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Non sub sampled Contourlet Transform Application in Image Denoising

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***Abstract— in this paper, we have covered the compressed sensing non sub sampled Contourlet transform (CSNSCT) and its application for noise reduction. The construction of this paper is including the non sub sampled pyramid structure and non sub sampled directional filter banks. The focus of this scheme is the inseparable two-channel non sub sampled filter bank (NSFB). We cover less stringent design state of the NSFB to design filters that lead to a NSCT with well frequency selectivity compared to the Contourlet transform. Here the paper presents a design framework based on the mapping approach that allows for a fast implementation based on a lifting or ladder structure, and only uses one-dimensional filtering in some cases. In addition, our design ensures that the corresponding frame elements are regular, symmetric, and compressed so that it needs a less samples to represent a signal than the Nyquest rate. We assess the performance of the CSNSCT in image Denoising application*.**

Keywords— Contourlet transform, image denoising, multidimensional filter banks, nonsubsampled filter banks

I. INTRODUCTION

A number of image-processing tasks are efficiently carried out in the domain of an invertible linear transformation. For example, image compression and denoising are efficiently done in the wavelet transform domain [1], [2]. An effective transform captures the essence of a given signal or a family of signals with few basic functions. The set of basic functions completely characterizes the transform and this set can be redundant or not, depending on whether the basic functions are linear dependent. By allowing redundancy, it is possible to enrich the set of basic functions so that the representation is more efficient in capturing some signal behaviour. Thus, most state-of-the-art wavelet denoising algorithms (see for example [3]–[4]) use an expansion with less shift sensitivity than the standard maximally decimated wavelet decomposition—the most common being the non subsample wavelet transform (NSWT) computed with the à trousalgorithm [5].In addition to shift-invariance, it has been recognized that an efficient image representation has to account for the geometrical structure pervasive in natural scenes. In this direction, several representation schemes have recently been proposed [6]–[11].The Contourlet transform [10] is a multidirectional and multi scale transform that is constructed by combining the Laplacian pyramid [12], [13] with the directional filter bank (DFB) proposed in [14].

The pyramidal filter bank structure of the Contourlet transform has very little redundancy, which are important for compression applications. However, designing good filters for the Contourlet transform is a difficult task. In addition, due to down samplers and up samplers present in both the Laplacian pyramid and the DFB, the Contourlet transform is not shift-invariant.

In this paper, we propose an over complete transform that we call the non sub sampled Contourlet transform (NSCT). Our main motivation is to construct a flexible and efficient transform targeting applications where redundancy is not a major issue (e.g., denoising). The NSCT is a fully shift-invariant, multiscale, and multidirectional expansion that has a fast implementation. The proposed construction leads to a filter-design problem that to the best of our knowledge has not been addressed elsewhere Non sub sampled Contourlet is similar to Contourlet. The different part is non sub sampled Contourlet removes the process of sub sampling.

It is made up of a nonsubsampled pyramid and a non sub sampled directional filter bank. Non sub sampled Contourlet can provide shift-invariant and higher redundancy besides of most of the excellent property which Contourlet can provide [19].This enables us to design filters with better frequency selectivity there by achieving better sub band decomposition. Using the mapping approach we provide a framework for filter design that ensures good frequency localization in addition to having a fast implementation through ladders steps. The NSCT has proven to be very efficient in image denoising and image enhancements we show in this paper.

The paper is structured as follows. In Section II, we describe the NSCT and its building blocks. We introduce a pyramid structure that ensures the multiscale feature of the NSCT and the directional filtering structure based on the DFB. The basic unit in our construction is the non sub sampled filter bank (NSFB) which is discussed in Section II. In Section III, we study the issues associated with the CSNSCT design and implementation problems.

Applications of the NSCT in image denoising and enhancement are discussed in Section IV. Concluding remarks are drawn in Section V .Notation: Throughout this paper, a two-dimensional (2-D) filter is represented by its Z-transform $H(z)$ where $z = [z_1, z_2]^T$. Evaluated on the unit sphere, a filter is denoted by $H(e^{j\omega})$where $e^{j\omega} = [e^{j\omega_1}, e^{j\omega_2}]$. If

$m = [m_1, m_2]^T$ is a 2-D vector, then $z^m = z_1^{m_1}, z_2^{m_2}$ .whereas if M is a 2×2 matrix, then $z^m =, z^{m_1}, z^{m_2}$ with m_1, m_2 the columns of M. In this paper we often deal with zero-phase 2-D filters. On the unit sphere, such filters can be written as polynomials in $cos\omega = (cos\omega_1, cos\omega_2)^T$. We thus write $F(x_1, x_2)$ for a zero-phase filter in which and denote x_1 and x_2 denote $cos\omega_1$ and $cos\omega_2$ respectively.

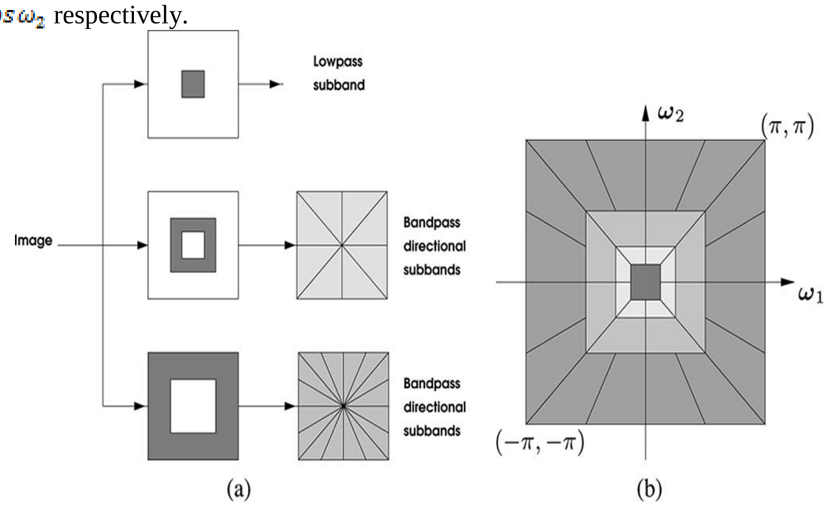


Fig.1. Non sub sampled Contourlet transform. (a) NSFB structures that implements the NSCT. (b) Idealized frequency partitioning obtained with the proposed structure [18].

Now the compressed sensing (compressive sample), it transforms the signal sampling to information sampling. It can sample signal with the sample rate much lower than which sample theorem requires. At the same time it can recovery signal completely.

II. COMPRESSED SENSING NONSUBSAMPLED CONTOURLET TRANSFORMS (CSNSCT) ALGORITHM

A. Non-sampled Contourlet transform

The Contourlet transform is a real 2-D image representation using cascade of Laplacian pyramid (LP) and a directional filter bank (DFB). The Contourlet transform can efficiently capture the intrinsic geometric structures such as contours in an image and can achieve better expression of image than the wavelet transform. Moreover, it is easily adjustable for detecting fine details in any orientation along curvatures, which results in more potential for effective analysis of images. However the Contourlet transform is lack of shift-invariance due to the down sampling and up sampling, In2006, Cunha et al. proposed the nonsubsampled Contourlet transformation (NSCT), which is a fully shift-invariant, multi-scale, and multi-direction expansion that has better directional frequency localization and a fast implementation. NSCT consists of two filter banks, i.e. the nonsubsampled pyramid filter bank (NSPFB) and the nonsubsampled directional filter bank (NSDFB) which split the 2-D frequency plane in the sub bands. The NSPFB provides nonsubsampled multi-scale decomposition and captures the point discontinuities. The NSDFB provides nonsubsampled directional decomposition and links point discontinuities into linear structures.

The different part is non sub sampled Contourlet removes the process of sub sampling. It is made up of a non sub sampled pyramid and a non sub sampled directional filter bank. Non sub sampled Contourlet can provide shift-invariant and higher redundancy besides of most of the excellent property which Contourlet can provide. Expanding basis function set can represent image information more flexible and more completely when certain amount of redundancy is admitted. Pyramid filters and DFB in non sub sampled Contourlet is non sub sampled, which derives two profits: the first, higher redundancy ensure the integrity of information and visual features in each sub-band which is transformed from original image by non sub sampled Contourlet. The second, according to multi-sample

Theory, there's no frequency spectrum mixing in low-pass sub-band by non sub sampled Contourlet, which can provide stronger direction decision. Suppose any signal in R^m can be expressed by linear combination of $N \times 1$ basis vector, and suppose these basis are standardized orthogonal. Let Vector

$\{\varphi_i\}_{i=1}^N \epsilon R^N$

be a column vector. to form a $N \times N$ basis matrix

$\Psi = [\Psi_1, \Psi_2, \ldots\ldots\ldots\ldots\ldots \Psi_N]$

Any time-domain signal x which is real, finite and discrete with one dimension can be expressed as:

$$x = \Psi\Theta = \sum_{i=1}^{N} \theta_i \Psi_i \qquad (1)$$

Where vector is coefficient of x which is sparsely represented (decomposed) in basis Ψ.

X and Θ are equal expresses of same signals, x is spatial express of the signal, and Θ is the express of signal. In Ψ-domain especially, if the non-zero number in vector Θ is K, it's called k-sparse. If Θ is sorted and attenuated with power law in basis Ψ, x is compressible. Generally speaking, signal is not sparse itself, but when it is transformed (like wavelet transform), the coefficients can be considered sparse. For example, when a signal is transformed by wavelet transform, we can reserve k bigger components of the coefficients, and set other n-k components

to zero(because their contribution to signal reconstruction is very small),and then obtain approximate reconstructed data by inverse wavelet transform. Thus x can be considered K-sparse in wavelet basis Ψ.
Let T be linear transformed by measurement matrix Φ□$R^{M\times N}$
Where measurement times M<<N, we can obtain measure result
Y=Φx=ΦΨΘ

(2)

Where=$\{y_i\}_{i=1}^{M}$ is considered as linear projection. The dimension of y is much lower than the dimension of x. When x is reconstructed by y, it can be exactly reconstructed with a high probability from measurement results by solving the optimal problem of $l_0$norm.

$\tilde{x} = \arg\min \|x\|_0$ s.t. y = Φx (3)
The problem of solving l0 norm is a NP-hard problem,
So we can change the problem to
$\tilde{x} = \arg\min \|x\|_1$s.t. y = Φx (4)

Where : the optimal problem with smallest norm. Algorithms like Match Pursuit, Orthogonal Match Pursuit, Gradient Projection, Chain Pursuit etc. are current solutions. In original CS algorithm of image processing, a N×N image was firstly transformed by certain transform, like DCT transform, or wavelet transform, then a measurement matrix F was formed,(measurement matrix could be random Gaussian matrix which obeys (0,1/N) distribution, or ±1 Bernoulli matrix. namely Noise let, etc.)All of the wavelet transform coefficients are measured by F, and then M×N measurement coefficients are obtained. When recovering image, original image can be recovered by F and M×N measurement coefficients with OMP algorithm. During the research, it is found that in original CS algorithm, wavelet decomposition level has significant impact on reconstruction results. The less the decomposition level is, the worse the reconstruction results is. With the increase of the decomposition reconstruction effects will be improved. That's because original image could be decomposed to low-frequency sub-band and high-frequency
Sub-band by wavelet decomposition. High-frequency sub-band can be considered sparse, but low-frequency sub-band is the approach signal of original image under different scales, it cannot be considered sparse. When measurement matrix F is multiplied by low-frequency and high-frequency coefficients together, the correlation among low-frequency approximate components coefficients will be damaged, which will deteriorate reconstruction results. When the number of wavelet decomposition level is 1, the reconstruction image is completely different from its origin. So, the wavelet decomposition level should be as large as possible. Even so, the recovered image quality is less than satisfactory. In this paper the CS algorithm is based on NSCT. The method of CS algorithm based on NSCT is as following:

1. Decomposing the N×N size image by NSCT, getting the coefficients of high-frequency and low-frequency sub-bands [19].
2. Selecting suitable value of M to get the measurement matrix F which is M × N / 2 size and Gaussian distribution, measuring the high-frequency sub-band coefficients.
3. Using OMP algorithm [19] to reconstruct the high-frequency sub-band coefficients, and combining the low-frequency sub-band coefficients to do inverse transformation of NSCT to get the recovery image.

III. PROPOSED ALGORTHM NONSUBSAMPLED CONTOURLETS AND FILTER BANKS

A. Non sub sampled Contourlet Transform

Fig. 1(a) displays an overview of the proposed NSCT. The structure consists in a bank of filters that splits the 2-D frequency plane in the sub bands illustrated in Fig. 1(b). Our proposed transform can thus be divided into two shift-invariant parts: 1) a non sub sampled pyramid structure that ensures the multiscale property and 2) a non sub sampled DFB structure that gives directionality.1) Non sub sampled Pyramid (NSP): The multiscale property of the NSCT is obtained from a shift-invariant filtering structure that achieves a sub band decomposition similar to that of the

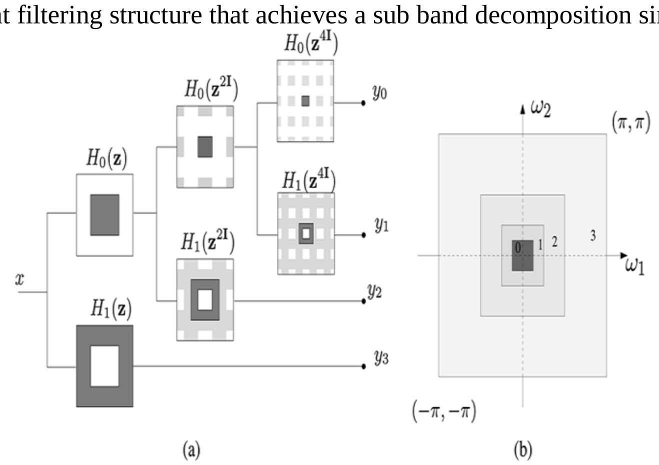


Fig.2. proposed non sub sampled pyramid is a 2-D multiresolution expansion similar to the 1-D NSWT. (a) Three-stage pyramid decomposition. The lighter gray regions denote the aliasing caused by up sampling. (b) Sub bands on the 2-D frequency plane [18].

Laplacian pyramid- This is achieved by using two-channel non sub sampled 2-D filter banks. Fig. 2 illustrates the proposed non sub sampled pyramid (NSP) decomposition with J=3 stages. Such expansion is conceptually similar to the one–dimensional (1-D) NSWT computed with the à trous algorithm [5] and has J+1 redundancy, redundancy, where J denotes the number of decomposition stages. The ideal pass band support of the low-pass filter at the jth stage is [-(π/2j)] , (π/2j)]2

The region accordingly, the ideal support of the equivalent high-pass filter is the complement of the low-pass, i.e., the region.

[(-π/2j-1), (π/2j-1)] 2/ [(-π/2j), (π/2j)] 2

The filters for subsequent stages are obtained by up sampling the filters of the first stage. This gives the multi scale property without the need for additional filter design. The proposed structure is thus different from the separable NSWT. In particular, one band pass image is produced at each stage resulting in J+1 redundancy. By contrast, the NSWT produces three directional images at each stage, resulting in 3J+1 redundancy. The 2-D pyramid proposed in [15] pp. 21 is obtained with a similar structure. Specifically, the NSFB of [15] is built from low-pass filter $H_0($. One then sets $H_1(z) = 1 - H_0($and the corresponding synthesis filters. $G_1(z) = G_0(z)$ =A similar decomposition can be obtained by removing the down samplers and up samplers in the Laplacian pyramid and then up sampling the filters accordingly. Those perfect reconstruction systems can be seen as a particular case of our more general structure. The advantage of our construction is that it is general and as a result, better filters can be obtained. In particular in our design and are low-pass and high-pass. Thus, they filter certain parts of the noise spectrum in the processed pyramid coefficients.2) Non sub sampled Directional Filter Bank (NSDFB): The directional filter bank of Bamberger and Smith [14] is constructed by combining critically-sampled two-channel fan filter banks and re sampling operations. The result is a tree-structured filter bank that splits the 2-D frequency plane into directional wedges. A shift-invariant directional expansion is obtained with a non sub sampled DFB (NSDFB). The NSDFB is constructed by eliminating the down samplers and up samplers in the DFB (see also [16]). This is done by switching off the down samplers/ up samplers in each two-channel filter bank in the DFB tree structure and up sampling the filters accordingly. This results in a tree composed of two-channel NSFBs. Fig. 3 illustrates a

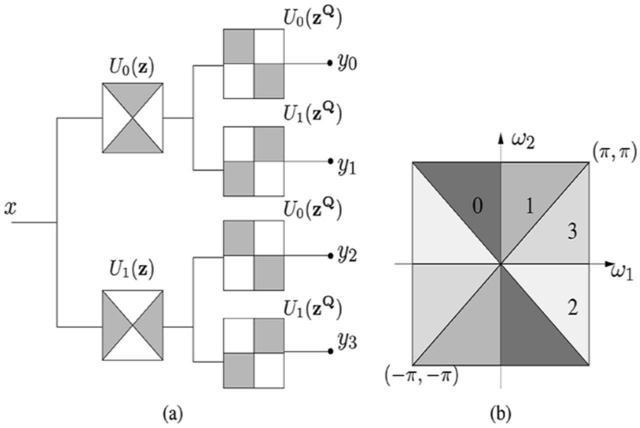


Fig.3. Four-channel non sub sampled directional filter bank constructed with two-channel fan filter banks. (a) Filtering structure. (b) Corresponding frequency decomposition [18].

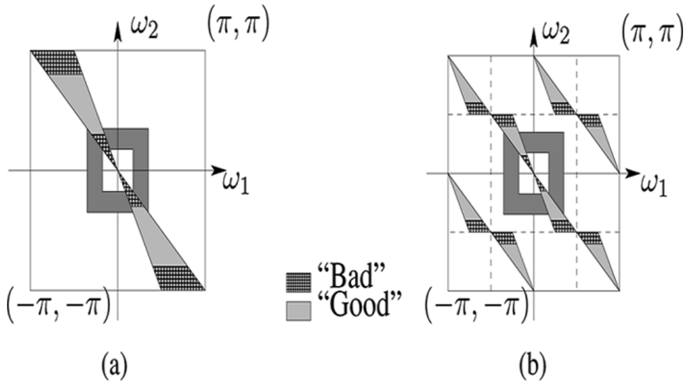


Fig.4. Need for up sampling in the NSCT. (a) With no up sampling, the high pass at higher scales will be filtered by the portion of the directional filter that has "bad" response. (b) Up sampling ensures that filtering is done in the "good" region [18].

Four channel decomposition. Note that in the second level, the fan filters $U_i(z^Q), i = ($have checker- board frequency support, and when combined with the filters in the first level give the four directional frequency decomposition shown in Fig. 3. The synthesis filter bank is obtained similarly. Just like the critically sampled directional filter bank, all filter banks in the non sub sampled directional filter bank tree structure are obtained from a single NSFB with fan filters [see Fig. 5(b)]. Moreover, each filter bank in the NSDFB tree has the same computational complexity as that of the building-block NSFB.

IV. IMAGE DENOISING

In order to illustrate the potential of the NSCT designed using the techniques previously discussed; we study additive white Gaussian noise (AWGN) removal from images by means of thresholding estimators.

1) Comparison to Other Transforms:

To highlight the performance of the NSCT relative to other transforms, we perform hard threshold on the sub band coefficients of the various transforms. We choose the threshold

$T_{i,j} = K\sigma_{n_{i,j}}$

for each sub band. This has been termed K-sigma thresholding. We set K=4 for the finest scale and K=3 for the remaining ones. We use five scales of decomposition for both

2) Comparison to Other Denoising Methods:

We perform soft thresholding (shrinkage) independently in each sub band. Following [6], we choose the threshold

$T_{i,j} = \sigma^2_{N_{i,j}} / \sigma_{n_{i,j,n}}$

Where σ_n denotes the variance of the -th coefficient at the -th directional sub band of the j-th scale, and $\sigma$is the noise variance at scale and direction. It is shown in [6] that shrinkage estimation with $T = \sigma^2 /$ and assuming X generalized Gaussian distributed yields a risk within 5% of the optimal Bayes risk. As studied in [35], Contourlet coefficients are well modelled by generalized Gaussian distributions. The signal variances are estimated locally using the neighbouring coefficients contained in a square window within each sub band and a maximum likelihood estimator. The noise variance in each sub band is inferred using a Monte Carlo technique where the variances are computed for a few normalized noise images and then averaged to stabilize the results. We refer to this method as local adaptive shrinkage (LAS). Effectively, our LAS method is a simplified version of the Denoising method proposed in [36] that works in the NSCT or NSWT domain. In the LAS estimator we use four scales for both the NSCT and NSWT. For the NSCT we use 3, 3, 4, 4 directions in the scales from coarser to finer, respectively. To benchmark the performance of the NSCT-LAS scheme, we have used two of the best Denoising methods in the literature: 1) Bivariate shrinkage with local variance estimation (BivShrink) [7]; 2) Bayes least squares with a Gaussian scale-mixture model (BLS-GSM) proposed in [8]. Table III (right columns) shows the results obtained.3 The NSCT coupled with the LAS estimator (NSCT-LAS) produced very satisfactory results.

In particular, among the methods studied, the NSCT-LAS yields the best results for the "Barbara" image, being surpassed by the BLS-GSM method for the other images. Despite its slight loss in performance relative to BLS-GSM, we believe the NSCT has potential for better results. This is because by comparison, the BLS-GSM is a considerably richer and more sophisticated estimation method than our simple local thresholding estimator. However, studying more complex Denoising methods in the NSCT domain is beyond the scope of the present paper. Fig. 11 displays the Denoised images with both BLS-GLM and NSCT-LAS methods. As the pictures show, the NSCT offers a slightly better reconstruction. In particular, the tablecloth texture is better recovered in the NSCT-LAS scheme. We briefly mention that in Denoising applications, one can reduce the redundancy of the NSCT by using critically sampled directional filter banks over the nonsubsampled pyramid. This results in a transform with J+1 redundancy which is considerably faster. There is however a loss in performance as Table IV shows. Nonetheless, in some applications, the small performance loss might be a good price to pay given the reduced redundancy of this alternative construction.

V. RESULT AND DISCUSSION

In In this work I have discussed about results obtain by implementing the algorithm and Denoised images obtained using NSS Contourlet transform with increases sampled rate. The diagrammatical representations used in our test wavelet, Contourlet, Nonsubsampled transform.

A. Digramitical Representation

this figure we compare the all methods.firstly we take original barbar image in first image after that add noise in this image we see in second figure then find denoise resuilt throgh wavelet in third image ,in fourth image find denoising result through contourlet transform,in fifth image we see the previous result and in last image we get the our proposed result.then find in each image SNR in increasing order.

A

B

C

D E F

Figure 5.1 (A) Original image, (B) Noisy image (9.54db), (C) Denoise using wavelete (12.79db), (D) Denoise contourlet(SNR=21.48db), (E) In previous method denoise using NSS counterlet transform (SNR=22.31db), (F) in proposed method denoise using NSS counterlet transform(SNR=25.27db)

B. Tabular Representation

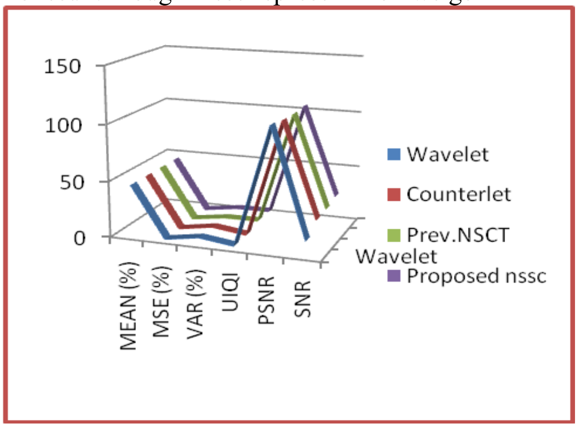
Tabular respresentation shows the comparison of performance parameters like MEAN, MSE, VARIENCE, UIQI, PSNR AND SNR for methods like wavelet,contourlet of base paper and our's result.we can find which method is best for denoising images

Table 5.1 Tabular representation of comparative analysis of different methods by different parameters.

Method	**MEAN (%)**	**MSE (%)**	**VAR (%)**	**UIQI**	**PSNR**	**SNR**
Wavelet	45.86	00.11	5.05	1.1915	107.5985	12.79
Counter let	45.86	00.14	05.07	1.1892	105.1103	21.48
Prev. NSCT	45.87	00.15	04.42	4.7865	104.5375	22.31
Proposed NSCT	45.87	00.14	04.42	4.7729	105.3738	25.27

C. Graphical Representation

Graphical representation shows the different result in some previous method base paper and our's result.Blue colour shows the wavelet transform method,red colour shows contourlet transform,green colour represent base paper result and last ond violet colour represent our's result.through these representation we get the best method of denoising.



VI. CONCLUSION

The design of the NSCT is reduced to the design of a non sub sampled pyramid filter bank and a non sub sampled fan filter bank. We exploit this new less stringent filter-design problem using a mapping approach, thus dispensing with the need for 2-D factorization. We also developed a lifting/ladder structure for the 2-D NSFB. This structure, when coupled with the filters designed via mapping, provides a very efficient implementation that under some additional conditions can be reduced to 1-D filtering operations. The results of simulation indicate that the NSCT provides better performance than the curvelets. There is a lot of scope with compressed sensing image processing using non sub sampled Contourlet transform [19]. A MATLAB toolbox that implements the NSCT can be downloaded from MATLAB Central (http://www.mathworks. com/matlabcentral/).

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