



# Design of Butterworth and Chebyshev1 Lowpass Filter for Equalized Group Delay

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**Abstract**— This paper presents a direct design of Infinite Impulse Response filters (IIR filters) which minimizes group delay without changing the magnitude response of filters. In this paper Butterworth and Chebyshev1 lowpass filters are designed by using allpass filters. The design specifications are passband and stopband frequencies and passband ripple and stopband attenuation. In this paper MATLAB programming is used for implementation of proposed algorithm. Experimental results show that the proposed method can effectively optimize the group delay of the designed Butterworth and chebyshev1 low pass filters.

**Keywords**— Allpass filter, Butterworth filter, Chebyshev1 filter, Equalized Group delay.

## I. INTRODUCTION

Filter is a frequency selective circuit that allows a certain band of frequency to pass while attenuating the others frequencies. Filters are classified as analog and digital. The Digital Filtering is one of the most powerful tools of DSP. The digital filters consist of software and hardware. The input and output signals in the digital filter is digital or discrete time variant. The procedure for designing digital filters involves the determination of a set of filter coefficients to meet a set of design specifications. Digital filters come in two flavors: FIR and IIR. As the terminology suggest, these classifications refer to the filters impulse response. By varying the weight of the coefficients and number of filter taps, virtually any frequency response characteristics can be realized with an FIR filter. FIR filters have a very useful property: they can exhibit linear phase shift for all frequencies. IIR filters have infinite impulse response. IIR filters have much better frequency response than FIR filters of the same order. In IIR filters their phase characteristics is not linear, which can cause a problem to the systems which need phase linearity. In this paper, a direct design of digital Butterworth and Chebyshev1 filter based on allpass filter is proposed. The design parameters of proposed method are shown in fig1[1]

Where  $w_p$ - Passband frequency,  
 $w_s$ - Stopband frequency,  
 $A_p$ - Passband ripples,  
 $A_s$ - Stopband attenuation.

## II. BUTTERWORTH FILTER

The butterworth filter has a maximally flat response, i.e., no passband ripple and roll-off of minus 20db per pole. Another name for it is “flat maximally magnitude” filters at the frequency of  $\Omega = 0$ , as the first  $2N - 1$  derivatives of the transfer function when  $\Omega = 0$  are equal to zero. [4]. The Butterworth filters achieve its flatness at the expense of a relatively wide transition region from passband to stopband with average transient characteristics. This filter is completely defined mathematically by two parameters i.e. cut off frequency and number of poles. Compared to chebyshev filter, the phase linearity of buttorworth filter is better. In other words, the group delay (derivative of phase with respect to frequency) is more constant with respect to frequency. This means that the waveform distortion of the butterworth filter is lower. This Butterworth filters have the following characteristics [3].

- The magnitude response is nearly constant (equal to 1) at lower frequencies. That means pass band is maximally flat.
- The response is monotonically decreasing from the specified cut off frequencies.
- The maximum gain occurs at  $\Omega= 0$  and it is  $|H(0)|= 1$ .
- Half power frequency, or 3db down frequency, that corresponds to the specified cut off frequencies.

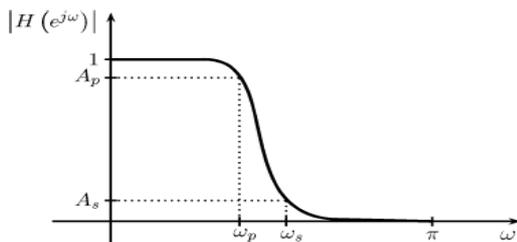


Fig.1- Design parameters

The magnitude squared response of low pass Butterworth filter is given by

$$|H(\Omega)| = 1 / \sqrt{1 + C^2 (\Omega / \Omega_c)^{2N}} \quad (1)$$

This equation is also expressed as

$$|H(\Omega)|^2 = 1 / (1 + C^2 (\Omega / \Omega_p)^{2N}) \quad (2)$$

Here  $|H(\Omega)|$  = Magnitude of analog low pass filter.  
 $\Omega_c$  = Cut-off frequency (-3db frequency)  
 $\Omega_p$  = Pass band edge frequency.  
 $C$  = Parameter related to ripples in pass band.  
 $N$  = Order of the filter.

The order of filter means the number of stages used in the design of filter. As the order of filter  $N$  increases, the response of filter is more close to the ideal response as shown in Fig.2.

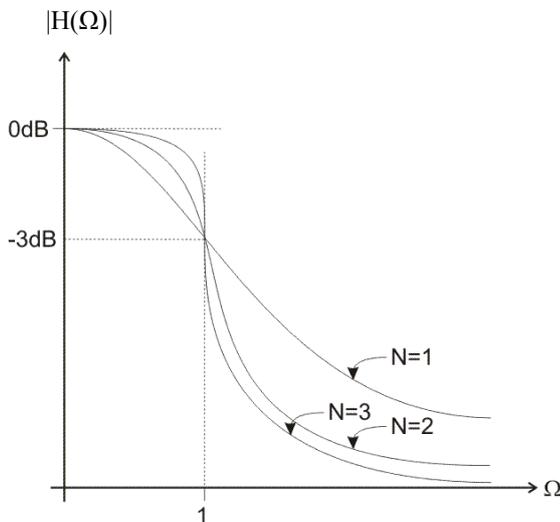


Fig.2- Effect of N on frequency response characteristics.

### III. CHEBYSHEV TYPE I FILTER

Chebyshev1 filters have a narrower transition region between the passband and the stopband. The sharp transition between the passband and the stopband of a chebyshev filter produces smaller absolute errors and faster execution speeds than a butterworth filter. The poles of chebyshev1 filter lies on an ellipse. ripple increase (band), the roll-off becomes sharper(good). The chebyshev filter is completely defined by three parameters-cut-off frequencies, number of poles and passband ripples. The chebyshev response is a mathematical strategy for achieving a faster roll off by allowing ripple in the frequency response. The chebyshev response is an optimal trade-off between these two parameters. The magnitude squared frequency response is given by

$$|H(\Omega)|^2 = 1 / (1 + C^2 T_N^2(\Omega / \Omega_p)) \quad (3)$$

Here  $|H(\Omega)|$  = Magnitude of analog low pass filter.  
 $C$  = Parameter related to ripples in pass band.  
 $T_N(x)$  = Chebyshev polynomial of order  $N$

The chebyshev1 polynomials are determined by using the equations

$$C_{N+1}(x) = 2x C_N(x) - C_{N-1}(x) \quad (4)$$

with  $C_0(x) = 1$  and  $C_1(x) = x$

The following figure shows the frequency response of a lowpass Chebyshev1 filter.

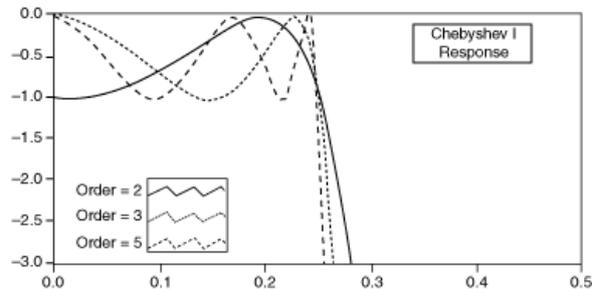


Fig.3- Effect of N on Chebyshev1 filter characteristics

### IV. ALLPASS FILTER

The allpass filter is an important building block for signal processing system. The magnitude response of an allpass filter is unity over its entire frequency range. In other words, all frequencies are “passed” in the same sense as in “lowpass”, “highpass” and “bandpass” filters. The phase response (which determines the delay versus frequency) is variable. The allpass filters are typically appended in a cascade arrangement following a standard IIR filter  $H_1(z)$  as shown in Fig 4

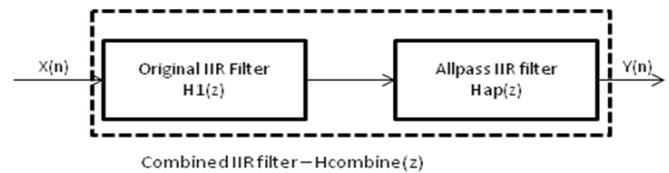


Fig 4- Cascade arrangement of original IIR filter and allpass filter.

The transfer function of IIR allpass filter with unity magnitude response for all frequencies is given by  $|A(e^{j\omega})|^2 = 1$  for all  $\omega$ . The Nth order causal real coefficient allpass transfer function is given by equation

$$A_N(z) = \frac{d_N + d_{N-1}z^{-1} + \dots + d_1z^{-N+1} + z^{-N}}{1 + d_1z^{-1} + \dots + d_{N-1}z^{-N+1} + d_Nz^{-N}} \quad (5)$$

If the denominator polynomials of  $A_N(z)$  is denoted by  $D_N(z)$  then equation (5) can be written as

$$A_N(z) = \pm z^{-N} D_N^*(z^{-1}) / D_N(z) \quad (6)$$

The numerator of the allpass filter is the mirror image polynomial of the denominator.

$$\overline{D}_N(z) = z^{-N} D_N^*(z^{-1}) \quad (7)$$

Equation (7) implies that poles and zeros of a real coefficient allpass function exhibit mirror image symmetry in the z plane.

**V. METHOD**

**A. Group Delay Equalization**

For too much variation in the obtained delay for signal processing requirement we can only modify the transfer function. But it should be done in such a way that the filters magnitude response is not destroyed. The filter G is connected with desired magnitude response |G(jw)|, in cascade with a circuit H, whose magnitude response is equal to unity |H(jw)|=1, but whose phase varies with frequency, for delay in equalization. The transfer function of this cascade configuration is Geq(s) = G(s) . H(s), or on the jw axis.[2].

$$Geq(jw)=G(jw)H(jw)= |G(jw)||H(jw)| e^{j\phi G(w)} e^{j\phi H(w)}$$

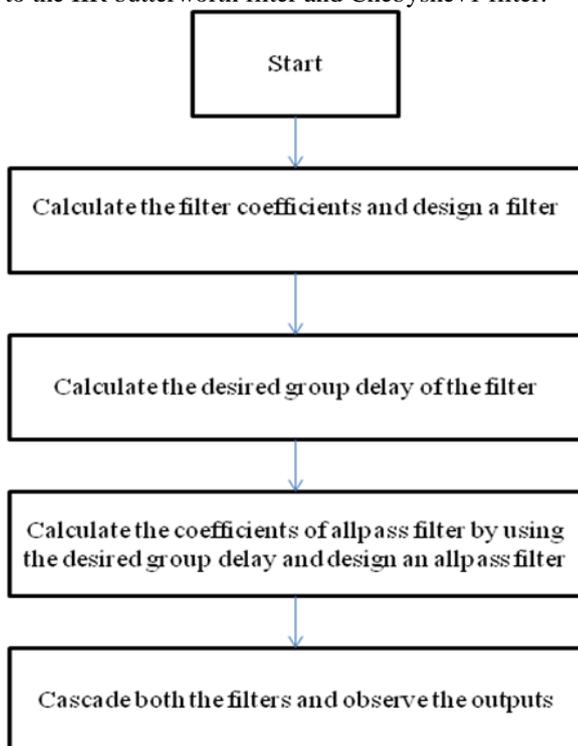
$$=|G(jw)| e^{j|\phi G(w)+\phi H(w)|} \tag{8}$$

As shown in equation [8] the magnitude multiply with no contribution from the allpass module because |H(jw)|=1, and that the phases add. The delay and add as well, as the group delay is obtained from the negative derivative of the phase.

$$T_{geq}(w)=-[d[\phi G(w)+ \phi H(w)]]/dw$$

$$=-d \phi G(w)/dw- d \phi H(w)/dw = T_{gG}+T_{gH} \tag{9}$$

The Tg in this process will increase because Tg is larger than either TgG and TgH. In this paper the group delay equalization is carried out with the aid of the allpass filter which is cascade connected to the IIR butterworth filter and Chebyshev1 filter.



Flow chart for Group delay equalization.

**B. Steps for Group delay equalization**

1. Initially set the passband frequency (wp), stopband frequency (rs), passband ripples(rp) and stopband ripples(rs).
2. Determine the order and wn of filters .In MATLAB, use the command buttord()for butterworth filter.  
[n,wn] = buttord(wp,ws,rp,rs)  
Where n is order of filter and wn is a cut off frequency.
3. Applying the command butter() to find the filter coefficients of butterworth filter .  
[b,a] = butter (n, wn, 'ftype')
4. Calculate the maximum group delay and desired group delay of Buttorworth filter.
5. Based on difference of the group delays calculate the coefficients and design an allpass filter.
6. Finally compute the coefficients of the desired butterworth filter
7. Plot the group delay of desired butterworth filter.

**RESULTS**

Specifications taken for the design of Butterworth filter are:  
 Sampling frequency=2000Hz.  
 Passband frequency wp =0.2π  
 Stopband frequency ws=0.4 π  
 Passband ripples Ap =1db  
 Stopband ripples As =45db

By using the above specifications following graphs have been obtained for Butterworth and Chebyshev1 filters.

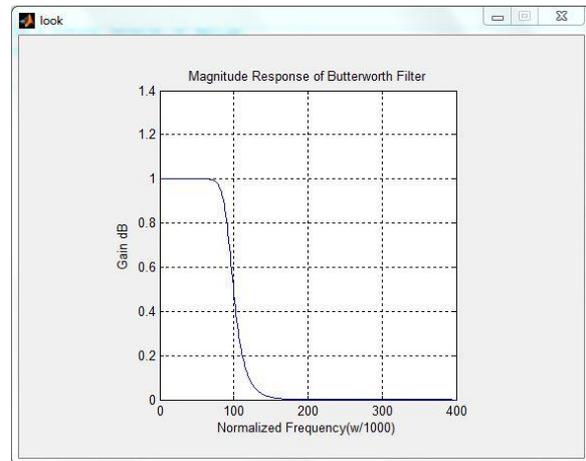


Fig 5 shows the magnitude response of Butterworth filter of order 4.

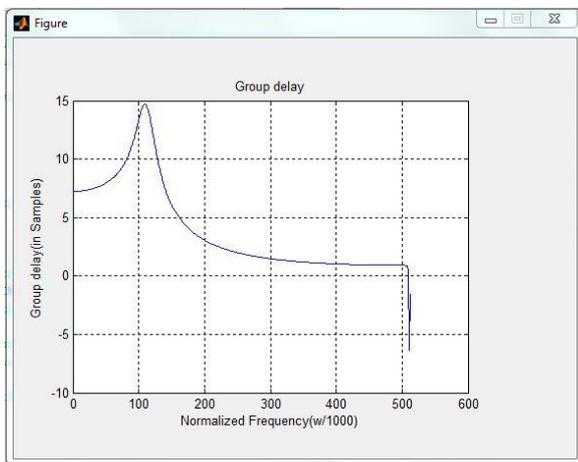


Fig 6 shows the Group delay of Original Butterworth filter.

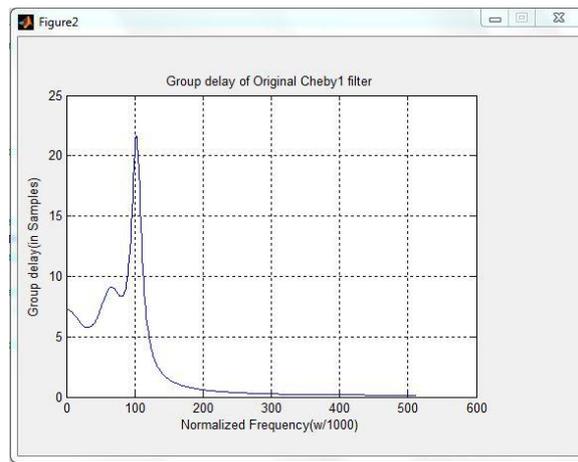


Fig 9 shows the Group delay of Original Chebyshev1 Filter.

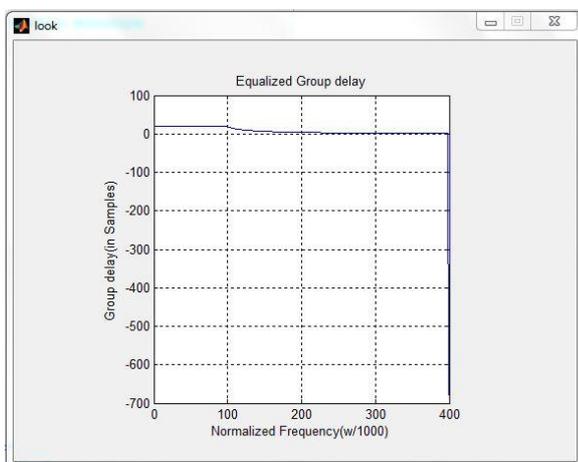


Fig 7 shows the Equalized Group delay of Butterworth filter.

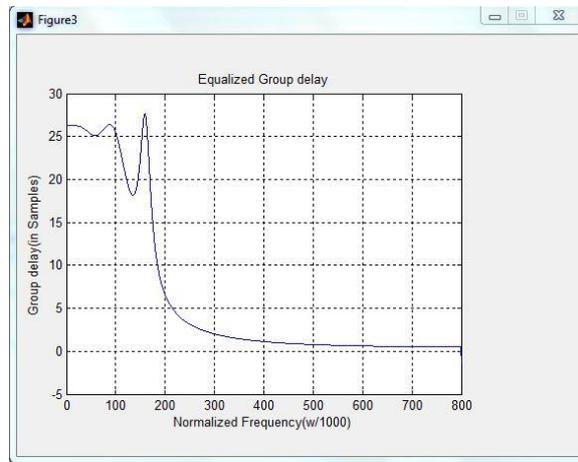


Fig 10 shows the Equalized Group Delay of Chebyshev1 filter

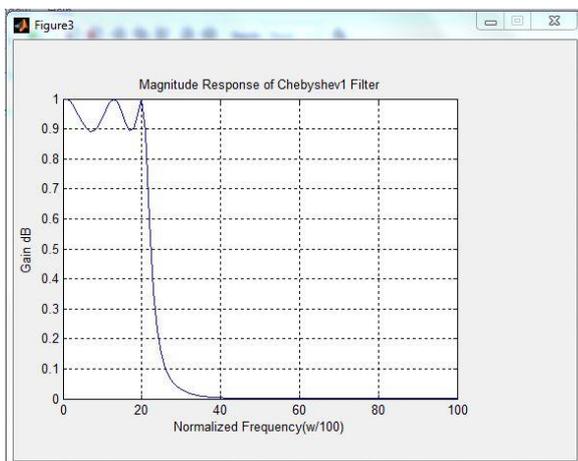


Fig 8 shows the Magnitude response of chebyshev1 filter of order 4.

The results in the above diagrams show that Equalized Group delay of Butterworth and Chebyshev1 filters, is almost flat group delay across the passband from 0 to 0.2, as compare to group delay of original Butterworth and Chebyshev1 filters respectively.

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