



## A Comprehensive Study of Cellular Automata

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**Abstract:** Cellular automata are a simple structure which lends itself to some remarkable ideas. They are simple to construct but have the complex behaviour. They can be studied in physics, mathematics, computer science, biology to study the natural process like self-reproduction. In this paper, we are going to show that how Pascal's triangle can be created using cellular automata and how the next generations can be created in the Game of Life.

**Keywords:-** Cellular Automata, Neighbourhood, active Cell, cellular Space, evolution of Next Generation.

### I. INTRODUCTION

The history of the cellular automata starts with the Stanislas Ulam. The curiosity of the Ulam in the evaluation of graphical constructions generated by some simple rules caused the evaluation of the Cellular Automata. There are several types of cellular automata in different dimensions, viz. one dimensional, 2 dimensional etc.

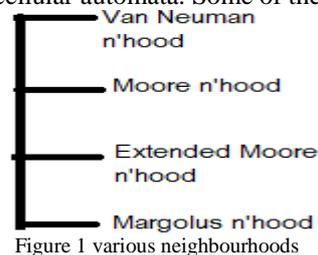
Cellular automata [1], [2], [7], [13], [14] can be seen where complicated patterns of behaviours can be produced by many simple components for a system.

1-Dimensional cellular automata consist of a row of "cells" and a set of rules. Whereas Ulam's 2-Dimensional cellular automata is a 2-Dimensional space which is divided into cells, much like a matrix. This 2-Dimensional [11] space can be visualized as a kind of grid where each cell is having two states- "ON and OFF". Starting from a pattern, a new generation is generated by following some neighbourhood rules. For say, if a cell comes into contact with a "ON or active" cell, it will also become an active cell; on the other hand, if a cell comes in contact with one or no active cell or four or more than four active cells, it will become inactive cell. [3] A set of definite rules determines the value at each site which can be calculated based on the values of the neighbouring cells.

The introduction of the cellular automata [15] by Van Neuman and Ulam was done in order to study some processes like self reproduction [13]. Any system having many discrete elements which are undergoing local deterministic interaction can be seen as cellular automata.

### II. TYPES of NEIGHBOURHOODS

Various types of neighbourhoods are there in cellular automata. Some of them are-



- In Van Neuman neighbourhood, north, south, east and west neighbourhoods are taken.

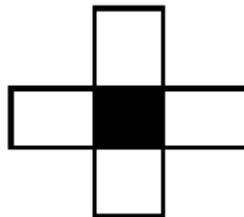


Figure 2 Van Neuman neighbourhood[11]

- In Moore neighborhood, along with the Van Neuman neighborhoods, diagonals are also added.

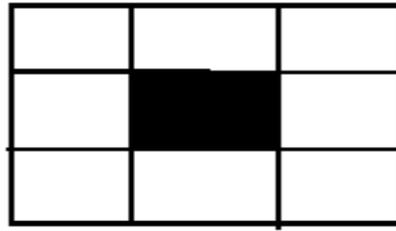


Figure 3 Moore neighbourhood [11]

- In Extended Moore case, the neighbourhood is extended by a distance of one beyond the one.

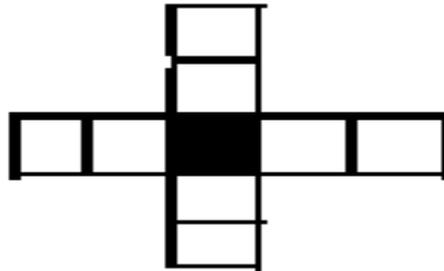


Figure 4 Extended Moore neighbourhood [ 11]

### III. PASCAL'S TRIANGLE FORMATION USING CELLULAR AUTOMATA

In mathematics, Pascal's triangle is a triangular array of binomial coefficients. Pascal's triangle can be formed using the addition modulo 2 formula which is the ordinary addition of two numbers where the sum is divided by 2, giving the resultant answer. The triangle thus formed has one major feature that the triangle pattern formed using modulo method inside any triangle looks like a sub triangle. If the Pascal's triangle is extended to infinite rows & every time the picture scale is reduced to half of its size, the resultant pattern looks like a self similar. That is to say, the [4] the picture or the triangle can be reproduced by taking its sub triangle and then magnifying it.

Let say, a state can be defined as

$$A(n) = \begin{cases} 1, & \text{if } n \text{ is odd} \\ 0, & \text{if } n \text{ is even} \end{cases}$$

Now, applying the addition modulo 2 formula, we can jump to the next state.

Let, the initial state can be defined as,

$$A(n) = \{1, \text{ if } n=0; 0 \text{ otherwise}\}$$

In every other state, we have an entry of 1 on both end and 0 present in between 1.

We know that every entry in the triangle is the sum of the adjacent entries which are just lying above, after which the addition modulo 2 is applied on that entry. After doing this, the triangle formation is done using the 1's whose value is the same as that of the Pascal's triangle. The two triangles remain independent of each other till they meet each other in the  $r^{\text{th}}$  row.

Various values calculated in the Pascal's triangle are [5]-

```

0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 1 0 1 0 0 0 0
0 0 0 1 0 0 0 1 0 0 0
0 0 1 0 1 0 1 0 1 0 0
0 1 0 0 0 0 0 0 0 1 0
    
```

The entries in the third row have two copies of the previous row except for the fact that the middle term will be formed when two previous row's 1's overlaid on each other. Therefore the row formed will have 1's on both ends whereas a 2 present in between them.

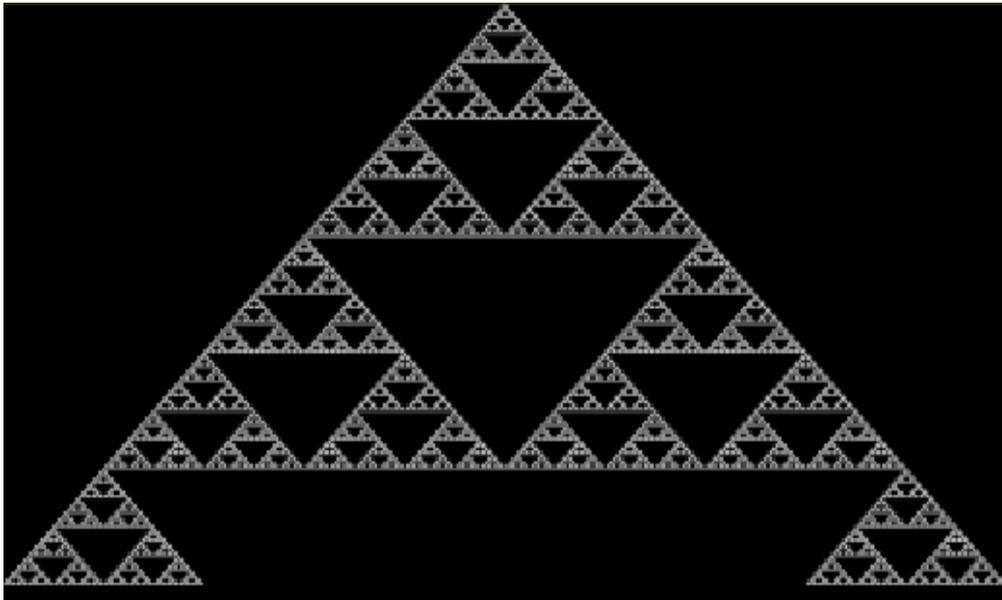


Figure 5 Formation of the Pascal's triangle [5]

#### IV. GAME of LIFE

The Game of Life was originally presented as a mathematical game [6], [12].

In Game of Life [13], all what is needed to do is to give some initial input, an initial configuration is created and this configuration evolves thereafter by its own. It is also known as a "Zero Player Game". Its evolution depends on the initial state. If the game is studied well, it can give us the opportunity to understand the cellular automata better.

Much like a 2-Dimensional cellular space, the Game of Life too made up of a grid of cells much like a matrix.

Certain rules to be followed in the Game of Life are-

For a populated space

- Each cell with one or no active neighbours dies.
- Each cell with four or more active neighbours dies.
- Each cell with two or three active neighbours survives.

For an unpopulated space

- Each cell with three neighbours becomes populated.

For the simplicity, let's take a simple example

Suppose we have a grid of 5X3, and we have given an initial input as follows-

00	01		03	04
10			13	14
20	21	22	23	24

Figure 5 Initial input [12]

For the sake of simplicity, we number the above grid starting from 00 to 24 respectively in three rows. In the above figure, cell number 02, 11, 12 are the active or ON cells.

Now by following the above mentioned rules, we can have the next generation of the cells as –

For the populated space

- Cell number 02, 11 and 12 has three active neighbours; therefore they will survive

For the unpopulated space

- Cell number 00, 10 and 20 has one active neighbour; therefore they will not get populated.
- Cell number 03, 13, 23, 22 and 21 has two active neighbours; therefore they will not get populated.

00				03	04
10				13	14
20	21	22	23	24	

Figure 6 First generation of cells [12]

- Cell number 01 has three active neighbours; therefore it will also get active in the next generation.

In the same way, next generations can be obtained for a number of designs. Some popular designs are

- Gosper Glider Gun
- Glider
- Small Exploder
- Exploder
- 10 cell row

## V. CELLULAR AUTOMATA APPLICATIONS

Next, we are giving some applications [7] of the cellular automata. These are -

### A. Cellular automata games

One of the major cellular automata game is the “Game of Life” given by James Conway. Apart from this, there are several other games created using the cellular automata. These games also provide some insights about the synchronization problem for example *firing squad* [8], *firing mob* [9], and *queen bee* [10].

### B. Parallel computing machine

The 2-Dimensional cellular automata are being used for image processing and pattern recognition. Toffoli and others developed a machine called CAM (Cellular Automata Machine) which operates in the autonomous mode. A higher order of magnitude at a comparable cost can be achieved using CAM as compared to the conventional computers.

### C. Cellular Automata for Physical and Biological systems

Cellular automata can be used to model several chemical processes like inter-diffusion of atoms of two materials.

In the biology, cellular automata models are being used for tumour development, developing drug therapy for HIV infections and various other things.

Other areas where cellular automata can be applied are-

- VLSI (very large scale implementation) implementations
- Pattern recognition

## VI. FUTURE WORK & CONCLUSION

In this paper, a sketch of different developments in the field of cellular automata is given. These developments provide a vast field for research in the field of cellular automata. Various tools can be used for the research. Using the theoretical concepts along with the tools, various new things can be done like with help of Cellular automata we can get a good help in 3D technology and 3D techno based games as it helps in the speed of showing graphics of the games as well as in the processing speed of the game which will also be fast as comapre to 2 D game. It will also give support to ANN technology because it will help to do parallel processing in easy way.

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