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Sparse Mixing Estimators for Single Image Super-Resolution

Mr. S.H. Jagtap*, Prof. M.M Patil, Prof. S.D. Ruikar
Department of E&Tc Engineering
SAE Kondhwa (Bk), Pune University
jagtap.somnath@gmail.com

Abstract - **This paper presents comparative study of different single image SR algorithms and takes deep drive on a new approach to single-image super-resolution, based upon Sparse Mixing Estimators. We introduce a class of inverse problem estimators computed by mixing adaptively a family of linear estimators corresponding to different priors. Sparse mixing weights are calculated over blocks of coefficients in a frame providing a sparse signal representation. They minimize an l^1 norm taking into account the signal regularity in each block. Adaptive directional image interpolations are computed over a wavelet frame with an algorithm, providing state-of-the-art numerical results [1]. This algorithm generates high-resolution images that are competitive or even superior in quality to images produced by other similar SR methods.**

Keywords— **Super-resolution (SR), Interpolation, Wavelet, mixing estimator, Sparse representation.**

I. INTRODUCTION

Super-Resolution (SR) is a technique to increase the resolution of an image or a sequence of images beyond the resolving power of the imaging system. The resolution of an image is defined as the amount of fine details that are visible. Nowadays Super-resolution image reconstruction is a active area of research, as it is capable of overcoming some resolution limitations of low-cost imaging sensors such as Cell phone cameras. This allows better utilization of the growing capability of high-resolution displays such as high-definition LCDs. Such resolution enhancing technology is important in medical imaging and satellite imaging where diagnosis or analysis from low-quality images can be extremely difficult.

Conventional approaches to generating a super-resolution image normally require as input multiple low-resolution images of the same scene, which are aligned with sub-pixel accuracy [2]. The SR task is cast as the inverse problem of recovering the original high-resolution image by fusing the low-resolution images, based on reasonable assumptions or prior knowledge about the observation model that maps the high-resolution image to the low-resolution ones. The fundamental reconstruction constraint for SR is that the recovered image, after applying the same generation model, should reproduce the observed low-resolution images. However, SR image reconstruction is generally a severely ill-posed problem because of the insufficient number of low-resolution images, ill-conditioned registration and unknown blurring operators and the solution from the reconstruction constraint is not unique [3], [4], [5], [6]. Various regularization methods have been proposed to further stabilize the inversion of this ill-posed problem. However, the performance of these reconstruction-based SR algorithms degrades rapidly when the desired magnification factor is large or the number of available input images is small. Another SR approach is based upon interpolation, while simple interpolation methods such as Nearest Neighbour interpolation, Bilinear interpolation, Bicubic interpolation and Cubic spline interpolation tend to generate overly smooth images with ringing and jagged artefacts, interpolation by exploiting the natural image priors will generally produce more favourable results [7].

Signal acquisition and restoration often requires solving an inverse problem. It amounts to estimate a signal f from measurements y, obtained through a linear operator U, and contaminated by an additive noise w

$$y = Uf + w$$

Image interpolation is an important example, where U is a sub-sampling operator. Many image display devices have zooming abilities that interpolate input images to adapt their size to high resolution screens. For example, high definition televisions include a spatial interpolator which increases the size of standard definition videos to match the high definition screen format and possibly improve the image quality. This paper introduces a general class of nonlinear inverse estimators obtained with an adaptive mixing of linear estimators, with applications to image interpolation.

Many families of linear interpolators have been studied, producing interpolation errors which depend upon the amount of aliasing. Bicubic or cubic spline interpolators most often provide nearly the best results among linear operators [8]. Nonlinear algorithms can take into account more adaptive image models which often improve linear interpolators, in which case they are loosely called super-resolution algorithms. Directional image interpolations take advantage of the geometric regularity of image structures by performing the interpolation in a chosen direction along which the image is locally regular. The main difficulty is to locally identify this direction of regularity. Along an edge, the interpolation direction should be parallel to the edge. Many adaptive interpolations have been, thus, developed with edge detectors [9] and by finding the direction of regularity with gradient operators [10], [11]. More global image models impose image smoothness priors such as a bounded total variation to optimize the interpolation [12]. Other image smoothness priors have also been used to compute interpolated images with alternate projections on convex sets [15]. These algorithms can provide a better image quality than a linear interpolator, but they also produce artefacts so that the resulting PSNR remains of the same order. The introductions of interpolators adapted to local covariance image models have lead to more precise estimators [16]. This approach has been improved by Zhang and Wu [17] by using autoregressive image models optimized over image blocks. In most cases, it currently provides the best PSNR for spatial image interpolation. Super-resolution interpolations can further be improved by using a sequence of images [18] or a comparison dataset [19] to perform the interpolation. While these approaches can be more accurate, they are much more demanding in computation and memory resources.

Prior information on the image sparsity has also been used for image interpolation. Wavelet estimators were introduced to compute fine scale wavelet coefficients by extrapolating larger scale wavelet coefficients [20]. A more general and promising class of nonparametric super-resolution estimators assumes that the high resolution signal *f* is sparse in some dictionary of vectors. This sparse representation is estimated by decomposing the low-resolution measurements *y* in a transformed dictionary. These algorithms, which are reviewed in Section II, have found important applications for sparse spike inversion in geophysics or image inpainting [21]. However, they do not provide state-of-the-art results for image interpolation.

Section III describes a new class of adaptive inverse estimators, calculated over a sparse signal representation in a frame. It is obtained with a sparse adaptive mixing of a family of linear Tikhonov estimators, which are optimized for different signal priors. Mixing linear estimators has been shown to be very effective for noise removal. However, these approaches do not apply to inverse problems because they rely on a Stein unbiased empirical estimator of the risk, which is then not valid.

Our inverse sparse mixing estimator is derived from a mixture model of the measurements *y*. It is computed in Section IV by minimizing an l^1 norm over blocks of frame coefficients, with weights depending upon quadratic signal priors. Section V describes a fast orthogonal block matching pursuit algorithm which computes the mixing weights. Linear mixture models have been studied over wavelet coefficients for image denoising [22].

II. SPARSE INVERSE PROBLEM ESTIMATION IN DICTIONARIES

Sparse super-resolution estimations over dictionaries provide effective nonparametric approaches to inverse problems, but whose flexibility may become a source of errors. These algorithms are reviewed with their application to image interpolation.

A signal $f \in R^N$ is estimated by taking advantage of prior information which species a dictionary $\Phi \in R^{N \times P}$ having *P* columns corresponding to vectors $\{\Phi_P\}_{1 \leq p \leq P}$ where *f* has a sparse approximation. This dictionary may be a basis or some redundant frame, with $P \geq N$. Sparsity means that *f* is well approximated by its orthogonal projection $f\wedge$ over a subspace $V\wedge$ generated by a small number $M = |\wedge|$ of column vectors $\{\Phi_P\}_{p \in \wedge}$ of Φ, which can be written

$$f\wedge = \Phi(c1\wedge) \quad (1)$$

Where $1\wedge$ is the indicator of the approximation set $\wedge$, c is the transform coefficient vector, $c1\wedge$ selects the coefficients in $\wedge$ and sets the others to zero, and $\Phi(c1\wedge)$ multiplies the matrix Φ with the vector $c1\wedge$.

Measurements are obtained with a linear operator *U*, with an additive noise *w*

$$y = Uf + w \quad (2)$$

If the number of measurements is larger than the size *M* of the approximation support $\wedge$, then sparse inversion algorithms try to estimate $\wedge$ and the coefficients c in $\wedge$ that specify the projection of *f* in the approximation space $V\wedge$. It results from (1) and (2) that

$$y = U\Phi(c1\wedge) + w` \text{ with } w` = U(f - f\wedge) + w \quad (3)$$

This means that *y* is well approximated by the same sparse set of coefficient vector *c* over $\wedge$ in the transformed dictionary $U\Phi$, whose columns are the transformed vectors $\{\Phi_P\}_{1 \leq p \leq P}$. This vector c is, thus, estimated by finding a sparse representation of *y* in $U\Phi$. Since *U* is not an invertible operator, this transformed dictionary is redundant, with columns vectors which are linearly dependent. It results *y* that has an infinite number of possible decompositions in this dictionary.

A sparse approximation $\tilde{y}=U\Phi\tilde{c}$ of y can be calculated with a basis pursuit algorithm which minimizes a Lagrangian penalized by an l^1 norm [23]

$$\frac{1}{2}\|y - U\Phi\tilde{c}\|^2 + \lambda\|\tilde{c}\|_1 \quad (4)$$

A sparse representation of y can also be calculated with faster greedy matching pursuit algorithms [22]. The resulting sparse estimation of f is $\tilde{f}=\Phi\tilde{c}$. The Restrictive Isometry Property of Candes and Tao [24] and Donoho [25] is a strong sufficient condition which guarantees that the penalized l^1 estimation is precise and stable. This restrictive isometry property is valid for certain classes of random operators U but not for structured operators such as a subsampling on a uniform grid. For structured operators, the precision and stability of this sparse inverse estimation depends upon the "geometry" of the approximation support $\wedge$ of f, which is not well understood mathematically, despite some sufficient exact recovery conditions proved by Tropp [26].

Several authors have applied this sparse super-resolution algorithm for image interpolation and inpainting. Curvelet frames and contourlet frames build sparse image approximations by taking advantage of the image directional regularity. Dictionaries of curvelet frames have been applied successfully to image inpainting [21]. For uniform grid interpolations, Table I in Section VI shows that the resulting interpolation estimations are not as precise as linear bicubic interpolations. Table I shows that a contourlet algorithm [27] can provide a slightly better PSNR than a bicubic interpolation, but these results are below the state-of-the-art obtained with adaptive directional interpolators [17]. Dictionaries of image patches have also been studied for image interpolations with sparse representations [3], but with little PSNR improvements compared to bicubic interpolations.

A source of instability of these algorithms come from their flexibility, which does not incorporate enough prior information. The approximation space $V\wedge$ is estimated by selecting the approximation support $\wedge$, with no constraint. A selection of M vectors, thus, corresponds to a choice of an approximation space among $\binom{P}{M}$ possible subspaces. It does not take into account geometric image structures which create dependencies on the choice of approximation vectors. Structured approximation algorithms use such prior information to restrict the set of possible approximation spaces [28]. Since approximation vectors often appear in groups, one can select simultaneously blocks of approximation vectors [29], which reduces the number of possible approximation spaces. The l^1 penalization in (4) is then replaced by a sum of the l^2 norm over each block, which results in a mixed l^1 and l^2 norm. This is also called a "group lasso" optimization [31]. These structured approximations have been shown to improve the signal estimation in a compressive sensing context for a random operator U [32]. However, for more unstable inverse problems such as image interpolation, this regularization is not sufficient to reach state-of-the-art results. The algorithm described below computes an adaptive signal representation in blocks, but to do so it also performs a strong linear regularization in each block, which is necessary to obtain accurate estimations for highly unstable inverse problems. In the context of image zooming, this linear regularization depends upon the directional image regularity.

III. MIXING ESTIMATORS OVER FRAME BLOCKS

Sparse super-resolution algorithms can be improved by using more prior information on the signal properties. This section introduces a general class of sparse inverse estimators, which are obtained as an adaptive mixing of linear Tikhonov estimators, over blocks of vectors in a frame.

A. *Linear Tikhonov Estimation*

A Tikhonov regularization optimizes a linear estimator by imposing that the solution has a regularity specified by a quadratic prior [23]. Suppose that f has a regularity which is measured by a quadratic regularity norm $\|R_{\acute{\emptyset}} f\|$, where is a linear $R_{\acute{\emptyset}}$ operator. Sobolev norms are particular examples where $R_{\acute{\emptyset}}$ is a differential operator. Let σ^2 be the variance of the noise w and Q be the number of samples. A Tikhonov estimator computes $\tilde{f}=U_{\acute{\emptyset}}^{+}y$ by minimizing $\|R\tilde{f}\|^2$ subject to

$$\frac{1}{Q}\|U\tilde{f} - y\|^2 \leq \sigma^2 \quad (5)$$

The solution of this quadratic minimization problem is also obtained by minimizing a Lagrangian

$$\frac{1}{2}\|U\tilde{f} - y\|^2 + \lambda\,\|R\tilde{f}\|^2 \quad (6)$$

In Bayesian terms, this Lagrangian is minus the log of the posterior distribution of the signal given the observations y, whose minimization yields a maximum a posterior estimator.

B. *Mixing Estimation Over Blocks*

An adaptive mixing estimation is computed from a family of Tikhonov estimators $\{U_{\acute{\emptyset}}^{+}\}_{\acute{\emptyset}\,\epsilon\,\Theta}$ optimized for different regularity operators $\{R_{\acute{\emptyset}}^{+}\}_{\acute{\emptyset}\,\epsilon\,\Theta}$. For image interpolation, $U_{\acute{\emptyset}}^{+}$ is an interpolator in a direction $\acute{\emptyset}$. A mixing estimation is obtained by weighting the different estimators, emphasizing some over others, depending upon the local signal regularity. For image interpolations, it amounts to choose preferential directions of interpolation.

A local mixture decomposition represents a signal y as a combination of block components plus a residue yr.

$$y = \sum_{B\,\epsilon\,ß} \tilde{a}(B) y_B + yr \quad (7)$$

Each block is selected if the mixing coefficient $\tilde{a}$ (B) is close to 1, which means that $\|\tilde{R}_{ByB}\|$ is small and it is removed if $\tilde{a}$ (B) is close to 0.

The residual signal yr does not have a preferential regularity with respect to any $R_{\acute{\emptyset}}$. A mixture estimator is defined from a mixture model (7) by inverting each block component y_B with U^+_B and the residue with a generic estimator U^+

$$\tilde{f} = \sum_{B \in ß} \tilde{a}(B) y_B + yr \quad (8)$$

Inserting $y_B = \tilde{\psi}(c1_B)$ together with (7) yields a mixing estimator which locally adapts the inverse operator to the signal regularity.

$$\tilde{f} = U^+ y + \sum_{B \in ß} \tilde{a}(B)(U^+_B - U^+)\ \tilde{\psi}(c1_B) \quad (9)$$

It computes a linear estimation with U^+ and this estimation is modified by adding the estimation difference relatively to the operator U^+_B adapted to each block *B*. For each block $B = B_{\acute{\emptyset},q}$ the estimator is $U^+_B = U^+_{\acute{\emptyset}}$ so (9) can be rewritten

$$\tilde{f} = U^+ y + \sum_{\acute{\emptyset} \in \Theta} (U^+_{\acute{\emptyset}} - U^+)\ \tilde{\psi}(\sum_{q \in \acute{\Gamma}\Theta} \tilde{a}(B_{\acute{\emptyset},q}) 1_{B\acute{\emptyset},q} C) \quad (10)$$

The generic linear estimator U^+ does not incorporate any prior knowledge concerning the signal regularity. In a Bayesian framework, U^+ is an estimator computed with a prior Gaussian distribution whose covariance is not conditioned on a particular block B. It can be computed from all the covariances $(R^*_B R_B)^{-1}$ $(R^*_{\acute{\emptyset}} R_{\acute{\emptyset}})^{-1}$ and from the probability distribution of $\acute{\emptyset}$, if it is known. For image interpolation, U^+ can be chosen to be a separable interpolation which is nearly isotropic, such as a bicubic interpolator.

IV. SPARSE MIXTURE ESTIMATION

The previous section explains to derive a mixing estimator $\tilde{f}$ of a signal *f* from a mixture model of the input data y = Uf + w. This mixture model is calculated from the frame coefficients c = ψy of *y* of and mixing coefficients $\tilde{a}$ (B) defined over a family blocks B.

$$y = \sum_{B \in ß} \tilde{a}(B) y_B + yr \text{ with } y_B = \tilde{\psi}(c1_B) \quad (11)$$

The main issue is to compute mixing coefficients $\tilde{a}$ (B) which optimize the inverse problem mixing estimator $\tilde{f}$. The mixing parameters $\tilde{a}$ (B) should be close to 1 if the local signal component y_B has a quadratic regularity that is compatible with the prior used to optimize the linear Tikhonov estimator U^+_B and, hence, if $\|\tilde{R}_{ByB}\|$ is small. A linear mixture estimator could, thus, be obtained by minimizing the residue energy $\| y_r \|$ penalized by the signal regularity over all blocks measured by

$$\sum_{B \in ß} \| \tilde{R}_B \tilde{a}(B) y_B \|^2 = \sum_{B \in ß} |\tilde{a}(B)|^2 \| \tilde{R}_B y_B \|^2 \quad (12)$$

However, this approach does not take advantage of another important prior. For each frame vector indexed by, there is typically only one block *B* which includes *p* and whose regularity operator $\tilde{R}_B$ is compatible with the regularity of *y* over *B*. As a consequence, all $\tilde{a}$(B) with $p \in B$ should be close to zero but one. According to the sparsity approach reviewed in Section II, the quadratic prior norm on in (12) is, thus, replaced by l^1an norm. Mixing coefficients are obtained by minimizing the residual norm penalized by the $\| y_r \|^2$ resulting weighted l^1 prior

$$Ł(\tilde{a}) = \frac{1}{2} \| y - \sum_{B \in ß} \tilde{a}(B) y_B \|^2 + \lambda \sum_{B \in ß} |\tilde{a}(B)| \, \| \tilde{R}_B y_B \|^2 \quad (13)$$

The minimization of such a quadratic function of the unknown $\tilde{a}$ (B) penalized by their l^1 norm can be computed with standard algorithms, such as an iterative thresholding [55].

As opposed to group lasso algorithms using mixed l^1 and l^2 norms, this minimization does not only recover the signal witha sparse set of blocks but it also regularizes the decomposition by imposing a signal regularity within each block. Moreover, it does not optimize a decomposition parameter for each frame coefficient but a single mixing parameter per block. The blocks *B* should not be too large to maintain enough flexibility in the choice of the U^+_B. However, each block $B = B_{\acute{\emptyset},q}$ of parameter $\acute{\emptyset}$ must also be large enough and have a shape that is adapted to $R_{\acute{\emptyset}}$ so that one can indeed "observe" the signal has this specific regularity in *B*. The means that the restriction of $\tilde{R}^*_{\acute{\emptyset}} \tilde{R}_{\acute{\emptyset}}$ over *B* must have eigenvalues that decrease like the first eigenvalues of the nonrestricted operator. For example, if $U^+_B = U^+_{\acute{\emptyset}}$ is a directional interpolation in the direction of $\acute{\emptyset}$ and if $R_{\acute{\emptyset}}$ measures the signal regularity in the direction of $\acute{\emptyset}$, then evaluating this regularity requires to use blocks that are sufficiently long in the direction of $\acute{\emptyset}$. To minimize the overall block size, one can use elongated blocks in the direction of $\acute{\emptyset}$.

The estimation also depends upon each grid $\acute{\Gamma}_{\acute{\emptyset}}$ of the position indexes of the blocks $B = B_{\acute{\emptyset},q}$. To reduce this grid effect, the estimation can be computed with several sets of translated grids. Each grid is translated by several vectors : $\{\acute{\Gamma}_{\acute{\emptyset},i}\}_{1 \le i \le I}$. $\acute{\Gamma}_{\acute{\emptyset},i} = + \Gamma_{\acute{\emptyset},i}$. For each *i*, mixing coefficients $\tilde{a}_i(B_{\acute{\emptyset},q})$ are computed with blocks *B* translated on the grid $\acute{\Gamma}_{\acute{\emptyset},i}$. The grids typically have a nonzero intersection and the final estimator is obtained by averaging these mixing coefficients

$$\tilde{a}(B) = \frac{1}{I} \sum_{i=1}^{I} \tilde{a}_i(B) \quad (14)$$

V. COMPUTATIONS AND ORTHOGONAL BLOCK MATCHING PURSUIT

To reduce the computation of mixing coefficients, an upper bound of the Lagrangian (13) is computed from the frame coefficients of *f* in each block. Efficient algorithms have been developed for l^1 minimization but they remain slow for image processing applications. An orthogonal block matching pursuit algorithm is introduced to approximate the optimization, with much less computations.

The Euclidean norm of coefficients in a block is written:

$$\|c\|_B^2 = \sum_{p \in B} |c(p)|^2$$

For frame coefficients c = *ψy*, we define a Lagrangian

$$Ł(\tilde{a}) = \frac{1}{2} \| c\,(1 - \sum_{B \in ß} \tilde{a}(B) 1_B) \|^2 + \lambda / A^2 \sum_{B \in ß} |\tilde{a}(B)| \, \| \tilde{R}_B c \|_B^2 \quad (15)$$

where *A* is the norm of the dual frame operator *ψ*.

$$Ł_1(\tilde{a}) = \frac{1}{2} (\| c \|^2 - \sum_{l=1}^{L} e(c, B_l))$$

At each iteration, to minimize $Ł_1(\tilde{a})$, an orthogonal block matching pursuit finds B_l which maximizes *e(c,B)* among all blocks that do not intersect with the previously selected blocks. The resulting algorithm is described in the following.

1) Initialization: set *l = 0* and compute

$$\forall B \in ß, \ e(c,B) = \| c \|_B^2 \, p(c,B)^2 \text{ and set } \tilde{a}(B) = 0 \quad (16)$$

2) Maxima finding

$B_l = \arg \max_{B \in ß} e(c,B)$ and set $\tilde{a}(B_l) = p(c,B_l)$

3) Energy update.

If $e(c,B_l) > T$ then eliminate all blocks that intersect with B_l.
$\forall B \in ß$ with $B \cap B_l \neq Ǿ$ set e(B) = 0 set l = l+1 and go back to step (2).
Otherwise stop.

This algorithm stops when there is no sufficiently energetic block compared to a precision threshold T, which is typically chosen to be proportional to the noise variance. In the image interpolation numerical experiments, we set $T = 0$ because the noise is neglected.

VI. RESULTS

In this section, we first demonstrate the SR results obtained by applying the previously mentioned methods and our method on different images. Then we show PSNRs and RMSEs of all SR methods.

Fig.4 (a) Fig.4 (b) Fig.4 (c) Fig.4 (d) Fig.4 (e) Fig.4 (f)

Fig. 4 shows results for Girl image.
Fig.4 (a) is down-sampled version of original image. Fig.4 (b) is High-Resolution image obtained from Nearest Neighbour Interpolation method. Fig.4 (c) is High-Resolution image obtained from Bilinear Interpolation method. Fig.4 (d) is High-Resolution image obtained from Cubic Spline Interpolation method and Fig.4 (e) is High-Resolution image obtained from SME method and Fig.4 (f) is the original High-resolution image.

Fig.5 (a) Fig.5 (b) Fig.5 (c) Fig.5 (d) Fig.5 (e) Fig.5 (f)

Fig. 5 shows results for Flower image.
Fig.5 (a) is down-sampled version of original image. Fig.5 (b) is High-Resolution image obtained from Nearest Neighbour Interpolation method. Fig.5 (c) is High-Resolution image obtained from Bilinear Interpolation method. Fig.5 (d) is High-Resolution image obtained from Cubic Spline Interpolation method and Fig.5 (e) is High-Resolution image obtained from SME method and Fig.5 (f) is the original High-resolution image.

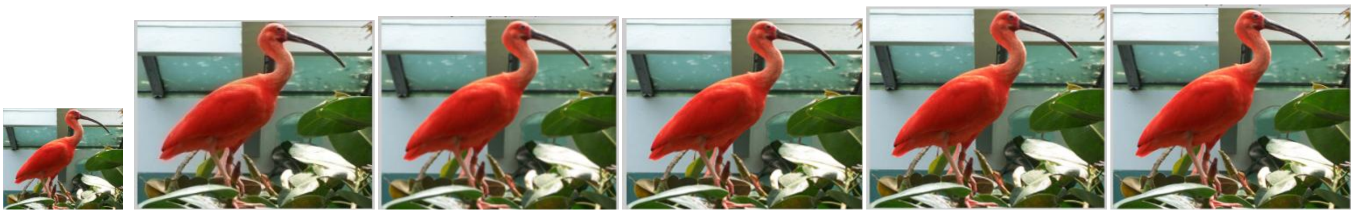

Fig.6 (a) Fig.6 (b) Fig.6 (c) Fig.6 (d) Fig.6 (e) Fig.6 (f)

Fig. 6 shows results for Bird image.
Fig.6 (a) is down-sampled version of original image. Fig.6 (b) is High-Resolution image obtained from Nearest Neighbour Interpolation method. Fig.6 (c) is High-Resolution image obtained from Bilinear Interpolation method. Fig.6 (d) is High-Resolution image obtained from Cubic Spline Interpolation method and Fig.6 (e) is High-Resolution image obtained from SME method and Fig.6 (f) is the original High-resolution image.

Table 1
PSNRs of reconstructed images using different methods.

	Nearest Neighbour Interpolation	**Bilinear Interpolation**	**Cubic Spline Interpolation**	**SME SR**
Girl Image	23.9094	24.9193	29.8088	30.3110

Flower Image	28.6315	29.7559	33.2613	33.6437
Bird Image	25.5594	27.1435	29.2133	30.4222

Table 2
RMSEs of reconstructed images using different methods.

	Nearest Neighbour Interpolation	**Bilinear Interpolation**	**Cubic Spline Interpolation**	**SME SR**
Girl Image	16.2582	14.4736	8.2433	7.7802
Flower Image	9.4398	8.2936	5.5396	5.3010
Bird Image	13.4452	11.2039	8.8283	7.6812

VII. CONCLUSIONS

In this paper we introduces a new class of adaptive estimators obtained by mixing a family of linear inverse estimators, derived from different priors on the signal regularity. Mixing coefficients are calculated in a frame over blocks of coefficients having an appropriate regularity and providing a sparse signal representation. Experimental results demonstrate the effectiveness of the this approach for different images.

REFERENCES

[1] Stephane Mallat, Guoshen Yu *"Super-Resolution With Sparse Mixing Estimators"* in IEEE transactions on image processing, VOL. 19, NO. 11, NOVEMBER 2010.

[2] S. Farsiu, M. D. Robinson, M. Elad, and P. Milanfar, *"Fast and robust multiframe super-resolution"* IEEE Trans. Image Process., vol. 13, no.10,pp. Oct.2004.

[3] Jianchao Yang, John Wright, Thomas S. Huang, Yi Ma *"Image Super-Resolution Via Sparse Representation"* in IEEE transactions on image processing, VOL. 19, NO. 11, NOVEMBER 2010.

[4] J. Yang, J. Wright, T. Huang, and Y. Ma *"Image super-resolutions sparse representation of raw image patches"* in Proc. IEEE Conf.Comput. Vis. Pattern Recognit., 2008, pp. 1–8.

[5] Qi Pan, Chengying Gao, Ning Liu, Jiwu Zhu *"Single Frame Image Super-resolution Based on Sparse Geometric Similarity"* Journal of Information & Computational Science 7: 3 (2010) 799–805

[6] Roman Zeyde, Michael Elad and Matan Protter *"On Single Image Scale-Up using Sparse-Representations"* The Computer Science Department Technion – Israel Institute of Technology – Haifa 32000

[7] H. S. Hou and H. C. Andrews, *"Cubic spline for image interpolation and digital filtering"* IEEE Trans. Signal Process., vol. 26, no. 6, pp. 508–517, Dec. 1978.

[8] R. Keys, "*Cubic convolution interpolation for digital image processing*" *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP–29, no. 6, pp. 1153–1160, Dec. 1981.

[9] J. Allebach and P. W. Wong, "*Edge-directed interpolation," in Proc. Int. Conf. Image Process"* 1996, vol. 3, pp. 707–710.

[10] S. D. Bayrakeri and R. M. Mersereau, "*A new method for directional image interpolation*" in *Proc. Int. Conf. Acoust., Speech, Signal Process.*, 1995, vol. 4, pp. 2383–2386.

[11] A. Biancardi, L. Cinque, and L. Lombardi, "*Improvements to image magnification*" *Pattern Recognit.*, vol. 35, no. 3, pp. 677–687, 2002.

[12] F. Malgouyres and F. Guichard, "*Edge direction preserving image zooming: A mathematical and numerical analysis*" *SIAM J. Numer. Anal.*, pp. 1–37, 2002.

[13] D. Calle and A. Montanvert, "*Super-resolution inducing of an image*" in *Proc. IEEE Int. Conf. Image Process.*, 1998, vol. 3, pp. 232–235.

[14] A. Biancardi, L. Cinque, and L. Lombardi, "*Improvements to image magnification*" *Pattern Recognit.*, vol. 35, no. 3, pp. 677–687, 2002.

[15] D. Calle and A. Montanvert, "*Super-resolution inducing of an image*" in *Proc. IEEE Int. Conf. Image Process.*, 1998, vol. 3, pp. 232–235.

[16] X. Li and M. T. Orchard, "*New edge-directed interpolation*" *IEEE Trans. Image Process.*, vol. 10, no. 10, pp. 1521–1527, Oct. 2001.

[17] X. Zhang and X. Wu, "*Image interpolation by adaptive 2-d autoregressive modeling and soft-decision estimation*" *IEEE Trans. Image Process.*, vol. 17, no. 6, pp. 887–896, Jun. 2008.

[18] M. Protter, M. Elad, H. Takeda, and P.Milanfar, "*Generalizing the nonlocal-means to super-resolution reconstruction*" *IEEE Trans. Image Process.*, vol. 18, no. 1, pp. 36–51, Jan. 2009.

[19] M. Elad and D. Datsenko, "*Example-based regularization deployed to super-resolution reconstruction of a single image*" *Comput. J.*, vol. 52, pp. 15–30,2007.

[20] W. K. Carey, D. B. Chuang, and S. S. Hemami, "*Regularity-preserving image interpolation*" *IEEE Trans. Image Process.*, vol. 8, no. 9, pp.

[21] M. Elad, J. L. Starck, P. Querre, and D. L. Donoho, "*Simultaneous cartoon and texture image inpainting using morphological component analysis (MCA)*" *Appl. Comput. Harmon. Anal*, vol. 19, pp. 340–358, 2005.

[22] J. Portilla and E. Simoncelli, "*Image restoration using gaussian scale mixtures in the wavelet domain*" in *Proc. IEEE Int. Conf. Image Process.*, 2003, pp. 965–968.

[23] A. N. Tikhonov, "*Solution of incorrectly formulated problems and the regularization method*" *Soviet Math. Dokl*, vol. 4, pp. 1035–1038,1963.

[24] D. L. Donoho, "*Compressed sensing*" *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, Apr. 2006.

[25] S. S. Chen,D. L.Donoho, andM.A. Saunders, "*Atomic decomposition by basis pursuit*" *SIAM J. Sci. Comput.*, vol. 20, pp. 33–61, 1999.

[26] J. A. Tropp, "*Just relax: Convex programming methods for identifying sparse signals in noise*" *IEEE Trans. Inf. Theory*, vol. 52, no. 3, pp. 1030–1051, Mar. 2006.

[27] A. Muñoz, T. Blu, and M. Unser, "*Least-squares image resizing using finite differences*" *IEEE Trans. Image Process.*, vol. 10, no. 9, pp. 1365–1378, Sep. 2001.

[28] J. Huang, T. Zhang, and D. Metaxas, "*Learning With structured sparsity*" *arXiv:0903.3002v1*, 2009.

[29] M. Stojnic, F. Parvaresh, and B. Hassibi, "*On the reconstruction of block-sparse signals with an optimal number of measurements*" *IEEE Trans. Signal Process.*, vol. 57, no. 8, pp. 3075–3085, Aug. 2009.

[30] H. Zou and T. Hastie, "*Regularization and variable selection via the elastic net*" *J. Roy. Statist. Soc. Ser. B*, vol. 67, no. 2, pp. 301–320,2005.

[31] M. Yuan and Y. Lin, "*Model selection and estimation in regression with grouped variables*" *J. Roy. Statist. Soc. Ser. B*, vol. 68, no. 1, pp. 49–67, 2006.

[32] R. G. Baraniuk,V. Cevher, M. F.Duarte, and C. Hegde, *"Model-Based Compressive Sensing" preprint*, 2008.

[33] I. Daubechies, M. Defrise, and C. De Mol, "*An iterative thresholding algorithm for linear inverse problems with a sparsity constraint*" *Comm. Pure Appl. Math.*, vol. 57, pp. 1413–1457, 2004.