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A Novel Method for Solving N-Queens Problem

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Abstract-In this paper, we discuss a new pattern observed while finding a unique solution to the famous N-Queens problem of placing N queens on an n X n chessboard such that none attacks the other. The basic idea for finding the solution for an n X n board is to apply the results of its preceding board, i.e., (n-1) X (n-1) board. The line of symmetry within a solution has also been explained and used to complete a solution from a half known solution.

Keywords- Polynomial time, N-queens, Combinatorial, exponential, heuristics

I. Introduction

The *N*-queens Problem, introduced in 1850 by Carl Gauss, is an extension of the original 8-queens problem. The problem requires us to find the placement of N queens on an NXN chessboard such that no queen is attacking the other. Only one queen must be positioned per row, per column and per diagonal. This problem belongs to the category of combinatorial problems, which require a lot of time and effort to be solved since there is no set formula for solving them. To find the most optimal solution, we need to consider every possible solution. There are methods such as heuristics to solve this problem. With increase in the size of the problem, the number of possibilities also increases exponentially. [1]

The N-Queens problem can have many distinct solutions for each value of N. In this paper, we present a pattern that is observed while finding the unique solutions for every value of N. Using this pattern, we can leverage the results of an NXN board to find a unique solution for an (N+1) x (N+1) board in polynomial time.

II. The Pattern

Solution for 4X4:

The N-Queens problem is has a solution for all N>3. Thus, the first solution is obtained when N=4, i.e., for a 4X4 board. There are two possible solutions for a 4-Queens problem, as shown in figure 1 and figure 2. We can see that one is a reflection of the other. Thus, we can build our pattern from figure 1 and extend it to figure 2 later.

	1	2	3	4
1			Q	
2	Q			
3				Q
4		Q		

Figure 1 : One solution for 4 Queens Problem
(Initial solution using which we can derive the solution for any value of N)

	1	2	3	4
1		Q		
2				Q
3	Q			
4			Q	

Figure 2 : Second solution for 4 Queens Problem (A Reflection of the previous Solution)

Solution for 5X5:

In the 4X4 solution board, add one row at the top and one column to the right to convert the board into a 5X5 one. The 5th queen must now be placed at the intersection of the newly added row and column, i.e., at position (1,5). This is one of the unique solutions for 5X5 board.

	1	2	3	4	5
1					Q
2			Q		
3	Q				
4				Q	
5		Q			

Figure 3 Solution for 5X5 problem

Solution for 6X6:

To the solution of the 5X5 board, add one new row after (6/2 = 3) rows and a new column after the rightmost column. The new board is now 6X6. Again, the 6th queen will be placed at the intersection of the newly added row and column, i.e., at position (4,6). This is a unique solution for the 6X6 board.

	1	2	3	4	5	6
1					Q	
2			Q			
3	Q					
4						Q
5				Q		
6		Q				

Figure 4 Solution for 6X6 problem

Solution for 7X7:

In the 6X6 solution board, add a new row at the top and a new column after the rightmost column to convert the board into a 7X7 one. The 7th queen must now be placed at the intersection of the newly added row and column, i.e., at position (1,7). This is a unique solution for the 7X7 board.

	1	2	3	4	5	6	7
1							Q
2					Q		
3			Q				
4	Q						
5						Q	
6				Q			
7		Q					

Figure 5 Solution for 7X7 problem

Observations:

In the discussed solutions, we observe two general rules for being followed by the solutions depending on the value of N. They are:

- Rule 1: We can see that for every odd value of N, we add a new row at the top and a new column after the rightmost column of the previous board's solution. The new queen is placed at the intersection of the newly added row and column, i.e., at position (1,N). Hence, the solution for NXN board is obtained using the solution of (N-1)X(N-1) board when N is odd.
- Rule 2: For even values of N, we add a new row after the $(N/2)^{th}$ row and a new column after the rightmost column of the previous board's solution. The new queen is placed at the intersection of the newly added row and column, i.e., at position ((N/2)+1,N). Hence, the solution for NXN board is obtained using the solution of (N-1)X(N-1) board when N is even.

Solution for 8X8:

The aforementioned two rules will work for all even and odd values of all N>3 except when N is even and also satisfies the following condition: ((N/2)-1)%3 = 0, i.e., when N=8, 14, 20,...etc.

According to the rule 2, the solution for 8X8 is shown in figure 6. But this solution is incorrect.

	1	2	3	4	5	6	7	8
1							Q	
2					Q			
3			Q					
4	Q							
5								Q
6						Q		
7				Q				
8		Q						

Figure 5 Incorrect Solution for 8X8 problem

In this case a small addition has to be made to the second rule. Once we have created the solution till figure 6, we must follow these additional steps to arrive at the correct solution:

1. Divide the board into two halves horizontally, i.e., upper half consisting of rows 1 to (N/2) and lower half consisting of rows from ((N/2)+1) to N.
2. In the upper half, shift all the queens upwards by one position, i.e., decrement their vertical coordinate (or row number) by 1. Don't change their horizontal coordinate (or column number). For the queen in the topmost row, shift it to the $(N/2)^{th}$ row. We can say that all the queens are rolling in the upward direction.
3. In the lower half, shift all the queens downwards by one position, i.e., increment their vertical coordinate (or row number) by 1. Don't change their horizontal coordinate (or column number). For the queen in the last row, shift it to the $((N/2)+1)^{th}$ row. We can say that all the queens are rolling in the downward direction.

Hence, the solution for 8X8 is obtained (figure 7).

	1	2	3	4	5	6	7	8
1					Q			
2			Q					
3	Q							
4							Q	

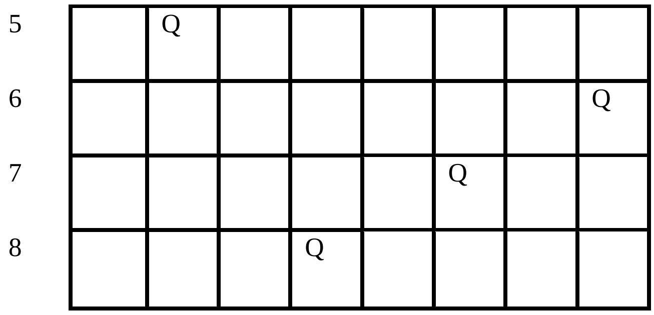

Figure 6 Correct Solution for 8X8 problem

III. Symmetry In the Solutions

Apart from this pattern, we also observed that for all even valued N, if we divide the board into two halves vertically, i.e., left half and right half, and we know the positions of all the queens in one half, we can complete the solution for the remaining queens by implementing the following steps:

- Take the reflection of the half whose solution is known (say left half) about the dividing vertical axis (figure 8).
- Divide this reflected half (the right half) into two halves horizontally, i.e., upper half and lower half.
- Take the reflection of this half (the right half) about the dividing horizontal axis (figure 9).
- These new reflections will be the final positions of the queens in the second half (figure 10).

Hence, the solution is completed from a half-known solution.

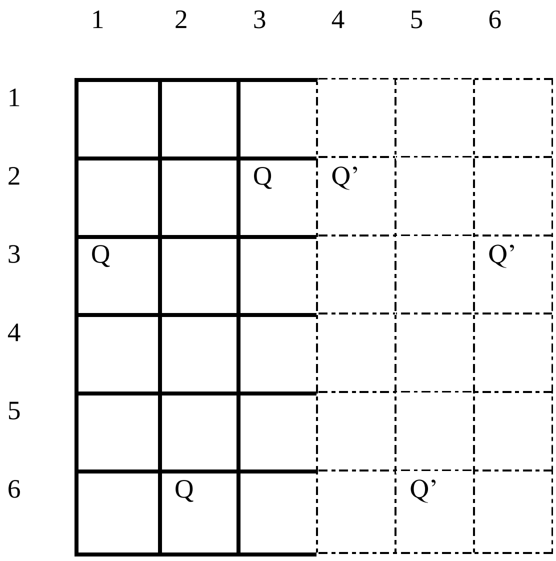

Figure 7 : Left half's solution is known. Its reflection (Q') is taken about the dividing vertical axis

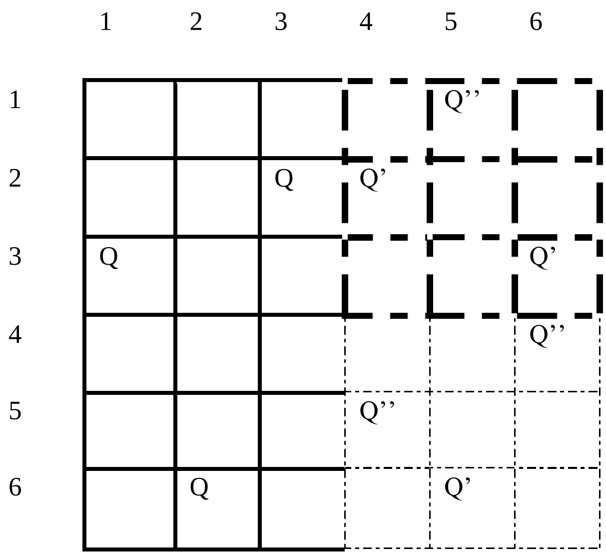

Figure 8 : Divide the right half into upper half and lower half. Take reflection (Q'') of Q' about the dividing horizontal axis

	1	2	3	4	5	6
1					Q	
2			Q			
3	Q					
4						Q
5				Q		
6		Q				

Figure 9 : Q" is the position of queens in the right half. Solution is complete for 6X6 board

IV. Conclusion

We have found a new pattern for finding a unique solution for the N-Queens problem for any value of N. This pattern is based on one initial solution for a 4X4 chessboard. For further research, we would like to find a similar pattern taking the second solution of a 4X4 board as the initial solution and building the unique solutions of succeeding boards on this second solution. We have also suggested a new way of finding the complete solution from an incomplete solution exploiting the symmetrical nature of the solutions when the value of N is even. We can extend this research into finding similar symmetrical patterns for odd valued N also.

References

[1] S. Pothumani, (2013), *Solving N Queen Problem Using Various Algorithms – A Survey*, “International Journal of Advanced Research in Computer Science and Software Engineering (IJARCSSE)”, Volume 3, Issue 2, February 2013, pp 247-250, ISSN: 2277 128X.

