



Design of FIR Filter Using GA and its Comparison with Hamming window and Parks McClellan Optimization Techniques

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Abstract— Digital filter are widely used in the world of communication and computation. On the other hand to design a digital finite impulse response (FIR) filter that satisfying all the required conditions is a challenge. In this paper, design techniques of low pass FIR filters using Hamming window method, Optimal Parks McClellan method and Genetic Algorithm method are presented. The magnitude response, phase response, stability, and filter coefficients are demonstrated for different design techniques. It has shown that filter design using GA is best because the transition bandwidth of GA is very less, possess very small amount of ripples in pass band and stop band and its phase response is better as compared with other techniques. Other advantages are discussed in paper. Design comparisons are presented to show the effectiveness of GA optimization method.

Keywords— Filter Design, FIR Filter, Genetic Algorithm, Optimal Parks McClellan, Hamming Window Method

I. INTRODUCTION

Digital filters are useful structures for digital signal processing applications and in signal analysis and estimation. An operation of digital filter design is calculation of filter transfer function coefficients that provide desired amplitude requirements. Two types of filters provide these functions are finite impulse response (FIR) filters and infinite impulse response (IIR) filters [2][13]. Typical filter applications include signal preconditioning, band selection, and low pass filtering.

1.1 FIR has following advantage over IIR Filter

2. FIR filter is Finite IR filter and IIR filter is Infinite IR filter.
3. FIR filters are non-recursive. That is, there is no feedback involved. Where as an IIR filter is recursive. There is feedback involved
4. The impulse response of an FIR filter will eventually reach zero. The impulse response of an IIR filter may very well keep "ringing" ad-infinitum.
5. IIR filters may be designed to accurately simulate "classical" analog filter responses where as FIR filters, in general, cannot do this.
6. FIR filter has linear phase and easily control where as IIR filter has no particular phase and difficult to control
7. FIR filter is stable and IIR filter is unstable
8. FIR filter depend only on I/P where as IIR filter depend upon both I/P and O/p
9. FIR filter consist of only zeroes and IIR filter consist of both poles and zeroes.

FIR filters are filters having a transfer function of a polynomial in z - and is an all-zero filter in the sense that the zeroes in the z -plane determine the frequency response magnitude characteristic [4]. The z transform of a N -point FIR filter is given by

$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n} \quad (1.1)$$

FIR filters are particularly useful for applications where exact linear phase response is required.

The FIR filter is generally implemented in a non-recursive way which guarantees a stable filter.

FIR filter design essentially consists of two parts

- (i) Approximation problem
- (ii) Realization problem

The approximation stage takes the specification and gives a transfer function through four steps. They are as follows:

- (i) A desired or ideal response is chosen, usually in the frequency domain.
- (ii) An allowed class of filters is chosen (e.g. the length N for a FIR filters).
- (iii) A measure of the quality of approximation is chosen.
- (iv) A method or algorithm is selected to find the best filter transfer function.

The realization part deals with choosing the structure to implement the transfer function which may be in the form of circuit diagram or in the form of a program.

There are essentially three well-known methods for FIR filter design namely:

- (1) The window method
- (2) The frequency sampling technique
- (3) Optimal Filter Design Method

II. The Window Method

The method the most used in digital filter design is Fourier series method. However, there is a problem in this method. The problem is that Fourier series method causes to Gibb's oscillations at cut-off frequency region.

In this method the desired frequency response specification $H_d(w)$, corresponding unit sample response $hd(n)$ is determined using the following relation

$$h_d(n) = 1/2\pi \int_{-\pi}^{\pi} h_d(w) e^{jwn} dw \tag{2.1}$$

$$H_d(w) = \sum_{n=-\infty}^{\infty} hd(n) e^{-jwn} \tag{2.2}$$

In general, unit sample response $hd(n)$ obtained from the above relation is infinite in duration, so it must be truncated at some point say $n= M-1$ to yield an FIR filter of length (i.e. 0 to $M-1$). This truncation of $hd(n)$ to length $M-1$ is same as multiplying $hd(n)$ by the rectangular window defined as

$$W(n) = a - b \cos(2p(n+1)/(N+1)) + c \cos(4p(n+1)/(N+1))$$

$$n = 0, 1, \dots, N-1$$

$$= 0 \text{ otherwise} \tag{2.3}$$

Thus the unit sample response of the FIR filter becomes

$$h(n) = h_d(n) w(n)$$

$$= hd(n) \quad 0 \leq n \leq M-1$$

$$= 0 \quad \text{otherwise} \tag{2.4}$$

Now, the multiplication of the window function $w(n)$ with $hd(n)$ is equivalent to convolution of $Hd(w)$ with $W(w)$, where $W(w)$ is the frequency domain representation of the window function

$$W(w) = \sum_{n=0}^{M-1} w(n) e^{-jwn} \tag{2.5}$$

Thus the convolution of $Hd(w)$ with $W(w)$ yields the frequency response of the truncated FIR filter

$$H(w) = 1/2\pi \int_{-\pi}^{\pi} H_d(v) W(w-v) dv \tag{2.6}$$

The frequency response can also be obtained using the following relation

$$H(w) = \sum_{n=0}^{M-1} h(n) e^{-jwn} \tag{2.7}$$

III. Optimal Parks McClellan

Although the rectangular windowing method provides the best mean-square approximation to a desired frequency response, just because it is optimal does not mean it is good. FIR filters designed with windows exhibit oscillatory behaviour around the discontinuity of the ideal frequency response and does not allow separate control of the pass band and stop band ripples[5]. An alternative FIR design technique is the Parks-McClellan algorithm which is based on polynomial approximations. Parks-McClellan method (also known as the Equiripple, Optimal, or Minimax method). To design a low pass filter, the desired frequency response is given by

$$H_d(w) = \begin{cases} 1 & 0 \leq w \leq w_p \\ 0 & w_s \leq w \leq \pi \end{cases} \tag{3.1}$$

For this system, consider $N = 51$, $W_s = 1$ rad/s and the corresponding frequency response, determined by

$$H_1(w) = h(25) + 2 \sum_{n=0}^{24} h(n) \cos w(25 - n) \tag{3.2}$$

There are various methods to determine the filter coefficients. Among these, the most widely used method is that of McClellan, Parks and Rabiner. Their program is

capable of designing optimal FIR filters including low-pass; high-pass, band-pass and band reject filters. We wish to design a low pass linear phase FIR frequency response $H1(\omega)$

IV. Genetic Algorithm

GA operates with a collection of chromosomes, called a population[11]. The population is normally randomly initialized. As the search evolves, the population includes fitter and fitter solutions, and eventually it converges, meaning that it is dominated by a single solution. Holland also presented a proof of convergence (the schema theorem) to the global optimum where chromosomes are binary vectors. GA use two operators to generate new solutions from existing ones: *crossover* and *mutation*. The crossover operator is the most important operator of GA. In crossover, generally two chromosomes, called *parents*, are combined together to form new chromosomes, called *offspring*. The parents are selected among existing chromosomes in the population with preference towards fitness so that offspring is expected to inherit good genes which make the parents fitter. By iteratively applying the crossover operator, genes of good chromosomes are expected to appear more frequently in the population, eventually leading to convergence to an overall good solution.

The mutation operator introduces random changes into characteristics of chromosomes [10]. Mutation is generally applied at the gene level. In typical GA implementations, the mutation rate (probability of changing the properties of a gene) is very small, typically less than 1%. Therefore, the new chromosome produced by mutation will not be very different from the original one. Mutation plays a critical role in GA. As discussed earlier, crossover leads the population to converge by making the chromosomes in the population alike. Mutation reintroduces genetic diversity back into the population and assists the search escape from local optima.

Reproduction involves selection of chromosomes for the next generation. In the most general case, the fitness of an individual determines the probability of its survival for the next generation. There are different selection procedures in GA depending on how the fitness values are used. Proportional selection, ranking, and tournament selection are the most popular selection procedures. Genetic algorithms find application in bioinformatics, phylogenetics, computational science, engineering, economics, chemistry, manufacturing, mathematics, physics and other fields. The genetic algorithm loops over an iteration process to make the population evolve. Each consists of the following steps:

4.1 SELECTION

The first step consists in selecting individuals for reproduction. This selection is done randomly with a probability depending on the relative fitness of the individuals so that best ones are often chosen for reproduction than poor ones.

4.2 REPRODUCTION

In the second step, offspring are bred by the selected individuals. For generating new chromosomes, the algorithm can use both recombination and mutations.

4.3 EVALUATION

Then the fitness of the new chromosomes is evaluated.

4.4REPLACEMENT

During the last step, individuals from the old population are killed and replaced by the new ones

In the basic Genetic Algorithm, to improve the fitness value of the chromosomes (represents a possible FIR filter) basic error functions are used. The chromosomes which have higher fitness values represent the better solutions. In the filter design the following error functions can be used:

Mean Squared Error (MSE), Least Mean Squared Error (LMS), Minimax Error or Mean Absolute Error (MAE) [5,6] The expressions of MSE and the MAE error functions are as follows,

$$MSE = \sum_f [Hd(f) - H(f)]^2 \tag{4.1}$$

$$MAE = \sum_f [Hd(f) - H(f)] \tag{4.2}$$

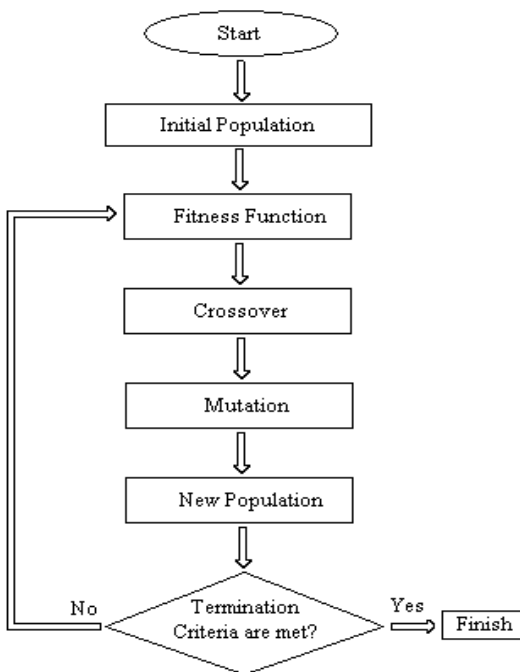


Fig:-1 A flowchart of the GA

When the error functions are used directly as they are in Equation (4.1) or Equation (4.2), it is seen that the magnitude response of the designed filter can not be efficiently optimized as expected and the minimum delay, namely minimum phase[11], can not be provided. It is assumed that the number of zeros that causes the non minimum phase is q and the error function used is $e(f)$, then the objective function to be minimized which is able to provide the minimum phase condition can be defined as,

$$\psi(f) = e(f) + w.q \tag{4.3}$$

w = weight parameter

In expression (4), w has to be selected high enough. As the number of zeros that causes the non minimum phase increases, the effect of these zeros on the error function will increase proportionally. Hence, by means of objective function the zeros which are located out of the unit circle are pulled into to the inside of the unit circle. When all the zeros are pulled inside the unit circle, the error function will be equal to the objective function since q=0.

The fitness evaluation function used in this section is given by Equation (4.4).

$$Fitness = 1/\psi(f) \tag{4.4}$$

After several trials, it is seen that the most appropriate value for the parameter w in Equation (4.3) is 100. When the value of w is chosen too high, the value of the zero term becomes dominant on objective function and therefore, the GA might have difficulties with converging and finally could not reach the optimum solution. When the value of w is chosen too low, the influence of the zero term on the objective function becomes too small and hence the GA ignores this zero term and the designed filter might become non-minimum phase.

V. Simulation and Results:-

In simulation number of sample point (N)=65, Filter Order=31, Ws=.4580, Wp=.341, Pc=.60, Pm=.01, Size=1000 and generation=30, crossover prob.= .60, mutation prob.= .01

5.1 Ideal Magnitude Response

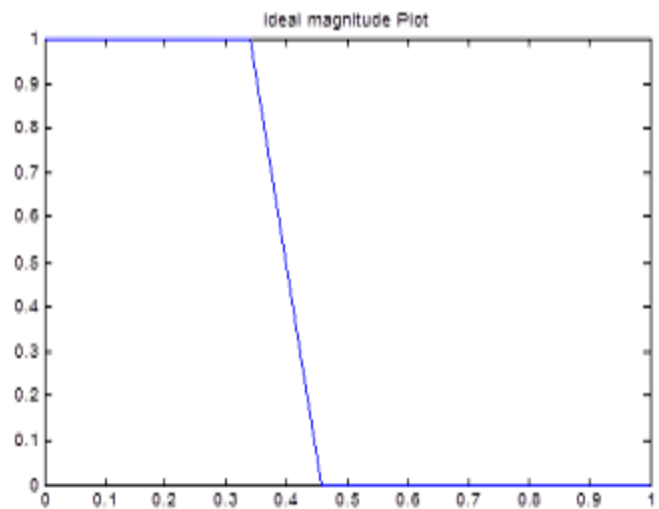


Fig:-2

5.2 Magnitude response of GA

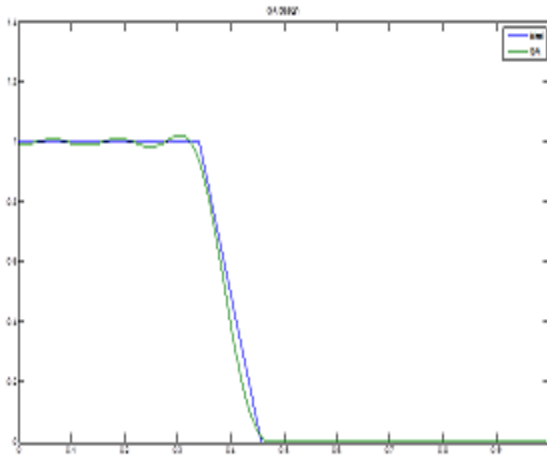


Fig:-3

5.3 Comparison between the GA, Parks McClellan and Hamming Window on the basis of Magnitude Response

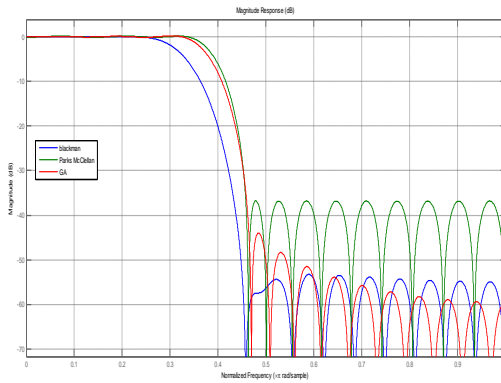


Fig:-4

5.4 Comparison between the GA, Parks McClellan and Hamming Window on the basis of Phase Response

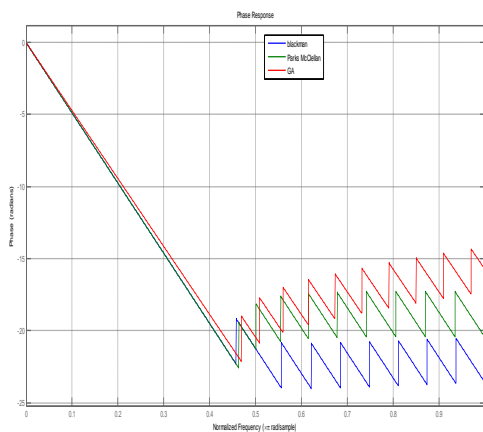


Fig:-5

5.4 Comparisons

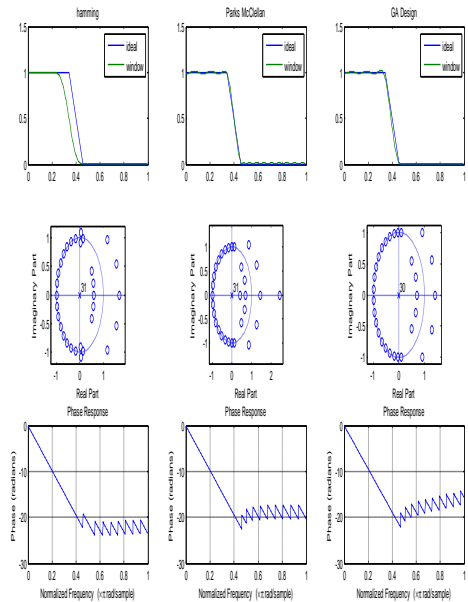


Fig:-6

VI. Conclusion

This paper present various optimization techniques and non optimization techniques for design of low pass FIR digital filters. In this paper we design low pass FIR digital filter with Hamming Window, Parks-McClellan algorithm and Genetic Algorithm techniques.

Design examples presented in the paper have indicated that GA is Best technique as compare to other techniques. Comparison is done on the basis of three factors 1.)Magnitude response 2) Phase response 3.) Phase delay

In this paper from magnitude response it shows that transition bandwidth is very less in GA and has flat pass band and stop band as compare to other techniques (Fig:-4) Phase response approaches to Zero in GA (Fig:-5) Most of the Zeros are laying inside the unit circle in GA. Therefore Phase delay is minimum(Fig:-6) Hence result from GA is best as compare to others techniques.

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