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New Aggregation Operator for Trapezoidal Fuzzy Numbers based on the Arithmetic Means of the Left and Right Apex Angles

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Abstract— **In previous work, authors have proposed two new aggregation operators for a class of LR fuzzy numbers known as triangular fuzzy numbers (TFNs) in which the L- and R- apex angles of the piecewise continuous linear membership function of the composite or resultant or aggregate TFN are the arithmetic means [1] and the geometric means [2] of the L- and R- apex angles of the individual TFNs. In this paper the concept of an aggregation operator for triangular fuzzy numbers based on the arithmetic means of the corresponding L- and R- apex angles is extended for a class of Interval Fuzzy Numbers known as Trapezoidal Fuzzy Numbers (TrFNs). The left and right side angles have been treated independently as was done for the L- and R- apex angles in the case of LR TFNs. Computation of the aggregate is demonstrated with a numerical example. Corresponding arithmetic and geometric aggregates have also been computed.**

Keywords— **LR Fuzzy Number, Interval Fuzzy Number, Triangular Fuzzy Number, Trapezoidal Fuzzy Number, Apex Angle, Left Apex Angle, Right Apex Angle, Aggregation Operator, Arithmetic and Geometric Mean**

INTRODUCTION

Many different types of fuzzy numbers are defined in the literature dealing with fuzzy logic and applications. In this paper only one class of fuzzy numbers i.e., Trapezoidal Fuzzy Numbers (TrFNs) which are a special class of interval fuzzy numbers are treated.

A. *Trapezoidal Fuzzy Numbers*

TFNs and TrFNs are extensively used in fuzzy applications owing to their simplicity. TrFNs are used in fuzzy applications where uncertainty exists on both (left and right) sides of an interval or range wherein the grade of membership or possibility of the values is 1. TrFNs are characterized by an ordered quartet of real numbers <a,b,c,d>. Figure depicts a TrFN <a,b,c,d> with values v on x-axis and grade of membership or possibility μ along y-axis. The left and right side apex angles are shown with arcs having one and two dashes respectively. Vertices of the trapezium are (a,0), (b,1), (c,1) and (d,0) moving clockwise. Line between (a,0) and (b,1) and between (c,1) and (d,0) are the membership functions for the values in the intervals [a, b] and [c, d] respectively. Membership function of the TrFN is the piecewise continuous linear function represented by the lines L_1, L_2 and L_3 respectively. Intuitive meaning of such TrFN is that the fuzzy number is approximately any of the values in the range or interval [b,c] [3]. The value of the fuzzy number varies in the range [a,d]. The possibility or membership grade of the number being a specified value v between a and d, $v \in [a, d]$ is represented by the ordinate of the projection of v on L_1, L_2 or L_3 accordingly as $a \le v \le b$ or $b \le v \le c$ or $c \le v \le d$. [a,b] is the range or interval which depicts or represents the possibilities of the number being a specific value less than b and [c, d] is the range or interval which depicts the possibilities of the number being a specific value greater than c. Possibility takes on the maximum value of 1 in the range or interval [b,c] and reduces with increasing distance on either side (left / right) of [b,c], becoming zero beyond a at the left and d at the right respectively.

Mathematically, a TrFN is defined as follows. Let $a, b, c, d \in \mathbf{R}$, $a < b < c < d$.
The fuzzy number tr: $\mathbf{R} \rightarrow [0, 1]$ denoted by

$$tr = \begin{cases} 0, & \text{if } x < a, \\ \dfrac{x-a}{b-a}, & \text{if } a \le x \le b, \\ 1, & \text{if } b \le x \le c, \\ \dfrac{x-d}{c-d}, & \text{if } m \le x \le u \\ 0, & \text{if } x > d \end{cases}$$

is called a trapezoidal fuzzy number [3][4][5][6].

FUZZY AGGREGATION

Aggregation operations on fuzzy numbers are operations by which several fuzzy numbers are combined to produce a single fuzzy number [7]. An excellent account of Mathematical Aggregation Operators is given in [8]

A. Arithmetic Mean

The arithmetic mean aggregation operator [7][9] defined on n TFNs $<a_1,b_1,c_1,d_1>$, $<a_2,b_2,c_2,d_2>$, ..., $<a_i,b_i,c_i,d_i>$, ..., $<a_n,b_n,c_n,d_n>$ produces the result $<\bar{a}, \bar{b}, \bar{c}, \bar{d}>$ where

$$\bar{a} = \frac{1}{n}\sum_1^n a_i \, , \; \bar{b} = \frac{1}{n}\sum_1^n b_i \, , \; \bar{c} = \frac{1}{n}\sum_1^n c_i \, , \text{ and}$$

$$\bar{d} = \frac{1}{n}\sum_1^n d_i$$

B. Geometric Mean

The geometric mean aggregation operator [7][10] defined on n TFNs $<a_1,b_1,c_1,d_1>$, $<a_2,b_2,c_2,d_2>$, ..., $<a_i,b_i,c_i,d_i>$, ..., $<a_n,b_n,c_n,d_n>$ produces the result $<\bar{a}, \bar{b}, \bar{c}, \bar{d}>$ where

$$\bar{a} = \left(\prod_1^n a_i\right)^{\frac{1}{n}}, \; \bar{b} = \left(\prod_1^n b_i\right)^{\frac{1}{n}}, \; \bar{c} = \left(\prod_1^n c_i\right)^{\frac{1}{n}}, \text{ and}$$

$$\bar{d} = \left(\prod_1^n d_i\right)^{\frac{1}{n}}$$

Other aggregation operators have also been defined in literature. For examples see [10][11][12][13][14].

C. Applications of Aggregation

The combination/aggregation/fusion of information from different sources is at the core of knowledge based systems. Applications include decision making, subjective quality evaluation, information integration, multi-sensor data fusion, image processing, pattern recognition, computational intelligence etc. An application of aggregation operators in fuzzy multicriteria decision making is discussed in [9][10]. Another application in sensor data fusion is discussed in [12].

D. Organization of the Paper

In this paper a new aggregation operator for TrFNs based on arithmetic means of the left and right side apex angles is proposed. The left and right side apex angles have been treated independently. The operator is described in the next section. A numerical example is given and the result from the proposed operator is presented alongside those obtained from arithmetic and geometric mean aggregate operators.

PROPOSED AGGREGATION OPERATOR

Consider the TrFN shown in Figure 1. If the value of this TrFN is $v \in [b,c]$ the corresponding possibility $\mu = 1$. The left side apex angle of this TrFN is $\angle apb$. The right side apex angle of this TrFN is $\angle drc$. The left and right side apex angles of the trapezoid refer to the apex angles subtended to the left and the right of the interval [b,c] respectively.

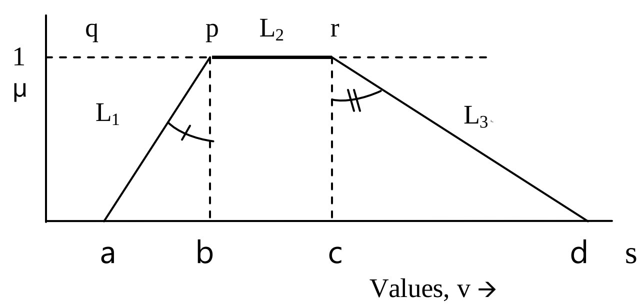

Figure 1: Trapezoidal Fuzzy Number

But, $\angle apb = \pi/2 - \angle bap$

and $\angle drc = \angle sdr - \pi/2$

Considering the left side and averaging over n TFNs we have

$$\frac{1}{n}\sum_1^n (\angle\, apb)_i = \frac{1}{n}\sum_1^n (\angle\, (\pi/2 - bap_i)$$

$$\frac{1}{n}\sum_1^n (\angle\, apb)_i = \pi/2 - \frac{1}{n}\sum_1^n (\angle\, bap)_i$$

The left side of the above equation represents the contributions of the left lines i.e., L_1's of all TrFNs to the aggregate apex angle. It can be seen that

$$tan\left(\frac{1}{n}\sum_{1}^{n}(\angle apb)_i\right) = \frac{1}{tan\left(\frac{1}{n}\sum_{1}^{n}(\angle bap)_i\right)}$$

It can be shown that

$$tan\left(\frac{1}{n}\sum_{1}^{n}(\angle bap)_i\right) = \left(tan\left(\frac{1}{n}\sum_{1}^{n}tan^{-1}((b_i - a_i))\right)\right)^{-1}$$

This represents the slope of the resultant aggregate left line $\bar{L}_1$. Similarly, it can be shown that the slope of the resultant fuzzy aggregate right line $\bar{L}_3$ is

$$-\left(tan\left(\frac{1}{n}\sum_{1}^{n}tan^{-1}((d_i - c_i))\right)\right)^{-1}$$

Under identical treatment, it can be shown that $\bar{b} = \frac{1}{n}\sum_{1}^{n} b_i$, $\bar{c} = \frac{1}{n}\sum_{1}^{n} c_i$. Subsequently it is possible to show that

$$\bar{a} = \frac{1}{n}\sum_{1}^{n} b_i - tan\left(\frac{1}{n}\sum_{1}^{n}tan^{-1}((b_i - a_i))\right), \text{ and}$$

$$\bar{d} = \frac{1}{n}\sum_{1}^{n} c_i + tan\left(\frac{1}{n}\sum_{1}^{n}tan^{-1}((d_i - c_i))\right)$$

NUMERICAL EXAMPLE

Consider the two trapezoidal fuzzy numbers **<1,1.5,3,4>** and **<5,6.5,9,11>** aggregate TrFN is computed as $\bar{b}$ =4; $\bar{c}$ =6;

$$\bar{a} = 4 - tan\left(\frac{1}{2}\left(tan^{-1}(1.5-1) + tan^{-1}(6.5-5)\right)\right) =$$

4-tan(41.437) =4-0.8828=3.117; Similarly, $\bar{d}$ can be computed as 7.3874.

Thus we have the aggregate as **<3.117,4,6,7.3874>** which is the expected result**.** The arithmetic mean aggregate is **<3,4,6,7.5>** and the geometric mean aggregate is **<2.24,3.12,5.20,6.6332>** respectively.

CONCLUSIONS

In this paper we have defined a new aggregate of TrFNs based on the arithmetic mean of the left and right side apex angles. The left and right side apex angles have been treated independently. A numerical example has been worked out. The aggregate is the resultant piecewise continuous linear membership function whose left and right side apex angles are the arithmetic means of the left and right side apex angles of the piecewise continuous linear membership functions of the individual TrFNs. The suitability of the aggregation operator proposed in this paper in different fuzzy logic applications involving fuzzy number aggregation remains to be explored.

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