,2,\ldots\ldots,i\}\backslash\{p, q\}$ that can dominate S_1. Let (p, q) be the edge in the graph G. Then in the graph G-e, p and q vertices totally dominates every vertex in S_1 except the two vertices p and q. Now consider a vertex x in S_1 which is adjacent to both p and q. Then clearly the set {p, q,} totally dominates the set S_1 in G-e. If there is no vertex x which is adjacent to both p and q then the graph G becomes disconnected, as the vertex x is isolated, which is a contradiction.

Let us assume that any two vertices {y, z}, y≠p and z≠q or y≠q and z≠p totally dominates the vertex set S_1 in G-e, this implies that y and z both are totally dominates the vertex set S_1 in G, a contradiction, because by the hypothesis p and q are the only two vertices which are totally dominates S_1 in G. Hence we can easily say that two vertices i.e., y and z cannot dominate S_1 in G-e.

Thus $TDS_1 = TDS \cup \{x\}$ becomes a dominating set of G-e. Since TDS is a minimum total dominating set in G, TDS_1 is also minimum in G-e, so that $\gamma_t(G-e) > \gamma_t(G)$. Hence $b_t(G)=1$.

2.4: ILLUSTRATION

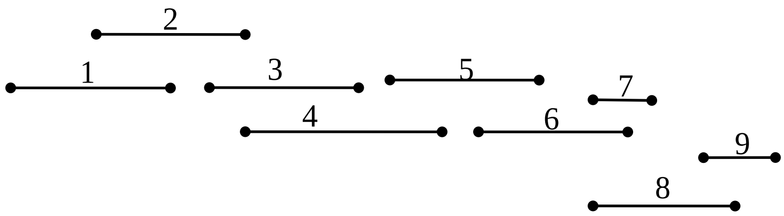

Fig. 4: Interval family I

The corresponding interval graph G is as follows,

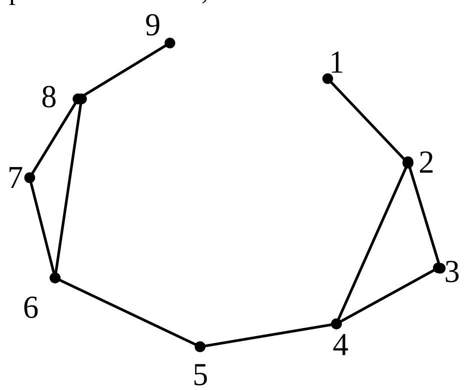

Fig. 5: Interval graph G

$S_1 = \{1,2,3,4,5\}$
$S_2 = \{6,7,8,9\}$
P = 2, q = 4, r = 8, s = 9
Total dominating set TDS = {2,4,8,9} and $\gamma_t(G) = 4$.

Remove the edge e = (2, 4) from G,

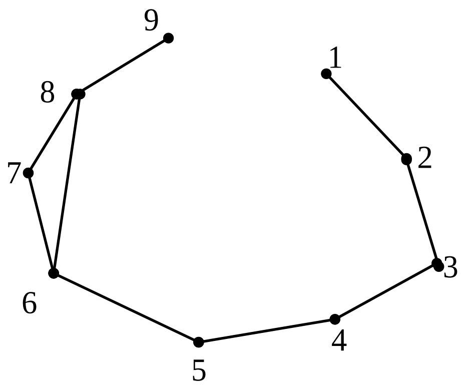

Fig. 6: Interval graph G-e

Total dominating set of G-e = TDS_1 = {2,3,4,8,9} and $\gamma_t(G-e) = 5$.

There fore $\gamma_t(G-e) > \gamma_t(G)$ and hence $b_t(G) = 1$.

THEOREM 2.5: Let G be an interval graph corresponding to an interval family I = {i_1, i_2, ……, i_n}. Let the total dominating set TDS of G consists of four vertices only, say p, q, r and s. Suppose p and q vertices totally dominates the vertex set $S_1 = \{1,2,\ldots,i\}$ and r and s vertices totally dominates the vertex set $S_2 = \{ i+1,\ldots,n\}$. Suppose there are two more vertices $\{u, v\} \subset S_1$ or S_2 respectively. Then $b_t(G)=1$.

PROOF: Let G be an interval graph corresponding to an interval family $I = \{i_1, i_2, \ldots\ldots, i_n\}$. Let the total dominating set TDS of G consists of four vertices only say p, q, r and s.Suppose there are two more vertices u, v $\subset S_1$ or S_2. The total dominating set TDS={p, q, r, s} and the vertices p and q totally dominates the set S_1 and the two vertices r and s totally dominates the vertex set S_2.

Let u, v $\subset$ S_1 such that u and v also totally dominates the vertex set S_1. Let $e_1 = (u, q)$, $e_2 = (v, q)$ and (p, v). Consider the graph G - $\{e_1, e_2, e_3\}$. In this graph the vertices u and q; v and q; p and v are not adjacent. Hence p and q cannot totally dominate the set S_1 in G - $\{e_1, e_2, e_3\}$. We require atleast three vertices in S_1 in G – $\{e_1, e_2, e_3\}$.

Therefore the total dominating set of G – e contains more than four vertices. Thus $\gamma_t(G-e) > \gamma_t(G).$ Hence $b_t(G) =3$.

Hence the proof of the theorem.

2.6: ILLUSTRATION

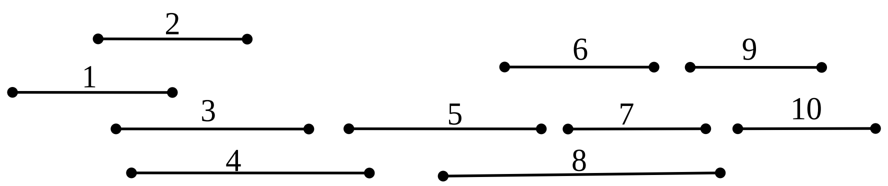

Fig. 7: Interval family I

The corresponding interval graph G is as follows,

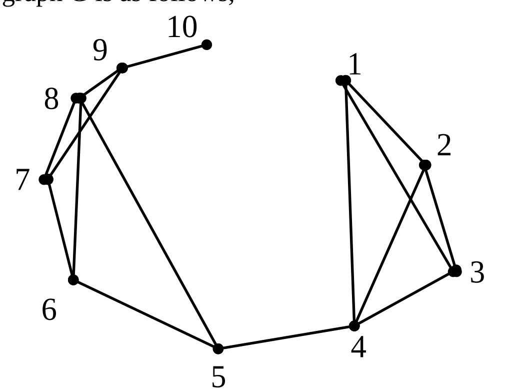

Fig.8: Interval graph G

$S_1 = \{1, 2, 3, 4\}$, $S_2 = \{5, 6, 7, 8, 9,10\}$, p = 2, q = 4, r = 8, s = 9, u = 1, v = 3.

Total dominating set TDS = {2, 4, 8, 9} and $\gamma_t(G) = 4.$

Remove the edges $e_1=(1, 4)$, $e_2=(3, 4)$, $e_3=(2, 3)$ from G then the corresponding interval graph G-e is as follows,

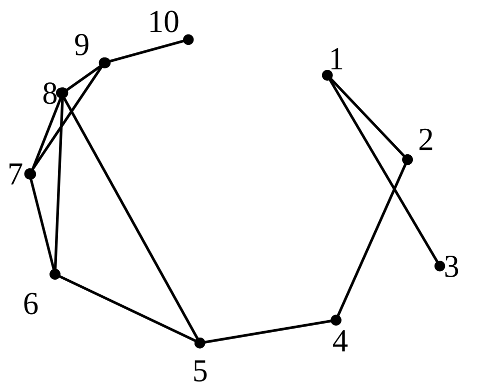

Fig.9: Interval graph G-e

Total dominating set of G-e= TDS_1={2,3,4,8,9} and $\gamma_t(G-e) = 5.$

Therefore $\gamma_t(G-e) > \gamma_t(G)$ and hence $b_t(G)=1$.

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