



# Combined Heat and Power Economic Dispatch Using Improved Differential Evolution Algorithm

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**Abstract**— *Combined Heat and Power Economic Dispatch (CHPED) plays a key role in economic operation of power systems. Due to complex characteristics, heuristic and evolutionary based optimization approaches have become effective tools to solve the CHPED problem. In this paper a new optimization approach based on improved differential evolution (IDE) has been proposed to solve the CHPED problem. IDE is an improved version of differential evolution optimization algorithm in which new solutions are produced in respect to global best solution. This approach leads the algorithm to optimum solution. The cause of CHPED problem difficulty is its constraints which are satisfied simply by using IDE. In order to demonstrate the applicability and efficiency of the proposed IDE based approach, it has been tested on a standard test system and obtained results are compared with those obtained using other existing methods. Simulation results show that the proposed approach is superior to the other existing methods.*

**Keywords**— *Combined Heat and Power Economic Dispatch, Improved Differential Evolution Algorithm, Optimization.*

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## I. INTRODUCTION

Increasing demand in the use of cogeneration systems that simultaneously produce heat and power is quite remarkable. Combined heat and power generation unit with industrial, commercial and residential applications is an efficient energy resource providing environmental advantages over other forms of conventional energy supply. Utilization of cogeneration units besides conventional power generating units and heat-only units to satisfy heat and electricity demands in an economical manner emphasizes on the need to combined heat and power economic dispatch (CHPED). In the pure economic dispatch problem, power demand is only taken and distributed on in-service generating units. In the CHPED problem, heat demand is also considered and mutual interdependency of heat and power production in cogeneration units introduces more complexity in the problem. In the CHPED problem, it is tried to find economical optimum solution considering the mentioned complexity of the constraints. Indeed, the purpose of the problem is to specify the output of the units to satisfy heat and power demands with minimum fuel cost [1].

Some researches worked in the field of the CHPED problem. Rooijers and van Amerongen [2] presented a two level strategy, the lower level solves economic dispatch problem for the given power and heat lambdas, and the upper level updates the lambda's sensitivity coefficients. The procedures are repeated until the heat and power demands are met. Also a customized branch-and-bound algorithm was also developed and solved the CHPED problem [3,4]. Alternatives to the traditional mathematical approaches: evolutionary computation techniques such as Genetic Algorithm (GA) [5], Improved Ant Colony (ACO) [6], Harmony Search (HS) [7], Evolutionary Programming (EP) [8], Mesh Adaptive Direct Search Algorithm (MADS) and Economic Dispatch Harmony Search (EDHS) have been successfully implemented in CHPED problem [9,10]. Self adaptive real coded genetic algorithm has been suggested in [11]. This variant of genetic algorithms uses tournament selection and simulated binary cross over and utilizes penalty factor for satisfying constraint. In [12] a new method has utilized in which the non-convex region is divided into two convex regions and then problem is solved. Also Augmented Lagrange Hopfield Network (ALHN) has been achieved to solve mentioned purpose [13].

In this paper, a new approach is proposed to solve the CHPED problem using an improved version of DE. Also a new heuristic penalizing method has been proposed to satisfy the constraints. Simulations has been done on sample case with and compared with other popular methods. Results show superiority of the improved differential method than other methods.

## II. PROBLEM FORMULATION

Combined heat and power economic dispatch determines heat and power output of generating units in a way that total production cost of the system is minimized while the power and heat demands and other constraints are satisfied. It is assumed that the system includes conventional power generating units, cogeneration units and heat-only units. In this problem, the power output of conventional and cogeneration units as well as heat production of heat-only units are the control (decision) variables which are restricted by their related upper and lower bounds. About the cogeneration units, related feasible operation regions are considered.

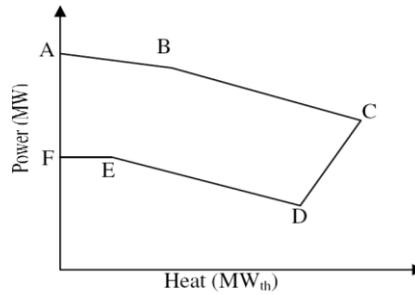


Fig. 1. The heat-power operating regions for a cogeneration unit

Fig.1 shows feasible operation region of a cogeneration unit. The feasible operation region is modeled by the boundary curve ABCDEF. Along the boundary curve of BC, increasing the power output leads to decreasing the heat production. While, the power output and heat production have variation with same direction during boundary curve of CD. Solution of the CHPED problem would be in this feasible region while the power and heat demands constraints are met.

The CHPED problem is formulated as an optimization problem including an objective function and its related constraints, expressed as follows:

$$\min f_{\text{cost}} = \sum_{i=1}^{Np} C_i(P_i) + \sum_{j=1}^{Nc} C_j(P_j, H_j) + \sum_{k=1}^{Nh} C_k(H_k) \quad (1)$$

Subject to

$$\sum_{i=1}^{Np} P_i + \sum_{j=1}^{Nc} P_j = P_D \quad (2)$$

$$\sum_{j=1}^{Nc} H_j + \sum_{k=1}^{Nh} H_k = H_D \quad (3)$$

$$P_i^{\min} \leq P_i \leq P_i^{\max} \quad i = 1, \dots, Np \quad (4)$$

$$P_j^{\min}(H_j) \leq P_j \leq P_j^{\max}(H_j) \quad j = 1, \dots, Nc \quad (5)$$

$$H_j^{\min}(P_j) \leq H_j \leq H_j^{\max}(P_j) \quad j = 1, \dots, Nc \quad (6)$$

$$H_k^{\min} \leq H_k \leq H_k^{\max} \quad k = 1, \dots, Nh \quad (7)$$

with:

$$C_i(P_i) = a_i + b_i P_i + c_i P_i^2 \quad i = 1, \dots, Np \quad (8)$$

$$C_j(P_j, H_j) = a_j + b_j P_j + c_j P_j^2 + d_j H_j + e_j H_j^2 + f_j P_j H_j \quad j = 1, \dots, Nc \quad (9)$$

$$C_k(H_k) = a_k + b_k H_k + c_k H_k^2 \quad k = 1, \dots, Nh \quad (10)$$

where,

- $f_{\text{cost}}$  Total fuel cost
- $C(\cdot)$  Production cost function of the unit
- $P_i$  Generated power of the  $i$ -th unit
- $H_j$  Heat production of the  $j$ -th unit
- $H_D$  Heat demand of the system
- $P_D$  Power demand of the system
- $Np$  Number of conventional power generating units
- $Nh$  Number of heat-only units
- $Nc$  Number of cogeneration units
- $P_i^{\min}, P_i^{\max}$  Lower and upper bounds for power output of unit  $i$
- $H_k^{\min}, H_k^{\max}$  Lower and upper bounds for heat production of unit  $k$
- $a_i, b_i, c_i$  Coefficients of fuel cost for the  $i$ -th conventional power generating unit
- $a_j, b_j, c_j, d_j, e_j, f_j$  Coefficients of fuel cost for the  $j$ -th cogeneration unit

$a_k, b_k, c_k$  Coefficients of fuel cost for the  $k$ -th heat-only unit.

Usually the power capacity limits of cogeneration units are functions of the unit heat productions and the heat capacity limits are functions of the unit power generations [14].

It is noticeable that the complication arising in the CHPED problem is the mutual dependencies of extra constraints than in pure economic dispatch.

### III. OPTIMIZATION ALGORITHM

Original DE algorithm is a simple population based evolutionary computational algorithm for global optimization. It is one of the accurate and fast meta-heuristic optimization algorithms that was introduced in 1995s by Price and Storn [15]. This evolutionary algorithm begins the search process by initial random population. DE includes three main operators, namely, mutation, crossover, and selection. Also, it has three control parameter, namely, population size ( $np$ ), scaling coefficient ( $F$ ), and crossover probability ( $CR$ ). In the subsequent sections, the implementation details of the DE are described [16,17].

1) *Initialization*: The DE algorithm searches in parallel using a group of members similar to the other evolutionary based heuristic optimization techniques. Each member corresponds to a candidate solution to the problem. In an  $n$ -dimensional search space, the structure of member  $k$  is represented as vector  $X_k = (x_{k,1}, x_{k,2}, \dots, x_{k,n})$  where the dimension represents the number of components. In the first stage of the DE optimization process, initial population contains  $np$  members should be created randomly.

2) *Mutation*: After the population is initialized, the operators of mutation, crossover and selection create the population of the next generation. At the generation  $t$ , the process for creation of a mutant solution ( $Y_k$ ) for each parent ( $X_k$ ) in the population can be expressed as follows:

$$Y_k(t) = X_{r3}(t) + F \cdot (X_{r1}(t) - X_{r2}(t)), \quad k=1, 2, \dots, np \quad (11)$$

where vector indices  $r1$ ,  $r2$ , and  $r3$  are randomly chosen, which  $r1, r2$ , and  $r3 \in \{1, \dots, np\}$  and  $r1 \neq r2 \neq r3 \neq k$ .  $X_{r1}$ ,  $X_{r2}$ , and  $X_{r3}$  are selected members for each parent vector.  $F$  is a user-defined constant known as the ‘scaling factor’, which is a positive and real number. The usual choice for  $F$  is a number between 0 and 1.

3) *Crossover*: In order to increase the diversity of the population, the crossover process is employed. At the generation  $t$ , the crossover operator creates a new solution (child) ( $Z_k$ ) using each parent ( $X_k$ ) and its related mutant vectors ( $Y_k$ ) as follows:

$$z_{kj}(t) = \begin{cases} y_{kj}(t) & \text{if } rand \leq CR \text{ or } j = jrand \\ x_{kj}(t) & \text{otherwise} \end{cases}, \quad \text{for } j = 1, \dots, n \quad (12)$$

where  $CR$  is ‘crossover probability’ which is a user-defined value usually selected from within the range [0, 1].  $CR$  controls the diversity of the population and helps the algorithm to escape from local optima.  $rand$  is a uniformly distributed random number within the range (0, 1) generated a new for each component  $j$ . Here,  $jrand \in [1, 2, \dots, n]$  and ensures that the trial vector gets at least one parameter from the mutant vector.

4) *Selection*: To keep the population size constant over subsequent generations, the selection operator is applied to determine which one of the child and the parent will survive in the next generation. This operator compares the fitness of the parent and the corresponding child and the fitter of the two solutions is then allowed to advance into the next generation. The selection process can be expressed as,

$$X_k(t+1) = \begin{cases} Z_k(t) & FIT(X_k(t)) \geq FIT(Z_k(t)) \\ X_k(t) & \text{otherwise} \end{cases} \quad (13)$$

where  $FIT(\cdot)$  is the fitness function.

5) *Stopping Criteria*: The overall optimization process is terminated if the iteration approaches to the predefined maximum iteration or other predetermined convergence criterion is satisfied.

In the original version of DE algorithm, new solutions are created by three previous random selected solutions. In the improved version of DE algorithm, it is tried to reach better solutions by changing the crossover mechanism of the original DE algorithm. To do this, new solutions are generated in respect to the global best solution. Therefore probability of obtaining optimum solutions will increase. Thus, instead of using three previous random solutions, two previous solutions and the global best solution ( $X_g$ ) are used. This process is expressed by [15]:

$$Y_k(t) = X_k(t) + rand \times (X_g - X_k(t)) + \mu \times (X_{r1}(t) - X_{r2}(t)) \quad (14)$$

where  $X_g$  is the best solution so far;  $rand$  is a uniformly distributed random number within the range (0, 1), and  $\mu$  is the constant number between 0 and 1.

#### IV. APPLICATION OF IDE IN SOLVING THE CHPED PROBLEM

In the proposed approach each solution can be considered as a vector. In the CHPED problem, power output and heat production of the generating units are decision/control variables. Therefore each solution  $f$  should contain these items as follow:

$$X_f = [P_{f,1}, \dots, P_{f,Np}, P_{f,Np+1}, \dots, P_{f,Np+Nc}, H_{f,1}, \dots, H_{f,Nc}, H_{f,Nc+1}, \dots, H_{f,Nc+Nh}] \quad (15)$$

Note that it is very important to create an initial population of solutions satisfying the equality constraints (2)-(3) and inequality constraints (4)-(7). That is, summation of all power outputs should be equal to the total system power demand and summation of all heat productions should be equal to the total system heat demand. In addition, the created elements at random (i.e., power output or heat production) should be located within their boundaries. To do this, the following procedure is suggested for any solution in the initial population:

Step 1) Set  $l = 1$ .

Step 2) Select a component  $x$  among set of the non-initialized power output components, randomly (i.e.  $x \in \{1, 2, \dots, Np + Nc\}$ ).

Step 3) Determine minimum and maximum allowed power outputs of the unit  $x$  (i.e.  $AP_x^{\min}$  and  $AP_x^{\max}$ , respectively) as follows:

$$AP_x^{\min} = \max\{P_x^{\min}, P_D - \sum_{y=1}^{NIP} P_y - \sum_{z=1, z \neq x}^{NNP} P_z^{\max}\} \quad (16)$$

$$AP_x^{\max} = \min\{P_x^{\max}, P_D - \sum_{y=1}^{NIP} P_y - \sum_{z=1, z \neq x}^{NNP} P_z^{\min}\} \quad (17)$$

where  $NIP$  is the set of initialized units (among the first  $Np+Nc$  components) and  $NNP$  is the set of non-initialized units (among the first  $Np+Nc$  components).

For the cogeneration units the maximum and minimum power output values in the related heat-power operation region is considered as  $P_z^{\max}$  and  $P_z^{\min}$  respectively.

Step 4) Create the value of the component (i.e., power output) in the range of  $[AP_x^{\min}, AP_x^{\max}]$  in a random fashion.

Step 5) If  $l = Np + Nc - 1$  go to Step 6; otherwise  $l = l + 1$  and go to Step 2.

Step 6) The final component value is equal to  $(P_D - \sum_{y=1}^{NIP} P_y)$ .

Step 7) Set  $l = 1$ .

Step 8) Select a component  $x$  among set of non-initialized heat productions, randomly (i.e.  $x \in \{Np + Nc + 1, \dots, Np + 2Nc + Nh\}$ ).

Step 9) Determine minimum and maximum allowed heat production of the unit  $x$  (i.e.  $AH_x^{\min}$  and  $AH_x^{\max}$ , respectively) as follows:

$$AH_x^{\min} = \max\{H_x^{\min}, H_D - \sum_{y=1}^{NIH} H_y - \sum_{z=1, z \neq x}^{NNH} H_z^{\max}\} \quad (18)$$

$$AH_x^{\max} = \min\{H_x^{\max}, H_D - \sum_{y=1}^{NIH} H_y - \sum_{z=1, z \neq x}^{NNH} H_z^{\min}\} \quad (19)$$

where  $NIH$  is the set of initialized units (among the last  $Nc+Nh$  components) and  $NNH$  is the set of non-initialized units (among the last  $Nc+Nh$  components).

For the cogeneration units the maximum and minimum heat production values (i.e.  $H_z^{\max}$  and  $H_z^{\min}$  respectively) can be determined according to their power outputs initialized in the previous steps.

Step 10) Create the value of the component  $x$  (i.e., heat production) in the range of  $[AH_x^{\min}, AH_x^{\max}]$  in a random fashion.

Step 11) If  $l = Nc + Nh - 1$  go to Step 6; otherwise  $l = l + 1$  and go to Step 2.

$$\min\{H_x^{\max}, H_D - \sum_{y=1}^{NIH} H_y\}$$

Step 12) Choose the final component as  $x$  and set its value to

Step 13) Stop the initialization process.

The developed initialization scheme always guarantees to produce solutions satisfying the inequality constraints (4)-(7) and equality constraint (2). But the resulting solution is not always guaranteed to satisfy the equality constraint (3) and there may be shortage in total heat demand. In such case, the solution is accepted but subject to repair to satisfy the heat demand constraints. To repair the solution, shortage heat value is loaded among heat-only units with free capacity.

In the CHPED problem, power output of power generation units and heat production of heat-only units should satisfy their related inequality constraints, i.e. Eq. (4) and Eq. (7), respectively. When the solutions are initialized, their values are generated between their upper/lower bounds. But in the process of generating new solutions, the resulting solutions are not always guaranteed to satisfy the inequality constraints due to random nature of algorithm. For satisfying inequality constraints, if any component of a solution exceeds from its upper bound, it will be set at the upper bound and if it decreases from its lower bound it will be set on the lower bound.

There are two equality constraints in CHPED problem which show the load and the heat demands satisfactions. The handling process is done after satisfying inequality constraints. As mentioned in section 4-1, the initial population is generated so that these equality constraints are satisfied. But in the search process, the resulting solution is not always guaranteed to satisfy equality constraints related to the heat and power demands. To handle these constraints solutions must be repaired. To do this, the surplus or shortage demand is divided among units. To balance the power demand, the difference between total power demand and generation is distributed among conventional power generating units. In case of power shortage/excessive, a conventional power generating unit is selected randomly and its power output is increased/decreased. This process is repeated until the shortage/excessive load is totally removed and the power demand equality constraint is met. In case of heat demand constraint violation, a heat-only unit is selected in random fashion and the same procedure is applies until the heat demand equality constraint is met.

c) Feasible operation region of cogeneration units

Power and heat output of cogeneration units are mutually dependent. Therefore these constraints are introduced as feasible operation region constraints that they are very difficult to be met. In this paper a new penalizing method is proposed in which infeasible solutions are penalized in respect to their violations from feasible regions. In this method if the output of a cogeneration unit is outside its feasible region, a penalty factor depending on the minimum distance between the cogeneration unit output and the feasible region border is employed. Fig. 2 shows this distance for an operating point outside the feasible region graphically.

If  $aH + bP + c = 0$  is the equation of the nearest region border of the cogeneration unit (line AB in Fig.2), the minimum distance will be calculated using equation 20. Then a penalty factor is calculated using equation 21.

$$d = \frac{|aH_0 + bP_0 + c|}{\sqrt{a^2 + b^2}} \tag{20}$$

$$Penalty = pf \cdot \sum_{j=1}^{N_c} d_j \tag{21}$$

where  $(H_0, P_0)$  is the output position of cogeneration unit,  $Penalty_i$  is the penalty factor related to  $i$ -th solution and  $pf$  is a constant value. Therefore penalty amount depends on distance directly, and more distance will result in more penalties and vice versa. Also, if the point  $(H_1, P_1)$  is in the output corner of feasible regions the infeasible solution will be penalized in respect to total distance between the point and two borders that creates the corner of feasible operation region ( $d_1$  and  $d_2$ ).

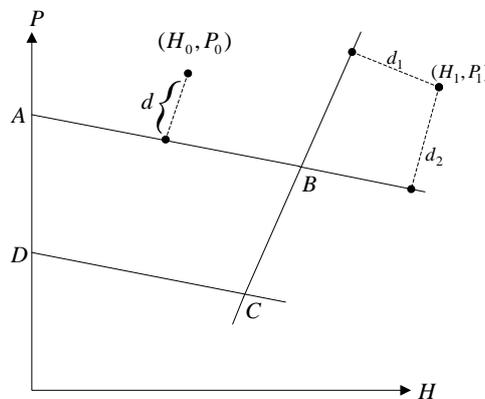


Fig. 2. Concept of penalty calculation for cogeneration units

The performance of solutions are assessed in the objective space and then assigned a scalar value known as fitness. In this paper, the fitness function of the CHPED problem is a combination of the objective function (1) and a penalty function related to the feasible operation region of cogeneration units as follows:

$$FIT = \frac{1}{f_{cost} + penalty}$$

In this paper, maximum iteration is selected as stopping criteria. When the iteration counter reaches to its maximum value, the algorithm is terminated and the solution with the best fitness is reported as the final solution of the CHPED problem.

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**Optimization Steps**

The CHPED problem is solved by IDE algorithm. Seven steps is proposed to solve the CHPED problem as follows:

Step 1: Initializing solutions randomly.

Step 2: Penalizing infeasible solutions.

Step 3: Evaluating solutions and determining global best solution.

Step 4: Creating new solutions.

Step 5: Constraint handling and penalizing infeasible solutions.

Step 6: Evaluating solutions, selecting and determining global best solution.

Step 7: If convergence criteria satisfied exit otherwise go to step 4.

Also Fig. 3 shows these steps graphically.

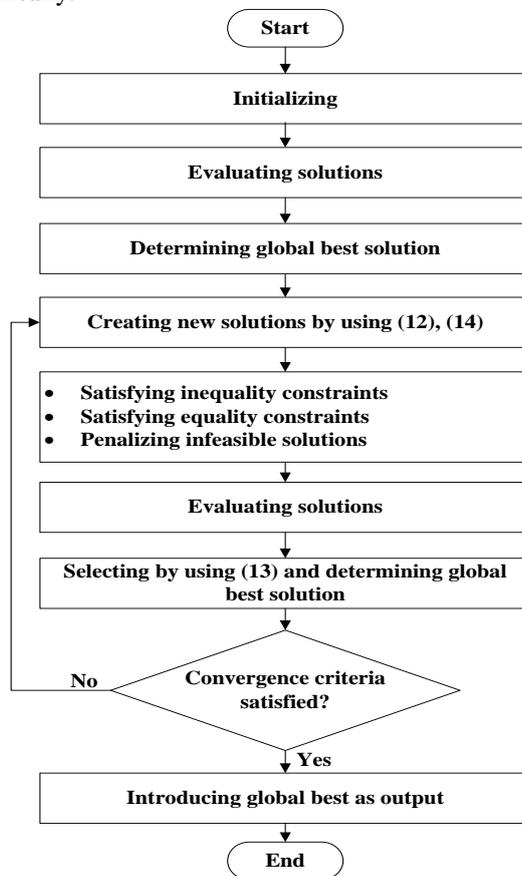


Fig. 3. Flowchart of proposed method

**V. NUMERICAL RESULTS**

To assess the efficiency of the proposed IDE based approach for the CHPED problem, a standard case is employed and the results are compared with those of other methods. For more comparison the original DE has been coded and implemented to the case studies by the authors.

The proposed approach has been coded in MATLAB language and executed on a 2-GHz Pentium IV personal computer with 1-GB of RAM.

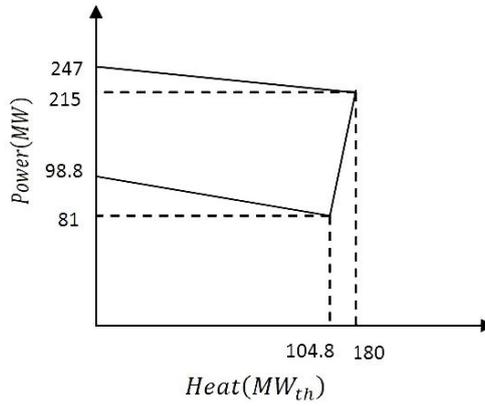


Fig. 4. Feasible region of unit 2

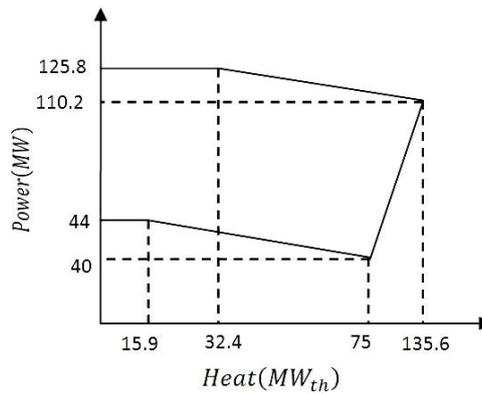


Fig. 5. Feasible region of unit 3

This case consist of: one heat unit, one power unit and two cogeneration units. Power and heat demands are 200 MW and 115 MW, respectively. The units' data are given in [11]. Feasible operation regions are showed in Fig. 4 and 5. Eqs 23 is the cost function of the case:

$$\min f_{cost} = C_1(P_1) + \sum_{j=2,3} C_j(P_j, H_j) + C_4(H_4) \quad (22)$$

where:

$$C_1(P_1) = 50P_1 \quad (23)$$

$$C_2(P_2, H_2) = 2650 + 14.5P_2 + 0.0345P_2^2 + 4.2H_2 + 0.03H_2^2 + 0.031P_2H_2 \quad (24)$$

(24)

$$C_3(P_3, H_3) = 1250 + 36P_3 + 0.0435P_3^2 + 0.6H_3 + 0.027H_3^2 + 0.011P_3H_3 \quad (25)$$

(25)

$$C_4(H_4) = 23.4H_4 \quad (26)$$

(26)

The CHPED problem is solved using the proposed IDE based approach with different characteristics. The maximum iteration number is set as 200.

In order to demonstrate the effect of population size on the performance of the optimization process, the proposed approach has been run with different population sizes and the obtained results among 100 trials are given in Table 1.

Table 1. 100 trials run result with different populations

No. of population	Best solution	Mean solution	Worst solution	Standard deviation	No. of hits to global solution	CPU time(s)
10	9257.08	9327.87	9840.11	102.43	1	0.044
25	9257.0	9257.0	9257.0	0	100	0.094

	7	7	7			
50	9257.0 7	9257.0 7	9257.0 7	0	100	0.17
75	9257.0 7	9257.0 7	9257.0 7	0	100	0.26
100	9257.0 7	9257.0 7	9257.0 7	0	100	0.33

As the results in these Tables indicate, the IDE based approach can reach to the best solution with all population sizes. These provide a robustness of the IDE regarding to the population size. However, the standard deviation of final solution solutions is greater with small population size. Total cost variation during IDE cycles for various population sizes is demonstrated in Fig. 6.

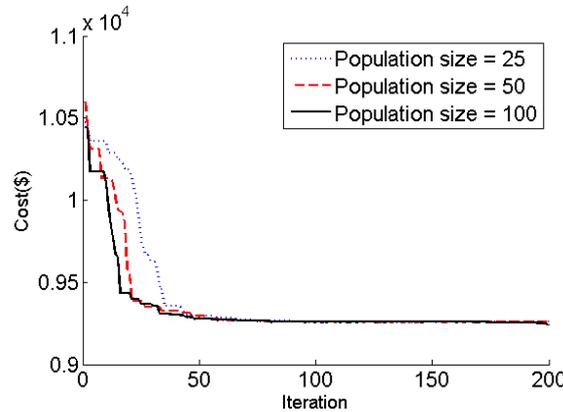


Fig. 6. Convergence process of the proposed algorithm with different populations

For more investigation DE has been executed by author the same as IDE. Fig. 7 displays the convergence diagram of DE and IDE in which proposed method is converged to optimum solution in less iteration rather than DE. From the figure it can be concluded that the IDE have better convergence characteristic. Also, convergence of all solutions and global best solution in 200 iterations is given in Fig. 8. Solutions are spread in search space at first and they are converged to global solution later.

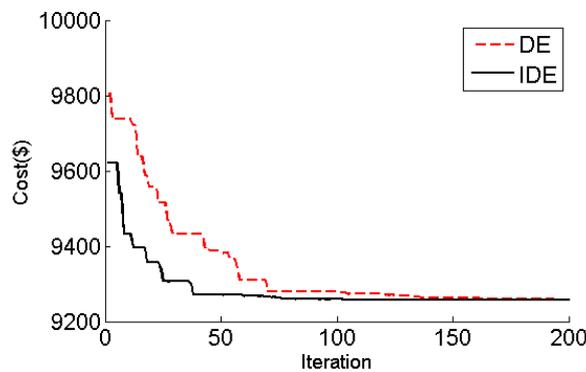


Fig. 7. Convergence diagram of DE and IDE algorithms

The CHPED solutions using the proposed approach and some other existing methods in this case are listed in Table 2. The reported results of IDE are obtained with 100 initial populations and 200 iterations. Comparing the obtained results with those of the other methods shows the superiority of the proposed approach. The IDE based approach can reach to the optimum global solution in lower time than the other methods.

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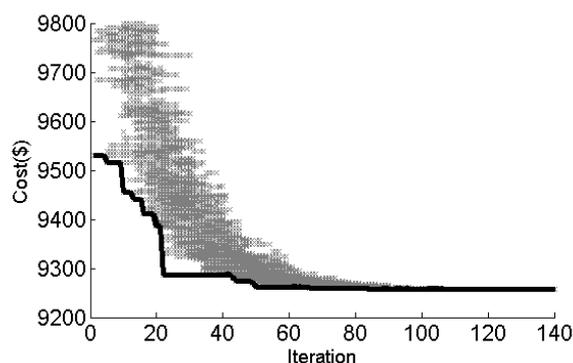


Fig. 8. Convergence of all solutions and global best solution

## VI. CONCLUSION

This paper presented an improved differential evolution algorithm for the complex problem of combined heat and power economic dispatch in power systems. The evolutionary mechanism of the IDE is more effective than the original DE and it has the advantage of being easy to comprehend, simple to implement so that it can be utilized for a wide variety optimization problems. The efficiency of the proposed IDE based CHPED method is proved by case studies on three test systems with different sizes. Results of the proposed IDE algorithm have been compared to those reported in the literature. Comparisons between original DE and IDE show that proposed approach is more efficient than DE to solve large scale test systems. Also comparisons with other methods clearly approved the effectiveness and the superiority of the proposed IDE approach over the other existing techniques in terms of solution quality.

## REFERENCES

- [1] Su CT, Chiang CL. An incorporated algorithm for combined heat and power economic dispatch. *Electr Power Syst Res*, vol. 69, no. 2-3, pp.187–95, 2004.
  - [2] Rooijers FJ, Amerongen V. Static economic dispatch for co-generation systems. *IEEE Trans Power Syst*, vol. 9, no. 3, pp. 1392–98, 1994.
  - [3] Makkonen S, Lahdelma R. Non-convex power plant modeling in energy optimization. *Eur J Oper Res*, vol. 171, no. 3, pp. 1113–26, 2006.
  - [4] Rong A, Lahdelma R. An efficient envelope-based branch-and-bound algorithm for non-convex combined heat and power production planning. *Eur J Oper Res*, vol. 183, no. 1, pp. 412–31, 2007.
  - [5] Song YH, Xuan QY. Combined heat and power economic dispatch using genetic algorithm based penalty function method. *Electr Power Comp Syst*, vol. 26, no. 4, pp. 363–72, 1998.
  - [6] Song YH, Chou CS, Stonham TJ. Combined heat and power dispatch by improved ant colony search algorithm. *Electr Power Syst Res*, vol. 52, no. 2, pp. 115–21, 1999.
  - [7] Vasebi A, Fesanghary M, Bathaee SMT. Combined heat and power economic dispatch by harmony search algorithm. *Int J Electr Power Energy Syst*, vol. 29, no. 10, pp. 713–19, 2007.
  - [8] Wong KP, Algie C. Evolutionary programming approach for combined heat and power dispatch. *Electr Power Syst Res*, vol. 61, no. 3, pp. 227–32, 2002.
  - [9] Hosseini SS, Jafarnejad A, Behrooz AH, Gandomi AH. Combined heat and power economic dispatch by mesh adaptive direct search algorithm. *Expert Syst Appl*, vol. 38, no. 6, pp. 6556-64, 2011.
  - [10] Khorram E, Jaberipour M. Harmony search algorithm for solving combined heat and power economic dispatch problems. *Energy Convers Manage*, vol. 52, no. 2, pp. 1550–4, 2011.
  - [11] Subbaraj P, Rengaraj R, Salivahanan S. Enhancement of combined heat and power economic dispatch using self adaptive real-coded genetic algorithm. *Appl Energy*, vol. 6, pp. 915–21, 2009.
  - [12] Geem ZW, Cho YH. Handling non-convex heat-power feasible region in combined heat and power economic dispatch. *Int J Electr Power Energy Syst* in press.
  - [13] Dieu VN, Ongsakul W. Augmented Lagrange Hopfield network for economic load dispatch with combined heat and power. *Electr Power Comp Syst*, vol. 37, no. 12, pp. 1289–304, 2009.
  - [14] Guo T, Henwood MI, Van Ooijen M. An algorithm for combined heat and power economic dispatch. *IEEE Trans Power Syst*, vol. 11, no. 4, pp. 1778–84, 1996.
  - [15] Storn R, Price K. *Differential evolution – A simple and efficient adaptive scheme for global optimization over continuous spaces*. Int Comput Sc Inst, Berkeley, CA, 1995.
  - [16] Varadarajan M, Swarup KS. Differential evolutionary algorithm for optimal reactive power dispatch. *Int J Electr Power Energy Syst*, vol. 30, no. 8, pp. 435–41, 2008.
- Zaharie D. Influence of crossover on the behavior of differential evolution algorithms. *Appl Soft Comput*, vol. 9, no. 3, pp. 1126–38, 2009.