



Modified ElGamal over RSA Digital Signature Algorithm (MERDSA)

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Abstract-Generally digital signature algorithms are based on a single hard problem like prime factorization problem, discrete logarithm problem, elliptic curve problem. If one finds solution of this single hard problem then these digital signature algorithms will no longer be secured and due to large computational power, this may be possible in future. There are many other algorithms which are based on the hybrid combination of prime factorization and discrete logarithms problem but different weaknesses and attacks have been developed against those algorithms. This paper also presents a new variant of digital signature algorithm which is based on two hard problems, prime factorization and discrete logarithm.

KEYWORD-Digital Signature; Discrete logarithm; Factorization; Cryptanalysis

I. INTRODUCTION

In modern cryptography [5], the security of digital signature algorithms are based on the difficulty of solving some hard number theoretical problems. These algorithms stay secure as long as the problem, on which the algorithm is based, stays unsolvable. The most used hard problems for designing a signature algorithm are prime factorization (FAC) [27] and Discrete Logarithm (DL) [6] problems. For improving the security, the algorithms may be designed based on multiple hard problems. Undoubtedly, the security of such algorithms is longer than algorithms based on a single problem. This is due to the need of solving both the problems simultaneously. Many digital signature algorithm have been designed based on both FAC and DL [8, 11, 12, 14, 17, 19, 26, 28, 30, 31] but to design such algorithms is not an easy task since many of them have been shown to be insecure [9, 18, 19, 20, 21, 29, 30, 31]. In 1994, He and Kiesler [11] proposed digital signature algorithms based on two hard problems-the prime factorization problem and the discrete logarithm problem. In 1995, Harn [9] showed that one can break the He-Kiesler algorithm if one has the ability to solve the prime factorization. Lee and Hwang [18] showed that if one has the ability to solve the discrete logarithms, one can break the He-Kiesler algorithm. Shimin Wei [31] showed that any attacker can forge the signature of He-Kiesler algorithm without solving any hard problem. In 2002, Z. Shao [28] presents an algorithm based on factoring and discrete logarithms. But later Tzeng [30] showed that Shao digital signature algorithm is not secure and there are many weaknesses. He then proposed a new signature algorithm [30] to overcome the weaknesses inherent in Shaos signature algorithm. In 2005, Shao [29] proved that Tzeng signature algorithm is not secure as if attackers can solve discrete logarithm problems, they can easily forge the signature for any message by using a probabilistic algorithm proposed by Pollard and Schnorr [24] and if attacker can factor the composite number, he can recover the private keys of legal signers. Therefore the security of Tzeng digital signature algorithm depends only one of the problem, prime factorization or discrete logarithm.

A signature scheme cannot be unconditionally secure, since Adv can test all possible signature for a given message m . So, given sufficient time, Adv can always forge Sender's signature on any message. Thus, our goal is to find signature schemes that are computationally or Provable secure. In this paper, a new variant of digital signature algorithm (DSA) is proposed which is based on the combined difficulties of integer factorization problem and discrete logarithm problem. Rest of the paper is organized as follows. Section II describes security threats against DL and FAC problem based algorithms. The proposed algorithm is described in section III. In section IV, security analysis is carried out for the proposed algorithm. Performance analysis of the proposed algorithm is discussed in section V. Finally, in section VI, paper is concluded.

II. SECURITY THREATS AGAINST DISCRETE LOGARITHM AND FACTORIZATION PROBLEM BASED ALGORITHMS

The ElGamal signature algorithm [6] is a digital signature algorithm which is based on the difficulty of computing discrete logarithms. The main threat against the ElGamal algorithm is that the strength of the algorithm solely depends on the discrete logarithm problem. If the discrete logarithm problem can be solved then it is possible to obtain the secret x from the public value g_x , and then one could sign messages as a genuine sender. In 1993 Daniel M. Gordon presented an algorithm [7] that could solve discrete

logarithms for small numbers in a finite field of prime order p , $GF(p)$, using the Number Field Sieve. Takuya Hayashi [10] presented an algorithm that can solve a 676-bit Discrete Logarithm Problem in $GF(36n)$ for n is any positive integer. It is clear from the work of Gordan and Hayashi that, in near future, it could be feasible to solve the discrete logarithms problem for large numbers in a polynomial time.

RSA Digital Signature algorithm (RSADSA) [27] proposed by Rivest, Shamir and Adleman, is a popular and well known digital signature algorithm. RSADSA is an asymmetric digital signature algorithm as it uses a pair of keys, one of which is used to sign the data in such a way that it can only be verified with the other key. Security of RSADSA algorithm is based on difficulty of solving the prime factorization problem. Many efforts have been made in past to solve the prime factorization problem [13, 23, 22, 25]. In 2002, Weger [4] described a new attack for solving prime factorization problem as if there is small difference between the prime factors of modulus then a polynomial time cryptanalysis for factoring modulus is possible. In 2003, Boneh and Brumley [1] demonstrated a more practical attack capable of recovering RSA factorizations over a network connection. This attack takes advantage of information leaked by the Chinese remainder theorem optimization used by many RSA implementations. RSADSA is not only vulnerable to the prime factorization attacks but also to the private key d . Paul Kocher [16] described that if an Adversary Eve knows Alice's hardware in sufficient detail and is able to measure the decryption times for several known cipher texts, she can deduce the decryption key d quickly. Next, there are many threats if the RSA private exponent is chosen small. The first significant attack on small private exponent RSA was Wiener's continued fraction attack [32]. Given only the public key (e, n) , the attack factors the modulus using information obtained from one of the convergent in the continued fraction expansion of e/n . It was shown by Coppersmith [13], that an RSA modulus with balanced primes could be factored given only $1/2$ of the most significant bits of one of the primes. It was later shown by Boneh, Durfee and Frankel [2] that $1/2$ of the least significant bits of one of the primes was also sufficient. A theoretical hardware device named TWIRL designed by Shamir and Tromer in 2003 [15], questioned the security of 1024 bit keys. Nowadays due to the availability of high end resources of computation the chances of the various types of attacks have increased. It is quite possible that an organization with sufficiently deep pockets can build a large scale version of his circuits and effectively crack an RSA 1024 bit message in a relatively short period of time. The RSADSA algorithm is also forgeable for chosen-message attack, since RSA is multiplicative; the signature of a product is the product of the signatures.

III. THE PROPOSED SIGNATURE ALGORITHM

This section proposes a new variant of digital signature algorithm based on the two NP-Complete problems named prime factorization and discrete logarithm. Following are the formal definitions of the problems:

Definition 1: (Discrete Logarithm problem :) If $y = g^x \pmod p$ such that p is a prime number and g is a primitive root in Z_p^* and a, y , and p are given then finding the value of x is a discrete logarithm problem. If g, x, p are large numbers then it is a hard number theoretic problem [6].

Definition 2: Prime factorization problem: For a given composite number n , such that $n = p \times q$; where p and q are prime numbers, finding p and q is a prime factorization problem. If a large, b -bit number is the product of two primes that are roughly the same size, then no algorithm has been published that can factor in polynomial time, i.e., that can factor it in time $O(b^k)$ for some constant k .

A new digital signature algorithm based on combined application of DL and FAC is described as follows:

A. Key Generation

- Choose a large prime p such that computing discrete logarithms modulo p is difficult and two large prime numbers p_1 and q_1 such that $p < n$ where $n = p_1 \times q_1$.
- Choose random numbers k and v such that $1 < k, v < p-1$.
- Choose random number b such that $1 < b < n-1$.
- Choose a primitive root g in Z^p .
- Calculate $\phi(n) = (p_1 - 1) \times (q_1 - 1)$.
- Choose e and x such that $e, x \in Z_{\phi(n)}^*$.
- Calculate d such that $d \times e \pmod{\phi(n)} = 1$.
- Calculate c such that $b^x \times c \pmod n = 1$.
- Calculate u, w , and t^* as follows: $u = g^k \pmod p$,
 $w = g^v \pmod p$,
 $t = u^w \pmod p$,
- Public key is (e, x, c, g) and private key is (k, v, t, b, d) .

B. Signature Generation

Step-1: Choose an integer z such that $1 < z < (p-1)$ and it is relative prime to $(p-1)$ i.e. $\gcd(z, p-1) = 1$. z should be different for every message m and is not public. Here $H(\cdot)$ is a one way hash function.

Step-2: Calculate

$$\begin{aligned} h &= g^z \text{ mod } p, \\ \gamma &= t \times w^h \text{ mod } p, \\ s_1 &= H(m)^d \text{ mod } n, \\ s_2 &= (H(m) \times b^s) \text{ mod } n, \\ s_3 &= (((H(m) - kw - hv) \times z^{-1})) \text{ mod } (p - 1)). \end{aligned}$$

If $\gamma = 0$ and/or $s_1 = 0$ and/or $s_2 = 0$ and/or $s_3 = 0$ and/or $H(m) \equiv (kw + hv) \text{ mod } (p - 1)$ then repeat step 1 and 2 else tuple $(\gamma, h, s_1, s_2, s_3)$ is the signature of m .

Here $-kw, -hv$ are additive inverse of kw and hv respectively and z^{-1} is the multiplicative inverse of z with respect to $\text{mod}(p - 1)$.

C. Signature Verification

- Calculates $H(m)$ using the received message m at receiver's end.
- If $g^{H(m)} \times s_1^{xx} \equiv (\gamma \times h^s \times s_2^2 \times c^s) \text{ mod } p$ then the signature is valid else reject the signature.

D. Proof of correctness

$$\begin{aligned} \text{L.H.S.} &= (g^{H(m)} \times s_1^{xx}) \text{ mod } p, \\ &= (g^{H(m)} \times (H(m)^d \text{ mod } n)^{xx}) \text{ mod } p, \\ &= (g^{H(m)} \times H(m)^x \text{ mod } n) \text{ mod } p, \\ &= (g^{H(m)}) \text{ mod } p \times (H(m)^x \text{ mod } n) \text{ mod } p, \\ \text{R.H.S.} &= (\gamma \times h^s \times s_2^2 \times c^s) \text{ mod } p, \\ &= ((t \times w^h \text{ mod } p) \times h^{(((H(m) - kw - hv) \times z^{-1})) \text{ mod } (p - 1)}) \times s_2^2 \times c^s) \text{ mod } p, \\ &= (((u^w \text{ mod } p \times w^h \text{ mod } p) \times (g^{H(m)} \times u^{-w} \times w^{-h}) \text{ mod } p) \times s_2^2 \times c^s) \text{ mod } p, \\ &= (g^{H(m)} \text{ mod } p \times ((H(m) \times b^s) \text{ mod } n)^x \times c^s) \text{ mod } p, \\ &= (g^{H(m)} \text{ mod } p \times ((H(m)^x \times b^{s \times x}) \text{ mod } n) \times c^s) \text{ mod } p, \\ &= (g^{H(m)} \text{ mod } p \times H(m)^x \text{ mod } n) \text{ mod } p, \\ &= g^{H(m)} \text{ mod } p \times (H(m)^x \text{ mod } n) \text{ mod } p, \\ &= \text{L.H.S.} \end{aligned}$$

Therefore, L. H. S. is equal to R. H. S.

IV. SECURITY ANALYSIS

In this section, security analysis of the proposed algorithm is carried out. We shall show that the security of proposed algorithm is based on solving both the problem; prime factorization and discrete logarithm, simultaneously. It is observed that if an oracle O breaks the FAC and DL then it can break the proposed algorithm also, if given the public key of the scheme and a message m_{adv} .

Theorem 1: If there is an ORACLE that can solve the prime factorization and discrete logarithm problem, then it can also break the proposed algorithm.

Proof: Let us the oracle O gives values of prime factor (p_1, q_1) of n and (k, v, z, w) from solving DL and FAC using (γ, h) . We know that $n = p_1 \times q_1$, and $\phi(n)$ is the Euler's totient function. Consider the equation $b^x \times c = \text{mod } n$ (1)

Where b and $x \in \mathbb{Z}_n$. Now from Diophantine equation for x and $\phi(n)$; $\exists u$ and v such that $xu - \phi(n)v = f$, where $f \in \mathbb{Z}_n$. Now a s in the proposed algorithm $\text{gcd}(x, \phi(n))=1$, so it is easy to solve equation (1) and the computation $b \equiv (c)^u \text{ mod } n$ gives the required value of b , since

$$\begin{aligned} b &= (1/c)u \text{ mod } n \\ &= (1/c) 1 + v \phi(n) \text{ mod } n; \\ &= (1/c) 1/x \text{ mod } n; \end{aligned}$$

Further, consider the equation

$$d \times e \equiv 1 \text{ mod } \phi(n) \quad (2)$$

where d is a private key element and e is a public key element. If Adv knows the prime factorization of modulus n then he can easily calculate $\phi(n)$ and hence using equation (2), private key d . Therefore, one can easily find the value of private key elements d and b .

Further, we know the value of z, k and v , hence the signature $(\gamma, h, s_1, s_2, s_3)$ of a message m_{adv} , can be generated as follows:

$$\begin{aligned}
 u &= g^k \text{ mod } p, \\
 w &= g^v \text{ mod } p, \\
 t &= u^w \text{ mod } p, \\
 h &= g^z \text{ mod } p, \\
 \gamma &= t \times w^h \text{ mod } p, \\
 s_1 &= H(m)^d \text{ mod } n, \\
 s_2 &= (H(m) \times b^{s_1}) \text{ mod } n, \\
 s_3 &= (((H(m) - kw - hv) \times z^{-1})) \text{ mod } (p - 1)).
 \end{aligned}$$

Therefore, the tuple $(\gamma, h, s_1, s_2, s_3)$ is a valid signature of message m_{adv} using the proposed algorithm.

There are some possible areas where an adversary (Adv) may try to attack on this new developed signature algorithm. Following are the possible attacks (not exhaustive) and the reasons why that would fail:

Key-Only Attack: Adv wishes to obtain private key (k, v, t, b, d) using all information that is available from the system. In this case, Adv needs to solve the prime factorization problem to find d and b from modulus $n = p_1 \times q_1$. Also he has to solve discrete logarithm problem to find z, k and v using γ, h and g . For finding b , Adv has to solve $b = c^{-1/x} \text{ mod } n$ which is NP-Complete for large b because Adv has to find prime factorization of modulus n to calculate x^{th} root of c^{-1} . Further, d can also be calculated easily, if factorization of modulus n is known. Therefore an Adv has to solve DL problem and FAC problem for finding the private key. This makes the proposed algorithm secure enough for this type of attacks.

Chosen-Message Attack: In this attack, Adv requires a sign on some messages of his choice by the authorized signatory. With the help of chosen-messages and corresponding signatures, Adv generates another message and can forge sender's signature on it. The RSADSA algorithm is forgeable for this attack. For attack on RSADSA, suppose, Adv asks signer to sign two legitimate messages m_1 and m_2 for him. Let us assume s_1 and s_2 are signatures of m_1 and m_2 respectively. Adv later creates a new message $m = m_1 \times m_2$ with signature $s = s_1 \times s_2$. Adv can then claim that signer has signed m . The chosen-message attack for the proposed algorithm is a matter of further research as there is no obvious method which shows that the proposed algorithm is vulnerable to this attack.

Known partial key and Message Attack: Let us assume that Adv is able to solve FAC problem hence, he knows the secret key component b and d . Therefore Adv is able to calculate the signature element s_1 and s_2 . Adv may also have i valid signatures $(\gamma_j, h_j, s_{1j}, s_{2j}, s_{3j})$ on message m_j where $j = 1, 2, \dots, i$ and public key (e, c, x, g) and he attempts to find secret keys (k, v, u, w, z_j) . Now, Adv has i equations as follows representing z_{j-1} as l_j :

$$\begin{aligned}
 s_{31} &= ((H(m_1)l_1 - kw l_1 - hv l_1) \\
 s_{32} &= ((H(m_2)l_2 - kw l_2 - hv l_2)
 \end{aligned}$$

$$s_{3i} = ((H(m_i)l_i - kw l_i - hv l_i)$$

In the above i equations, there are $(i + 3)$ variables namely k, w, v and l_j where $j = 1, 2, \dots, i$ which are not known by the Adv. Hence, k, w, v and l_j stay hard to detect because for Adv, there are $i+3$ unknowns to be found from i equations.

Blinding: In this attack, in case of RSADSA suppose Adv wants sender's signature on his message m . For this Adv try the following: he picks a random $r \in Z^n$ and calculates $m' = r^e \times m \text{ mod } n$. He then asks sender to sign the message m' . Sender may provide his signature s' on the message m' . But we know that $s' = (m')^d \text{ mod } n$. Adv now computes $s = s'/r \text{ mod } n$ and obtains sender's signature s on the original m . This technique, called blinding, enables Adv to obtain a valid signature on a message of his choice by asking Sender to sign a random blinded message. Sender has no information as to what message he is actually signing. So, RSA is vulnerable to this attack. Again an intensive research is required to check whether the proposed algorithm is vulnerable to Blinding or not. Currently, best of authors' efforts it seems not vulnerable for Blinding.

V. PERFORMANCE ANALYSIS

Using the criterion presented in [3], the complexity of each method is estimated as a function of number of bit operations required. The basic exponential operation here is $a^b \text{ mod } n$ and time complexity of this operation is $O(\log b \times M(n))$, where $M(n)$ is the complexity of multiplying two n bit integers. In the proposed algorithm signature generation requires 4 modular exponentiation and signature verification requires 5 modular exponentiation which leads to the complexity of the algorithm to be $O(4 \times \log^3 n)$ and $O(5 \times \log^3 n)$ for signature generation and verification respectively as here $b = O(n)$ and time complexity of multiplying two n bit integers is $O(\log^2 n)$. If the complexity of proposed DSA compared with other DSA algorithms of same category (i.e. DSA algorithms that are based on multiple hard problems) then we see that the Dimitrios Poulakis signature algorithm [26] requires 6 modular

exponentiation in signature generation and 2 modular exponentiation in signature verification. Ismail E. S signature algorithm [14] requires 5 modular exponentiation in signature generation and 5 modular exponentiation in signature verification. Shimin Wei signature algorithm [31], requires 5 modular exponentiation in signature generation and 5 modular exponentiation in signature verification. So it is clear that the complexity of the proposed algorithm is competitive equivalent to most of the digital signature algorithms which are based on prime factorization and discrete logarithm.

VI. CONCLUSION

In this paper, a new variant of digital signature algorithm is proposed which is based on the two hard problems called prime factorization and discrete logarithm. It is shown that one has to solve both the problems simultaneously for crypt-analysis of this algorithm. The performance of the proposed algorithm is found to be competitive to the most of the digital signature algorithms which are based on multiple hard problems.

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