Volume 2, Issue 6, June 2012 ISSN: 2277 128X
International Journal of Advanced Research in Computer Science and Software Engineering
Research Paper
Available online at: www.ijarcsse.com

Closed Recurrent Neural Network for Four Neurons

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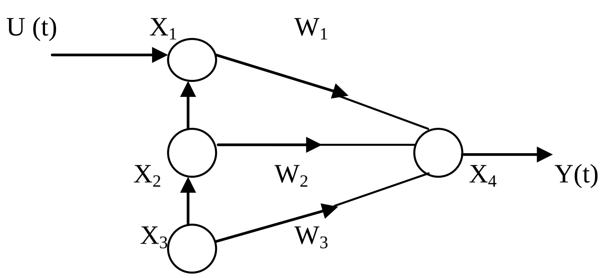
***Abstract*— In this paper we present four node recurrent neural networks system with three weight parameters. This closed recurrent neural network generates a limit cycle. In this system for which the equation of equilibria involves transcendental function tan h(X) and its replace σ (X) and its iterates.**

***Keywords*— Neural network, limit cycle, nonlinear dynamics, learning systems, transcendental function and equilbrium.**

I. INTRODUCTION

The neural network based models have various momentous advantages over the more traditional parameter models. The first time neural networks model is used for market forecasting, i.e. to identity orderlities in asset price movements such as fluctuations of general stock price [3]. Neural networks with at least one feedback loop are known as recurrent neural network [7].Recurrent neural network have often been occupied for nonlinear dynamical system identification control[2][4],and have been shown to be more powerful than pure feed forward neural networks. Feldkamp and puskorius[6] have shown that a class of recurrent neural networks displays adaptive behaviour. A one-layer recurrent neural network for solving a class of constrained nonsmooth optimization problems with piecewise-linear objective functions is studied by [5].

Our works describe a recurrent neural network for four neurons. Here we use monotone dynamical system theory to show that for a well defined set of parameters, every orbit of the recurrent neural network is asymptotic to a period orbit and this model is shown to possess a stable limit cycle under some condition.



In this figure, U(t) is the input and Y(t) is the output of the neural network. This recurrent neural network described of the system of nonlinear differential equations

$$\dot{X}_1 = -X_1 + \tan h\,(X_2\,(t)) \quad (1)$$
$$\dot{X}_2 = -X_2 + \tan h\,(X_3\,(t)) \quad (2)$$
$$\dot{X}_3 = -X_3 + \tan h\,(X_4\,(t)) \quad (3)$$
$$\dot{X}_4 - X_4 + W_1 \tan h\,(X_1\,(t)) + W_2 \tan h\,(X_2\,(t)) + W_3 \tan h\,(X_3\,(t)) \quad (4)$$

$$Y\,(t) = \tan h(X_4\,(t))$$

Where $X(t) \in R^n$ is the state $W_i \in R$, i =1,2,3 are the network parameter of weights U(t) is the input and Y(t) is the output the nonlinear transcendental function for the neurons. When we linearization at X = 0 we find two pair of complex conjugate poles with positive real pole and one negative real pole, then this is a particular type of instability of the equilibrium. At X=0 combined with boundness of the solution forces the system to generate a limit cycle [1].

The freedom in choosing W_1, W_2 and W_3 in the system (1)-(4) Allows us to determine the position of the pole of linearized System and so influences the amplitude and frequency of the corresponding limit cycle.

The linearization of (1)-(4) at X =0 given by

$$\dot{z}_1 = -z_1 + z_2$$
$$\dot{z}_2 = -z_2 + z_3$$

$$\dot{z}_3 = -z_3 + z_4$$
$$\dot{z}_4 = -z_4 + W_1 z_1 + W_2 z_2 + W_3 z_3$$

Let
$$A = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ W_1 & W_2 & W_3 & -1 \end{pmatrix}$$

The roots of the characteristic polynomial B -λI = 0 is define by

$$(-1-\lambda)^3 - (-1-\lambda)(W_2 + W_3) + W_1 = 0$$

Let
$$f(\lambda) := -[1+\lambda^3 + 3\lambda^2 + 3\lambda] + (1+\lambda)(W_2 + W_3) + W_1 = 0 \quad (5)$$

Differentiating (5)

$$f'(\lambda) = 3\lambda^2 + 6\lambda - (W_2 + W_3 - 3) = 0 \quad (6)$$

The roots of (6) are given by.

$$\lambda_1 = -1 + \frac{1}{\sqrt{3}}\sqrt{W_2 + W_3}, \lambda_2 = -1 - \frac{1}{\sqrt{3}}\sqrt{W_2 + W_3}$$

substituting the value of λ_1 in (5) we get

$$= 1 + \left[-1 + \frac{1}{\sqrt{3}}\sqrt{W_2 + W_3}\right]^3 + 3\left[-1 + \frac{1}{\sqrt{3}}\sqrt{W_2 + W_3}\right]^2 + 3\left[-1 + \frac{1}{\sqrt{3}}\sqrt{W_2 + W_3}\right] - \left[1 + \left[-1 + \frac{1}{\sqrt{3}}\sqrt{W_2 + W_3}\right]\right]$$
$$(W_2 + W_3) - W_1 = 0$$

$$4(W_2 + W_3)^3 = 27W_1^2$$

If $W_2 = W_3$

$$\Rightarrow 27W_1^2 = 32 W_2^3 \quad (7)$$

According to Ruiz, Owens and Townley [1]A necessary and sufficient condition for f(λ) to have a pair of complex conjugate roots is the that $f(\lambda_1)\, f(\lambda_2) > 0$,.From (5) we obtain

$$f(\lambda_1)\, f(\lambda_2) = \left[1 - \frac{(W_2 + W_3)}{3}\right] > 0 \quad (8)$$

$$\Rightarrow \quad W_2 + W_3 < 3 \quad (9)$$

Let
$$B = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ W_1 & W_2 & W_3 & -1 \end{pmatrix}$$

The roots of the characteristic polynomial B -λI = 0 is define by

$$(-1-\lambda)^3 + (1+\lambda) W_3 - (W_1 + W_2) = 0$$

Let
$$\emptyset(\lambda) := -[1+\lambda^3 + 3\lambda^2 + 3\lambda] + (1+\lambda)(W_3) - (W_1 + W_2) = 0 \quad (10)$$

Differentiating (10)

$$\emptyset'(\lambda) := 3\lambda^2 + 6\lambda - (W_3 - 3) = 0 \quad (11)$$

$$\lambda_3 = -1 + \frac{1}{\sqrt{3}}\sqrt{W_3}, \quad \lambda_4 = -1 - \frac{1}{\sqrt{3}}\sqrt{W_3}$$

substituting the value of λ_1 in (10) we get

$$= 1 + \left[-1 + \frac{1}{\sqrt{3}}\sqrt{W_3}\right]^3 + 3\left[-1 + \frac{1}{\sqrt{3}}\sqrt{W_3}\right]^2 + 3\left[-1 + \frac{1}{\sqrt{3}}\sqrt{W_3}\right] - \left[1 + \left[-1 + \frac{1}{\sqrt{3}}\sqrt{W_3}\right]\right] W_3 - (W_2 + W_3) = 0$$

$$W_3(2W_2 + 3W_3)^2 = 27W_1^2$$

If $W_2 = W_3$

$$\Rightarrow 27W_1^2 = 25W_3^3 \quad (12)$$

$$\emptyset(\lambda_1)\, \emptyset(\lambda_2) = \left[1 - \frac{(W_3)}{3}\right] > 0 \quad (13)$$

$$\Rightarrow \quad W_3 < 3 \quad (14)$$

Equation (9) and (14) put condition W_2, W_3 for the system of equations (1)-(4) to have a limit cycle at the origin.

Theorem: 1 If the system (1) to (4) tan h(X) replaced by σ (X), where σ (X) = X + 0(X^3). Assume inequalities. $27 W_1^2 = 32 W_2^3$ or $27 W_1^2 = 25 W_2^3$ hold. Then the system has a unique equilibrium point at the origin if and only if

$$8[\sigma_4(X_4)]^2 X_4 < [\sigma_3(X_4) + \sigma_2(X_4)]^3$$
Or
$$25[\sigma_4(X_4)]^2 X_4 < [\sigma_3(X_4) + \sigma_2(X_4)]^3$$

respectively.

Proof: We assume that X= (X_1, X_2, X_3, X_4) is an equilibrium points (1)-(4) with tan h (X) replaced by σ (X). Then satisfies.

$$X_1 = \sigma(X_2) \quad (15)$$
$$X_2 = \sigma(X_3) \quad (16)$$

$$X_3 = \sigma(X_4) \quad (17)$$
$$X_4 = W_1\sigma(X_1) + W_2\sigma(X_2) + W_3\sigma(X_3) \quad (18)$$

With the $W_1, W_2, W_3 \in R$ clearly, $X = 0$ satisfies(15) to (18) and is therefore and equilibrium points. We define σ_k the k^{th} composition of $\sigma(X)$ on X. Substituting the first three equation of (15)-(18) in to forth and $W_2 = W_3$ obtain.

$$X_4 = W_1\sigma_4(X_4) + W_2\sigma_3(X_4) + W_3\sigma_2(X_4) \quad (19)$$

We shall prove that for W_1, W_2, W_3 satisfying inequalities $27 W_1^2 = 32 W_2^3$, $X_4 = 0$ is the unique solution of (19). For any fixed (19) define a line with abscissa $a = X_4 / \sigma_4(X_4)$ and ordinate $b = (X_4) / [\sigma_3(X_4) + \sigma_2(X_4)]$ in the parameter space (W_1, W_2, W_3) it follows that (15)- (18)has a unique solution at zero.

If $W_1 \le 0$ then the branch of solution of

$$27 W_1^2 - 32 W_2^3 = 0 \quad \text{or} \quad 27 W_1^2 = 25 W_2^3 \quad (20)$$

Let $y = \sqrt{W_2}$ and eliminate W_1 from (19) and (20) to obtain.

Let
$g(y) = -4\sqrt{2}/3\sqrt{3}\, y^3\sigma_4(X_4) + y^2\sigma_3(X_4) + y^2\sigma_2(X_4) - X_4 < 0$
Or
$g(y) = -5/3\sqrt{3}\, y^3\sigma_4(X_4) + y^2\sigma_3(X_4) + y^2\sigma_2(X_4) - X_4 < 0$ (21)

And note that $g(y) < 0$ for large and $g(0) = -X_4 < 0$
So, we only need to show that the g(y) is negative at its local maximum.
The local maximum y_{max} of (21) is

$$y_{max} = \sqrt{3}\left[\frac{\sigma_3(X_4) + \sigma_2(X_4)}{2\sqrt{2}\sigma_2(X_4)}\right]$$

Or

$$y_{max} = 2\sqrt{3}\left[\frac{\sigma_3(X_4) + \sigma_2(X_4)}{5\,\sigma_2(X_4)}\right] \quad (22)$$

And we should have that
$g(y_{max}) = -4\sqrt{2}/3\sqrt{3}\, y^3\sigma_4(X_4) + y^2\sigma_3(X_4) + y^2\sigma_2(X_4) - X_4 < 0$
Or
$g(y_{max}) = -5/3\sqrt{3}\, y^3\sigma_4(X_4) + y^2\sigma_3(X_4) + y^2\sigma_2(X_4) - X_4 < 0$ (23)

using (22) we simplify (23) to obtain that

$$8\,[\sigma_4(X_4)]^2\ X_4 < [\sigma_3(X_4) + \sigma_2(X_4)]^3$$
Or
$$25\,[\sigma_4(X_4)]^2\ X_4 < [\sigma_3(X_4) + \sigma_2(X_4)]^3$$

Hence W_1, W_2 and W_3 satisfying inequalities (7) or (12). The system (15)-(18) has a unique solution at X= 0.

Theorem 2: The system given by (1)-(4) has a unique equilibrium point at the origin subject to $27 W_1^2 = 32 W_2^3$ or $27 W_1^2 = 25 W_2^3$ holds.

Proof: The equation (21) which we rewrite in this specific case as
Let $h(X_4) := -4\sqrt{2}/3\sqrt{3}\, y^3 \tanh_4(X_4) + y^2\tanh_3(X_4) + y^2\tanh_2(X_4) - X_4$
Or Let $h(X_4) := -5/3\sqrt{3}\, y^3\tanh_4(X_4) + y^2\tanh_3(X_4) + y^2\tanh_2(X_4) - X_4$ (24)

Now h(0) and $h(X_4) < 0$ for $X_4 > 0$, since $\tanh_k(X_4)$, $k \in N$ is bounded so it sufficient to show that the derivative of $h(X_4)$ and $h'(X_4)$, remains non positive for all values of parameter y. from (24) we calculate $h'(X_4)$ which is given by

$$h'(X_4) = -4\sqrt{2}/3\sqrt{3}\, y^3\,\mathrm{sech}^2{}_4(X_4)\,\mathrm{sech}^2{}_3(X_4)\,\mathrm{sech}^2{}_2(X_4) + y^2\,\mathrm{sech}^2{}_3(X_4)\,\mathrm{sech}^2{}_2(X_4) + y^2\mathrm{sech}^2{}_2(X_4) - 1$$

$$= \mathrm{sech}^2{}_2(X_4)\,[-4\sqrt{2}/3\sqrt{3}\, y^3\,\mathrm{sech}^2{}_4(X_4)\,\mathrm{sech}^2{}_3(X_4) + y^2\,\mathrm{sech}^2{}_3(X_4) + y^2] - 1$$
Or
$$h'(X_4) = \mathrm{sech}^2{}_2(X_4)\,[-5/3\sqrt{3}\, y^3\,\mathrm{sech}^2{}_4(X_4)\,\mathrm{sech}^2{}_3(X_4) + y^2\,\mathrm{sech}^2{}_3(X_4) + y^2] - 1 \quad (25)$$

observe that $h'(X_4) < 0$ for every X_4 if $y = 0$ or y is large.We should have now that $h'(X_4) \le 0$ for all values of y. To check this consider the right hand side of (25) as a function of y, say h(y), X_4 as a parameter. We differentiate of h(y) with respect to y, obtain the maximum y_{max} and verify $h(y_{max}) \le 0$. Differentiating h(y) with respect toy and solving that we have that

$$y_{max} = \left[\frac{\sqrt{3}\ \mathrm{sech}_3^2(X_4) + 1}{2\sqrt{2}\,\mathrm{sech}_4^2(X_4)\,\mathrm{sech}_3^2(X_4)}\right]$$
Or
$$y_{max} = \left[\frac{2\sqrt{3}\ \mathrm{sech}_3^2(X_4) + 1}{5\,\mathrm{sech}_4^2(X_4)\,\mathrm{sech}_3^2(X_4)}\right]$$

Substituting (25) in to right side of (26) and arranging terms we obtain that $h'(y_{max}) \le 0$ if and only if

$$\frac{[1 - \tanh_2^2(X_4)][2 - \tanh_3^2(X_4)]^3}{8\,[1 - \tanh_4^2(X_4)][2 - \tanh_3^2(X_4)]^2} \le 1$$

Or
$$\frac{4\,[1 - \tanh_2^2(X_4)][2 - \tanh_3^2(X_4)]^3}{25\,[1 - \tanh_4^2(X_4)][2 - \tanh_3^2(X_4)]^2} \le 1$$

Since $\tanh_4(X_4) < \tanh_3(X_4) < \tanh_2(X_4) < \tanh(X_4)$ it is easy to conclude that (26) is valid.

II. CONCLUSIONS

The neurotic equations modelling of this class recurrent neural network shows limit cycle at the origin. The theorem (1) and (2) prove that the system has unique equilibrium point at the origin under certain conditions on weight parameters W_1, W_2 and W_3. This idea is used in manufacturing engineering to control the system.

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