



## Performance Evaluation of Direction of Arrival Estimation Using MUSIC and ESPRIT Algorithms for Mobile Communication Systems

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**Abstract**— Source direction of arrival (DOA) estimation is one of the challenging problems in many applications. Such applications can be seen in wireless communications, radar, radio astronomy, sonar and navigation. For example, in commercial applications it is necessary to identify the direction of an emergency cell phone call in order to dispatch a rescue team to the proper location. The resolution of a source direction of arrival estimation can be enhanced by an array antenna system with innovative signal processing. Super resolution techniques take the advantage of array antenna structures to better process the incoming waves. These techniques also have the capability to identify the direction of multiple targets. This report investigates performance of the DOA estimation algorithms namely; MUSIC and ESPRIT on the uniform linear array (ULA) in the presence of white noise [3]. The performance of these DOA algorithms for a set of input parameters such as number of snapshots, number of array elements, signal-to-noise ratio are investigated. The simulation results showed that the resolution of the DOA techniques MUSIC and ESPRIT improves as number of snapshots; number of array elements, signal-to-noise ratio and separation angle between the two sources are increased. We will study the methods of MUSIC (Multiple Classification Signal) and ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques). MUSIC-based and ESPRIT-based algorithms, when used on uniform linear array with Omni-directional antenna elements, always have high performance. The accurate estimation of direction which is also known as DOA of the incident signals is very significant to produce beam form antenna. The DOA estimation techniques with array antennas are applied in wide areas of research fields and have received considerable attention in literature.

**Keywords**— MUSIC, ESPRIT, DOA, Communication Systems

### I. INTRODUCTION

Smart antennas have emerged as one of the most promising directions in supporting maximum communication link throughput. We have investigated the impact of smart antennas on a complex mobile network such as a railroad wireless communications system. The objective is to analyze the selection of a Direction-Of-Arrival (DOA) estimation algorithm which provides the maximum efficiency when deployed in our railroad test beds for wireless vehicular communication. Our findings are discussed to provide an in-depth understanding of how different algorithms should be selected to support efficient network operations. In any communication system, it is desirable to deliver maximum throughput. In achieving this objective, many hardware, physical layer and cross layer design techniques have been studied in the literature. Among these approaches, smart antennas are one of the most promising directions. Smart antennas have two primary aspects: position estimation and beamforming. An array of antenna elements is employed to receive multiple versions of the same signal from a distant source, at slightly different locations. The signals are then processed to indicate the position of the source. In a radial coordinate system, the azimuthal and elevation angles represent that position. Then, the array steers the principal lobe of the beam pattern towards the estimated Direction-of-Arrival (DOA). This way, the maximum power emitted by the antenna is directed towards the desired source, resulting in a very high effective antenna gain in that direction. In some papers, the authors show that the throughput can be improved by using smart antennas in a network. However, when designing a communication infrastructure for complicated networks and test beds, the mathematical derivation and analysis does not provide all the details. Therefore, use of test bed simulation tools to predict smart antenna performance is needed. Work includes theoretical analysis as well as simulation studies using MATLAB.

The initial step and one of the key aspects in integrating smart antenna systems into our previous works is the selection of an appropriate Direction-of-Arrival algorithm. Many research works have investigated different designs of a DOA estimation algorithm. The choice of an algorithm depends on the specifications and requirements of the project itself. In our study, we can be flexible for antenna parameters like number of antenna elements and number of source samples used for estimation. The gain produced by the antenna system depends not only on the property of the antenna but also on its location within the environment. Thus the benefit of using directional smart antennas in terms of gain is better evaluated by computer simulation or field testing. The objectives are :

- To develop a program in MATLAB for the estimation of direction of arrival of signals impinge on the antenna array by using MUSIC and ESPRIT algorithm.

- To evaluate and compare the performance of MUSIC and ESPRIT in variation of samples and elements used in the antenna array.

This paper introduces the field of Direction Of Arrival Algorithms such as MUSIC and ESPRIT and is divided into four main sections. In the first section, the reader is introduced to DOA, its definitions. The second section deals with the various direction of arrival algorithms including MUSIC and ESPRIT. And last section deals with the MUSIC and ESPRIT algorithm with the simulation results for different parameters.

## II. DIRECTION OF ARRIVAL (DOA)

In signal processing literature, direction of arrival denotes the direction from which usually a propagating wave arrives at a point, where usually a set of sensors are located. This set of sensors forms what is called a sensor array. Often there is the associated technique of beamforming which is estimating the signal from a given direction. Various engineering problems addressed in the associated literature are:

- Find the direction relative to the array where the underwater sound source is located.
- Direction of different sound sources around you are also located by you using a process similar to those used by the algorithms in the literature.
- Radio telescopes use these techniques to look at a certain location in the sky.
- Recently beamforming has also been used in RF applications such as wireless communication. Compared with the spatial diversity techniques, beamforming is preferred in terms of complexity.

On the other hand beamforming in general has much lower data rates. In multiple access channels (CDMA, FDMA, TDMA), beamforming is necessary and sufficient. Various techniques for calculating the direction of arrival, such as Angle of Arrival (AoA), Time Difference of Arrival (TDOA), Frequency Difference of Arrival (FDOA), or other similar associated techniques.

*Beamforming :*

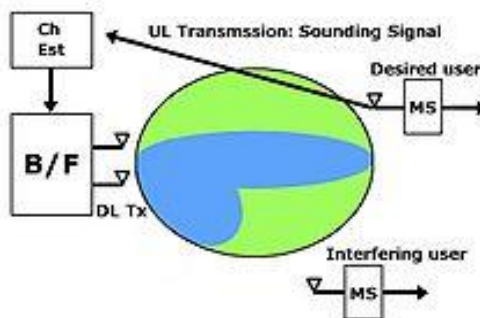


Fig 1 : Beamforming

Beamforming or spatial filtering is a signal processing technique used in sensor arrays for directional signal transmission or reception. This is achieved by combining elements in a phased array in such a way that signals at particular angles experience constructive interference while others experience destructive interference. Beamforming can be used at both the transmitting and receiving ends in order to achieve spatial selectivity. The improvement compared with omnidirectional reception/transmission is known as the receive/transmit gain (or loss). Beamforming can be used for radio or sound waves. It has found numerous applications in radar, sonar, seismology, wireless communications, radio astronomy, acoustics, and biomedicine. Adaptive beamforming is used to detect and estimate the signal-of-interest at the output of a sensor array by means of optimal (e.g., least-squares) spatial filtering and interference rejection. The DOA algorithm must satisfy the following conditions:

- High accuracy particularly at low SNRs
- Low computational intensity
- Low memory
- Efficient for a 2 microphone array with 4cm separation (set by beamforming requirements) [11].

## III. VARIOUS DOA METHODS

The high demand on the usage of the wireless communication system calls for higher system capacities. The system capacity can be improved either enlarging its frequency bandwidth or allocating new portion of frequency spectrum to wireless services. But since the electromagnetic spectrum is a limited resource, it is not easy to get new spectrum allocation without the international coordination on the global level. One of the approaches is to use existing spectrum more efficiently, which is a challenging task. Efficient source and channel coding as well as reduction in transmission power or transmission bandwidth or both are possible solutions to the challenging issue. This chapter provides a detailed overview of the various methods available for estimation of Direction-Of-Arrival (also called Angle-Of-Arrival) of a radio signal using an antenna array. Promising methods for determining the position of a mobile user are also given. The use of adaptive arrays is critical in position location applications; there is intense interest in determining the Direction-Of-Arrival of RF signals in wireless systems. Direction of arrival(DOA) estimation algorithms are used to improve the performance of an antenna by controlling the directivity of the antenna to reduce effects like interference, delay spread, and multi path fading.

The technique used for estimating directions of arrival of signals using an antenna array has been study of interest in recent years. Direction of arrival methods has been widely studied in the literature and can be grouped into two categories. In the first category, they are called as classical methods which provides a representation of the sources field (power and angular positions of the sources) by projecting the model vector (directional vector) on the space of observation without considering the determination of the number of sources. However, these conventional methods do not get a good resolution. The second category is known as "parametric" or high-resolution method which requires prior knowledge of the number of uncorrelated sources before estimating their characteristics (angular position, power). The estimation problem is first solved by estimation methods of the number of sources. Then a high resolution method is applied to estimate the angular position of these sources. These high-resolution methods are known to be more robust than conventional techniques.

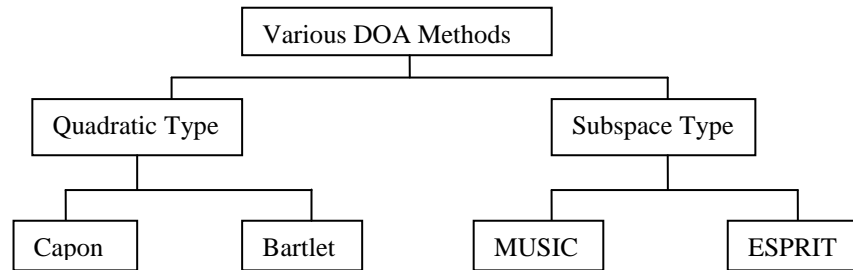


Fig 2: Various DOA Methods

The DOA algorithms are classified as :

*Quadratic Type :*

The Bartlett and Capon (Minimum Variance Distortionless Response) are quadratic type algorithms. The both methods are highly dependent on physical size of array aperture, which results in poor resolution and accuracy.

*Subspace Type :*

Subspace based DOA estimation method is based on the eigen decomposition. The subspace based DOA estimation algorithms MUSIC and ESPRIT provide high resolution; they are more accurate and not limited to physical size of array aperture. Capon and MUSIC algorithm performances is analyzed based on number of snapshots, number of users, user space distribution, number of array elements, and signal to noise ratio. In the design of adaptive array smart antenna for mobile communication the performance of DOA estimation algorithm depends on many parameters such as number of mobile users and their space distribution, the number of array elements and their spacing, the number of signal samples and SNR.

*Capon Method:*

Capon, in 1969, used a maximum likelihood (ML) method to solve for a minimum variance distortion response (MVDR) of an array. It finds the maximum likelihood estimate of the power arriving from a point source in direction  $\theta$  assuming that all other sources are interference i.e. maximization of signal to interference ratio. The expression for the power spectrum is given by [9]

$$P_c(\theta) = \frac{1}{\mathbf{a}^H(\theta) \mathbf{R}_{xx}^{-1} \mathbf{a}(\theta)}$$

*Bartlett Method :*

It is the one of the earliest methods of spatial analysis. In this method a rectangular window of uniform weighting is applied to the time series data to be analyzed. For bearing estimation problems using an array, this is equivalent to applying equal weighting on each element. Bartlett method is also called Ordinary Beamforming Method (OBM). This method estimates the mean power  $P_B(\theta)$  by steering the array in  $\theta$  direction. The power spectrum in bartlett method is given by

$$P_B(\theta) = \frac{\mathbf{S}_\theta^H \mathbf{R} \mathbf{S}_\theta}{L^2}$$

Where,

‘ $\mathbf{S}_\theta$ ’ denotes the steering vector associated with the direction  $\theta$ ,

‘ $\mathbf{R}$ ’ is the array correlation matrix.

‘ $L$ ’ denotes the number of elements in the array

In DOA estimation, a set of steering vectors  $\{ \mathbf{S}_\theta \}$  associated with various direction  $\theta$  is often referred to as the array manifold. In practice, it may be measured at the time of array calibration. From the array manifold and an estimate of the array correlation matrix,  $P_B(\theta)$  is computed. Peaks in  $P_B(\theta)$  are then taken as the directions of the radiating sources [2]. The array-based Direction-Of-Arrival (DOA) estimation techniques considered here are broadly divided into four different types:

- conventional techniques,
- subspace based techniques,
- maximum likelihood techniques and
- the integrated techniques which combine property restoral techniques with subspace based approaches.

**Conventional Methods :** These are based on classical beamforming techniques and require a large number of elements to achieve high resolution.

**Subspace Based Methods :** These are high resolution sub-optimal techniques which exploit the eigen structure of the input data matrix.

**Maximum Likelihood Techniques :** These are optimal techniques which perform well even under low signal-to-noise ratio conditions, but are often computationally very intensive.

**Integrated Techniques :** A promising method for CDMA is the integrated approach which uses property-restoral based techniques to separate multiple signals and estimate their spatial signatures from which their Directions-Of-Arrival can be determined using subspace techniques.

**Conventional Methods for DOA Estimation :** Conventional methods for Direction-Of-Arrival estimation are based on the concepts of beamforming and null-steering and do not exploit the nature of the model of the received signal vector  $u(k)$  or the statistical model of the signals and noise. Given the knowledge of the array manifold, an array can be steered electronically. Conventional DOA estimation techniques electronically steer beams in all possible directions and look for peaks in the output power. The conventional methods discussed here are the delay-and-sum method (classical beamformer) and Capon's minimum variance method.

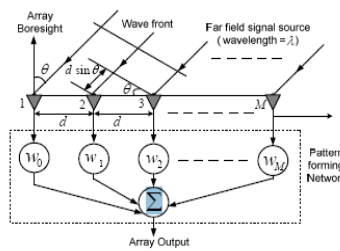
**Delay-and-Sum Method :** The delay-and-sum method, also referred to as the classical beamformer method or Fourier method, is one of the simplest techniques for DOA estimation.

**Subspace Methods for DOA Estimation :** Though many of the classical beamforming based methods such as Capon's minimum variance method are often successful and are widely used, these methods have some fundamental limitations in resolution. Most of these limitations arise because they do not exploit the structure of the input data model. Schmidt and Bienvenu and Kopp were the first to exploit the structure of a more accurate data model for the case of sensor arrays of arbitrary form. Schmidt derived a complete geometric solution to the DOA estimation problem in the absence of noise, and extended the geometric concepts to obtain a reasonable approximation to the solution in the presence of noise. The technique proposed by Schmidt is called the Multiple Signal Classification (MUSIC) algorithm, and it has been thoroughly investigated since its inception. The geometric concepts upon which MUSIC is founded form the basis for a much broader class of subspace-based algorithms.

Apart from MUSIC, the primary contributions to the subspace-based algorithms include the Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) proposed by Roy et al. and the minimum-norm method proposed by Kumaresan and Tufts. 1979 is a high resolution multiple signal classification technique based on exploiting the eigenstructure of the input covariance matrix. MUSIC is a signal parameter estimation algorithm which provides information about the number of incident signals, Direction-Of-Arrival (DOA) of each signal, strengths and cross correlations between incident signals, noise power, etc. While the MUSIC algorithm provides very high resolution, it requires very precise and accurate array calibration. The MUSIC algorithm has been implemented and its performance has been experimentally verified.

**IV. SIGNAL MODEL**

The following signal model is applicable for both MUSIC and ESPRIT algorithms. The data model assumes that the signal impinging upon an array of sensors to be narrow-band and emitted from a point source in the far field. Consider a number of plane waves from  $M$  narrow-band sources impinging from different angles  $\theta_i, i = 1, 2, \dots, M$ , impinging into a uniform linear array (ULA) of  $N$  equi-spaced sensors, as shown in Fig. 3.1.1. At a particular instant of time  $t, t=1, 2, \dots, K$ , where  $K$  is the total number of snapshots taken, the array output will consist of the signal plus noise components.



**Fig 3 : A Plane Wave Incident On A Uniform Linear Array Of N-Equi-Spaced Sensors**

The signal vector  $x(t)$  can be defined as:

$$x(t) = \sum_{m=1}^M a(\theta_m) \cdot S_m(t)$$

where  $s(t)$  is an  $M \times 1$  vector of source waveforms, and for a particular source at direction  $\theta$  from the array boresight;  $a(\theta)$  is an  $N \times 1$  vector referred to as the array response to that source or array steering vector for that direction. It is given by:

$$a(\theta) = [1 \quad e^{-j\phi} \quad \dots \quad e^{-j(N-1)\phi}]^T$$

where  $T$  is the transposition operator, and  $\phi$  represents the electrical phase shift from element to element along the array. This can be defined by:  $\phi = (2\pi / \lambda) d \cos \theta$  where  $d$  is the element spacing and  $\lambda$  is the wavelength of the received signal. The signal vector  $x(t)$  of size  $N \times 1$  can be written as:  $x(t) = A(\theta) \cdot S(t)$  where  $A(\theta) = [a(\theta_1) \dots a(\theta_M)]$  is an  $N \times M$  matrix of steering vectors and  $S(t) = [S(t_1) \dots S(t_M)]$  is an  $N \times M$  matrix of source vector. The model described in

above equation can never explain the observed data; this is may be due to noise and modeling errors. Therefore to account for these effects, an additive noise term  $w(t)$  is included. Hence the array output consists of the signal plus noise components, and it can be defined as:  $x(t) = A(\theta) \cdot S(t) + w(t)$  where  $x(t)$  and  $w(t)$  are assumed to be uncorrelated and  $w(t)$  is modeled as temporally white and zero-mean complex Gaussian process. Above equation can be written in matrix form of size  $N \times K$  as:  $X = A \cdot S + W$  where  $S = [s(1) \dots s(K)]$  is an  $M \times K$  matrix of source waveforms and  $W = [w(1) \dots w(K)]$  is an  $N \times K$  matrix of sensor noise. The spatial correlation matrix  $R$  of the observed signal vector  $x(t)$  can be defined as:

$$R_{xx} = E[x(t) \cdot x(t)^H]$$

$$R_{xx} = E[A(\theta) \cdot S(t) \cdot S(t)^H \cdot A^H(\theta)] + E[w(t) \cdot w(t)^H]$$

$$R_{xx} = A(\theta) \cdot R_{ss} \cdot A^H(\theta) + R_{ww}$$

$$= A(\theta) \cdot R_{ss} \cdot A^H(\theta) + \sigma_n^2 I$$

where  $\sigma_n^2$  is the variance of noise and  $I$  is the identity matrix. For this signal model, the correlation matrix  $R_{xx}$  will have  $M$  signal eigenvalues, and  $N-M$  noise eigenvalues. Let  $E_s$  be the matrix constructed of the corresponding  $M$  signal eigenvectors  $E_s = [e_1 \ e_2 \ \dots \ e_M]$ , and  $E_n$  be the matrix containing the remaining  $N-M$  noise eigenvectors  $E_n = [e_{M+1} \ e_{M+2} \ \dots \ e_N]$ . In real array measurements, the covariance matrices are unknown and they can be estimated from a finite amount of measurement called snapshots. Therefore the natural estimate of the correlation matrix or the sample covariance matrix is given by

$$\hat{R}_{xx} = \sum_{k=1}^K X(k) \cdot X(k)^H$$

Or

$$\hat{R}_{xx} = \frac{1}{K} \sum_{k=0}^{K-1} X_k \cdot X_k^H$$

where  $K$  is the number of samples or observation vectors, and  $X$  is the  $K \times M$  complex envelopes matrix of  $M$  measured signals [3].

#### V. MUSIC ALGORITHM AND ITS SIMULATION RESULTS

MUSIC is an acronym which stands for Multiple Signal classification. It is high resolution subspace DOA technique which gives the estimation of number of signals arrived, hence their direction of arrival. The algorithm is based on exploiting the eigenstructure of input covariance matrix. The incident signals are somewhat correlated creating non diagonal signal correlation matrix. The algorithm is used to describe experimental and theoretical techniques involved in determining the parameters of multiple wave fronts arriving at an antenna array from measurements made on the signal received at the array elements. MUSIC deals with the decomposition of covariance matrix into two orthogonal matrices, i.e., signal-subspace and noise-subspace. Estimation of DOA is performed from one of these subspaces, assuming that noise in each channel is highly uncorrelated. This makes the covariance matrix diagonal. The steps of the algorithm are summarized as follows:

Step 1: Collect input samples  $X$ ,  $k = 0 \dots N - 1$  and estimate the input covariance matrix

$$\hat{R}_{xx} = \frac{1}{K} \sum_{k=0}^{K-1} X_k \cdot X_k^H$$

Step 2: Perform eigen decomposition on  $\hat{R}_{xx}$

$$\hat{R}_{xx} E = E \Lambda$$

where  $\Lambda = \text{diag}\{\lambda_0, \lambda_1, \dots, \lambda_{M-1}\}$  are the eigen values and  $E = \text{diag}\{q_0, q_1, \dots, q_{M-1}\}$  are the corresponding eigenvectors of  $\hat{R}_{xx}$ .

Step 3: Estimate the number of signals  $\hat{L}$  from the multiplicity  $K$ , of the smallest eigenvalue  $\min \lambda$  as equation  $\hat{L} = M - K$ .

Step 4: Compute the MUSIC spectrum by the following Eq.

$$\hat{P}_{MUSIC}(\theta) = \frac{A^H(\theta) \cdot A(\theta)}{A^H(\theta) E_n E_n^H A(\theta)}$$

Step 5: Find the  $\hat{L}$  largest peaks of  $\hat{P}_{MUSIC}(\theta)$  to obtain estimates of the Direction -Of- Arrival [3].

#### Simulation Results :

The MUSIC techniques for DOA estimations are simulated using MATLAB tool. The performance of the algorithms has been analyzed by considering the effect of changing a number of parameters related to the signal environment as well as the antenna array. A uniform linear array with  $M$  elements has been considered in our simulation experiments and all inputs were made fixed when the effects of changing a parameter value was investigated. In these simulations, it is considered a linear array antenna formed by 10 elements that are evenly spaced with the distance of  $\lambda / 2$ . The noise is considered to be additive, having the 0.1 variance value. The simulation has been run for two signals coming from different angles  $\theta_1 = -25^\circ$  and  $\theta_2 = 35^\circ$  for different value of snapshots, SNR, and array elements. These two signals are considered to have equal amplitudes. The simulation results for MUSIC algorithm for different parameters are explain below:

Case.1: MUSIC Spectrum For Varying The Values Of SNR : The effect of changing the SNR with three different values (5, 15, and 25) dB is shown in Fig.4 It is clear that as SNR value increases, the ratio between the output peaks and the noise level at the output of the array increases proportionally.

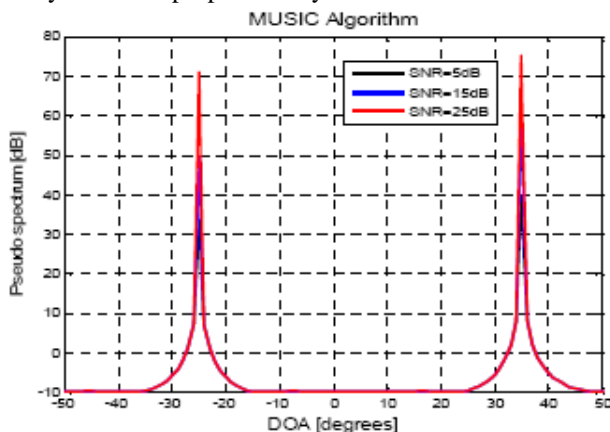


Fig 4 : The Effect Of Varying The SNR On MUSIC Algorithm

Case.2: MUSIC Spectrum For Varying The Horizontal Angle Separation : Figure 5 shows the effect of varying horizontal angle separation on MUSIC algorithm. Sharper peaks increases as the angle separation between signals increases, since MUSIC is a high resolution technique; it is capable to resolve the signal from two users.

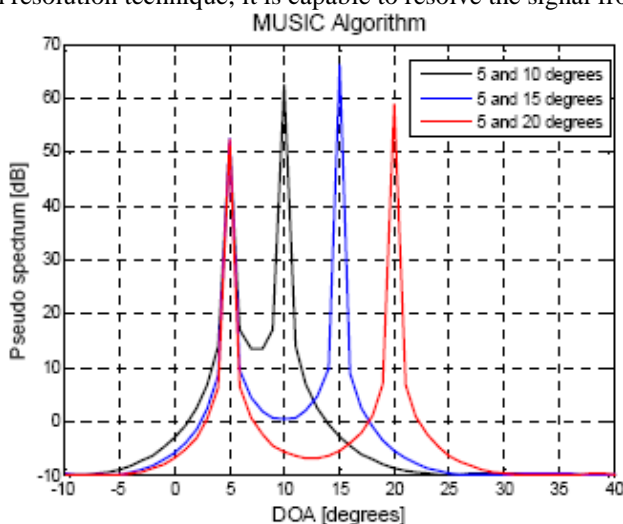


Fig 5 : Effect Of Varying Number Of Elements On ESPRIT Algorithm

Case.3: MUSIC Spectrum For Varying Number Of Snapshots : Figure 6 shows MUSIC spectrum as a function of the number of snapshots with  $K=100$ ,  $K=500$  and  $K=1000$  and keeping all parameter constant. We can notice that by the increasing the number of snapshots peaks in the MUSIC spectrum become further sharper for higher number of snapshots e.g. with  $K=1000$ .

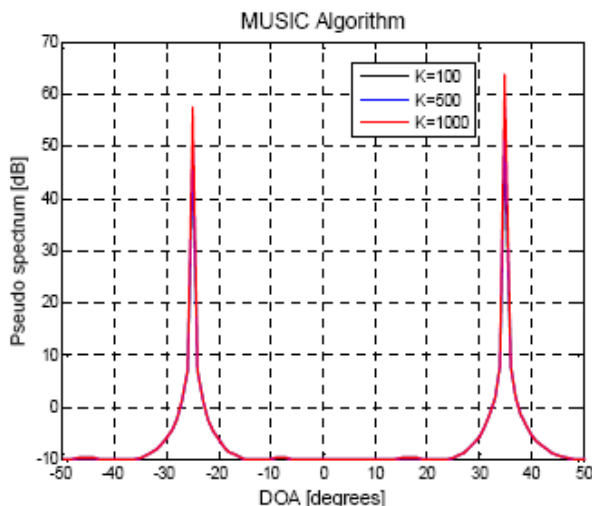


Fig 6: The Effect Of Varying The Number Of Snapshots On MUSIC Algorithm



Case.4: MUSIC Spectrum For Varying The Number Of Array Sensor : The effect of increasing the number of array sensor on the performance of the MUSIC algorithm can be shown in Fig.7. Small reduction in beamwidths and the noise level is observed.

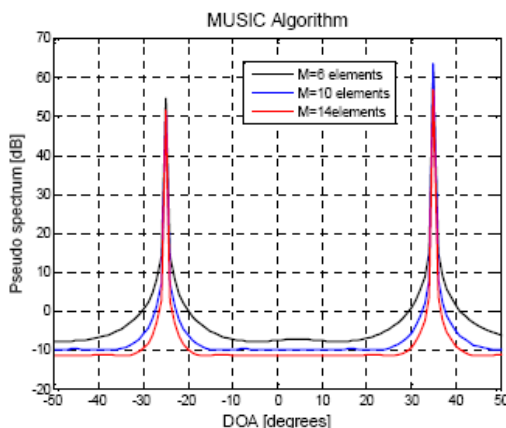


Fig 7: Effect Of Varying Number Of Sensors On MUSIC Algorithm

Case 5: Element Spacing Of The Sensor Array: The spacing between the elements of the sensor array must be increased resulting in a better resolution of the estimated peaks, as shown in Fig.8 for which  $d=0.5\lambda$ . Small peaks appeared in the response at angles of -500 and 650 in case of  $d=0.75\lambda$  this is due to grating lobes [3].

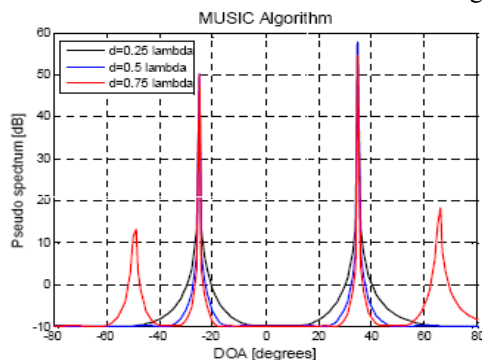


Fig 8: Effect Of Element Spacing d On MUSIC Algorithm

### VI. ESPRIT ALGORITHM AND ITS SIMULATION RESULTS

ESPRIT stands for Estimation of Signal Parameters via Rotational Invariance Techniques which is another subspace based DOA estimation algorithm. It does not involve an exhaustive search through all possible steering vectors to estimate DOA and dramatically reduces the computational and storage requirements compared to MUSIC. The goal of the ESPRIT technique is to exploit the rotational invariance in the signal subspace which is created by two arrays with a translational invariance structure. ESPRIT assumes that there are  $D < M$  narrowband sources centered at the centre frequency  $0 f$  as shown in Fig.9.

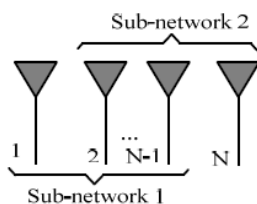


Fig 9 : Principle Of ESPRIT Algorithm

ESPRIT further assumes multiple identical arrays called doublets and these arrays are displaced translationally but not rotationally. The steps of the ESPRIT algorithm are summarized as follows:

- Step 1: Obtain an estimate  $\hat{R}_{xx}$  of  $R_{xx}$  from the measurements  $X$ .
- Step 2: Perform eigen decomposition on  $\hat{R}_{xx}$ .

$$\hat{R}_{xx} = V\Lambda V^H$$

where  $\Lambda = \text{diag}\{\lambda_0, \lambda_1, \dots, \lambda_{M-1}\}$  are the eigen values and  $E = \text{diag}\{q_0, q_1, \dots, q_{M-1}\}$  are the corresponding eigenvectors of  $\hat{R}_{xx}$ , respectively.

Step 3: Using the multiplicity,  $K$ , of the smallest eigen-value  $\lambda_{\min}$ , estimate the number of signals  $L^*$ , as  $L^* = M - K$ .

Step 4: Obtain the signal subspace estimate  $\hat{V}_s = [V^0, \dots, V^{L-1}]$  and decompose it into sub-array matrices,

$$\hat{V}_s = \begin{bmatrix} \hat{V}_0 \\ \hat{V}_1 \end{bmatrix}$$

Step5: Compute the eigen decomposition ( $\lambda_1, \dots, \lambda_{2L}$ )

$$\hat{V}_{01}^H \hat{V}_{01} = \begin{bmatrix} \hat{V}_0^H \\ \hat{V}_1^H \end{bmatrix} \begin{bmatrix} \hat{V}_0 \\ \hat{V}_1 \end{bmatrix} = V \Lambda V^H$$

And partition v into  $L \times L$  sub- matrices.

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$$

Step 6: Calculate the eigen values of

$$\psi = -V_{12} V_{22}^{-1},$$

$$\hat{\Phi}_k = \text{eigen values of } (-V_{12} V_{22}^{-1}), k = 0, \dots, L-1$$

Above equation indicates that if we are able to estimate eigenvalue of which are diagonal elements of  $\hat{\Phi}_k$  we can estimate DOA.

Step7: Estimate the Angle-Of-Arrival as

$$\theta_k = \cos^{-1} \left[ \frac{\arg(\hat{\Phi}_k)}{\beta d} \right]$$

As seen from the above discussion, ESPRIT eliminates the search procedure inherent in most DOA estimation methods; ESPRIT produces the DOA estimates directly in terms of the eigen values [3].

*Simulation Results :*

The ESPRIT technique for DOA estimations is simulated using MATLAB tool. The performance of the algorithms has been analyzed by considering the effect of changing a number of parameters related to the signal environment as well as the antenna array. A uniform linear array with M elements has been considered in our simulation experiments and all inputs were made fixed when the effects of changing a parameter value was investigated. In these simulations, it is considered a linear array antenna formed by 10 elements that are evenly spaced with the distance of  $\lambda / 2$ . The noise is considered to be additive, having the 0.1 variance value. The simulation has been run for two signals coming from different angles  $\theta_1 = -25^\circ$  and  $\theta_2 = 35^\circ$  for different value of snapshots, SNR, and array elements. These two signals are considered to have equal amplitudes. The simulation results for ESPRIT algorithm for different parameters are explain below:

Case.1: The Effect Of Varying The SNR :The effect of varying the SNR, with higher values of SNR, the performance of ESPRIT algorithm will be better than lower values of SNR as shown in Fig 10.

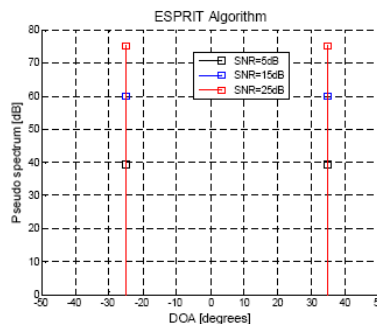


Fig 10: Effect Of Varying SNR On ESPRIT Algorithm

Case.2: The Effect Of Varying Number Of Elements: ESPRIT algorithm has been tested for different number of array elements. The spectrum of the algorithm signals as shown in Fig 11 provides almost similar DOA estimates for different array elements with lesser variance than MUSIC algorithm.

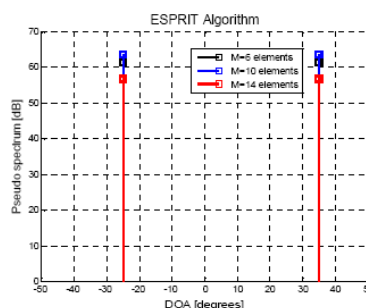


Fig 11: Effect Of Varying Number Of Elements On ESPRIT Algorithm



Case.3: The Effect Of Varying The Horizontal Angular Separation Between Users: ESPRIT algorithm has been tested for different angular separation. From Fig. 12 it is observed that for this algorithm DOA detection decreases as angular separation between arriving signals increases.

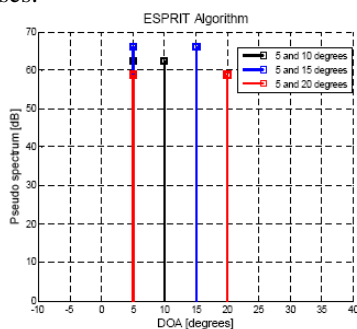


Fig 12: Effect Of Varying Horizontal Angular Separation Between Two Users On ESPRIT Algorithm, Users Located At 10,15, & 20 Degrees

Case.4: Effect Of Varying Number Of Snapshots: ESPRIT algorithm has been tested for different number of snapshots as shown in Fig. 13. It can be seen that the spectrum and DOA estimates for this algorithm are almost close to the actual DOA even at lower number of snapshots. Therefore, it outperforms MUSIC technique.

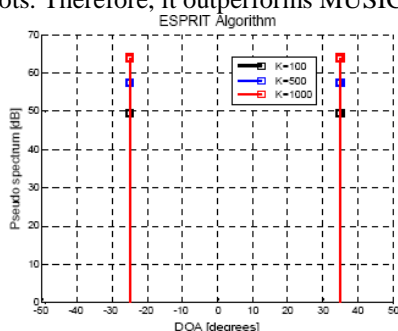


Fig 13: Effect Of Varying Number Of Snapshots On ESPRIT Algorithm

Case 5: Element Spacing Of The Sensor Array: Figure 14 shows the ESPRIT for an element spacing of  $d=0.25\lambda$ ,  $d=0.5\lambda$  and  $d=0.75\lambda$ , respectively. When the elements of the sensor array are placed too close to each other, mutual coupling effects dominate resulting in inaccuracies in the estimated angles of arrival, as shown in Fig.12 for which  $d=0.25\lambda$ . To overcome this problem, the spacing between the elements of the sensor array must be increased resulting in a better resolution of the estimated peaks, as shown in Fig. 12 for which  $d=0.5\lambda$  [3].

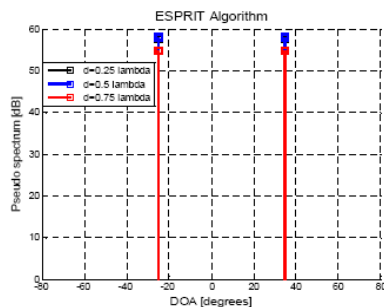


Fig 14: Effect Of Element Spacing d On ESPRIT

## VII. CONCLUSIONS

This presents results of direction of arrival estimation using MUSIC and ESPRIT algorithms. The two methods have greater resolution and accuracy than the other considered classical methods like Bartlett and Capon. Extensive computer simulations were performed to demonstrate the effect of various parameters on the performance of the MUSIC and ESPRIT algorithms and their ability to resolve incoming signals accurately and efficiently. From the simulated results, it is observed that both investigated algorithms provide an accurate estimation of the DOA with improved resolution power than other classical DOA techniques. The simulation results show that their performance improves with more elements in the array, with large snapshots of signals and greater angular separation between the signals. These improvements are seen in form of the sharper peaks and a smaller error in angle detection. The results obtained from these two algorithms add new possibility of user separation through SDMA and can be widely used in the design of smart antenna system. The results also improve and accelerate the design of wireless networks.

## REFERENCES

- [1] S. S. Balabadratrani, "Performance Evaluation Of Direction Of Arrival Estimation Using MATLAB".
- [2] S. Raga, S. Bhat, V. Katgi and S. Gowda, "Simulation Of Classical And Subspace Angle Of Arrival Algorithms For Mobile User Detection", World Journal of Science and Technology, pp. 115-119, 2012.

- [3] K. A. George, V. Tsoulos and Andy Nix, “ Performance Evaluation Of Direction-Of-Arrival Estimational Algorithm For Mobile Communication Systems”, IEEE, pp 1055-1059, 2003.
- [4] C. L. Srinidhi, Dr. S. A. Hariprasad, “Comparative Study on Performance Analysis of High Resolution Direction of Arrival Estimation Algorithms” , IJARCT, pp 67-79, Vol. 1, Issue 4, June 2012.
- [5] T. B. Lavate, V. K. Kokate & A. M. Sapkal, “Performance Analysis Of Music And Esprit DOA Estimation Algorithms For Adaptive Array Smart Antenna In Mobile Communication” , IJCN, pp 152-158, vol. 2, issue 3, 2012.
- [6] J. G. Arceo-olague, D. H. Covarrubias-rosales, J. M. Luna-rivera, A. Ángeles-Valencia, “Efficient Adaptive Algorithms For DOA Estimation In Wireless Communications” , IJCNS, pp. 173-176, Feb 2010.
- [7] P. L. Shrestha, M. Hempel, P. Mahasukhon, Tao Ma and H. Sharif, “ Performance Analysis for Direction of Arrival Estimating Algorithms”, IEEE 75th VTC Spring, VETECS.2012.6240151, 2012.
- [8] T.S. Dhope, “Application of MUSIC, ESPRIT and ROOT MUSIC in DOA Estimation”
- [9] Md. Bakhar, Vani. R. M. and P. V. Hunagund, “Comparative Studies Of Direction Of Arrival Algorithms For Smart Antenna Systems”, World journal of science and technology 2011, 1(8): 20-25
- [10] L. K. S. Daldorff, D. S. Turaga, O. Verscheure and A. Biem, “ Direction Of Arrival Estimation Using Single Tripole Radio Antenna” , IEEE, pp. 2149-2152, 2009.
- [11] [www.wikipedia.com](http://www.wikipedia.com)