Description of Attraction-Repulsion Forces by Probabilistic Fuzzy Logic for Handling Uncertainty in Swarm Aggregations

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Abstract—Attraction-\-repulsion forces are widely used to describe the interactions among individuals in natural and artificial swarm aggregations from macro to nano scale. In most of the previous works, it is assumed that these forces are mathematically describable by deterministic functions. But there exist two crucial problems for mathematical modeling of attraction-\-repulsion forces in real swarms especially in micro and nano scales. First, taking sampling data by measuring the available forces among individuals is difficult and costly; therefore, system identification techniques cannot be used for finding crisp mathematical models. Instead, experts can usually describe forces linguistically based on the acquired knowledge from observations. Second, due to the presence of non\-deterministic uncertainty in addition to deterministic uncertainty, forces among individuals cannot be described deterministically; thus, the stochastic behaviour of forces should be considered in mathematical modeling. In contrast to the prior researches, this paper benefits from probabilistic fuzzy logic to address both of the above-mentioned problems. Simulation results demonstrate that the proposed approach is an efficient method for handling uncertainty in swarm aggregations.

Keywords—Swarm Aggregation - Attraction-\-Repulsion Forces – Deterministic Uncertainty –Non-deterministic Uncertainty - Probabilistic Fuzzy Logic

I. INTRODUCTION

In recent years, considerable researches have been performed on multi-agent systems and specially swarm intelligence. This field of research is currently recognized as a promising applied paradigm in distributed artificial intelligence. Swarm intelligence have a very wide range of applications from mathematics, physics and biology such as modeling the dynamics of ants, bees, moving particles and micro-organisms to engineering problems like distributed computing, wireless sensor networks, traffic control, and swarm robotics. More importantly, swarm intelligence can be considered as a powerful approach for realizing collective artificial intelligence in swarms of very simple nano-scale agents that is a hot research topic in today's nanorobotics ([1]-[5]).

Previous studies on the behaviour of natural swarms in physics and biology demonstrates that the cooperative behaviour among particles or individuals in a swarm have very important benefits in reaching the final goal like finding food, avoiding enemy, reaching nest, etc. Ant colony, bees, bacteria, fishes and birds are all well-known samples of natural swarms. The central question in studying the behaviour of swarms is: How can a swarm reach its final goal intelligently while its individuals are very simple and they individually have not any notable intelligence? Studies in swarm intelligence have answered this question. Indeed, the mystery of success in a swarm lies in the interactions among individuals. A famous sentence of “1+1=3” points to this fact that good local interactions among very simple individuals in a swarm can lead to an emerging collective intelligence, called swarm intelligence, which can globally generate complex self-\-organized patterns.

Inspired by the dynamics of particle swarms in physics and biology, in recent decades, special focus has been performed on artificial attraction-\-repulsion forces to mathematically model the interactions among individuals in a swarm. In this paradigm, the force that two individuals apply to each other is a function of their distance from each other. Previous researches demonstrate that this approach can be widely used for modeling the dynamics of swarms from macro to nano; however it is promisingly applicable for natural and artificial micro and nano scale swarms. Some of the previous works are reviewed.

Different studies have been performed on the effect of interactions among individuals on the dynamics of natural and artificial swarms ([6]-[9]). Gazy and Passino ([10]-[12]) mathematically modeled the emerging collective behaviour of swarms by attraction-\-repulsion forces among individuals and introduced the fundamental concept of swarm stability as an analytical tool for studying swarm dynamics. Moreover, Liu and Passino [13] considered swarm stability in the presence of uncertainty, in which, noise takes effect on the interactions among individuals. Many researches have been done on mathematical modeling of swarm aggregation by attraction-\-repulsion forces and different models have been proposed ([14]-[25]). These models, as basic algorithms, can be broadly applied to simulate multi-agent and multi-robot systems ([26]-[28]). For example in [29], attraction and repulsion forces are employed for target tracking and obstacle
avoidance, respectively. In comparison to the earlier works, in [30], fuzzy logic is employed for modeling the dynamics and stability of swarm aggregation, where force functions are described by fuzzy if-then rules.

In most of the previous works, it is assumed that these forces can be mathematically modeled by deterministic functions, either crisp or fuzzy. But in this paper, we show that there exist two crucial problems for mathematical modeling of attraction-repulsion forces in natural swarms especially in micro and nano scales which cannot be simultaneously addressed by previous works. In contrast, this paper proposes a novel approach based on probabilistic fuzzy logic to deal with both of these problems. Simulation results demonstrate that the proposed approach is an efficient method for handling both types of deterministic and non-deterministic uncertainty in swarm aggregations.

II. PROPOSED APPROACH

A. Conventional Model of Swarm Aggregation

Fig. 1 shows a swarm of $N$ individuals, in which, interactions among individuals can be described by attraction-repulsion forces. The force between each two individuals is a function of the distance between them. The velocity of each individual is highly affected by the forces that other individuals apply to it. According to the prior researches in the literature ([10]-[13]), the governing equation of motion for each individual in the swarm of Fig. 1 is:

$$\ddot{x}_i = \sum_{j=1, j \neq i}^{N} F(\|x_i - x_j\|)(x_i - x_j), \quad i = 1, ..., N \tag{1}$$

where $\dot{x}_i = [x_{1i}, x_{2i}, ..., x_{ni}] \in \mathbb{R}^n$ is the position of individual $i$ in $n$-dimensional space, $x_i$ is the position of individual $j$, $F(\cdot): \mathbb{R} \to \mathbb{R}$ is the force that individual $j$ applies to individual $i$ and $\dot{x}_i$ is the velocity of individual $i$.

![Fig1. Inter-individual interactions in form of attraction-repulsion forces in the presence of both types of uncertainty.](Image)

Different functions have been already proposed for $F(\cdot)$ in the literature ([6]-[30]). Inspired by these works, in this paper, we focus on the following force function:

$$F(d_{ij}) = f_r(d_{ij}) - f_a(d_{ij}) + f_n(d_{ij}) \tag{2}$$

where $d_{ij} = \|x_i - x_j\|$ is the Euclidean distance between individuals $i$ and $j$, $f_r$ and $f_a$ are positive force functions which describe repulsion and attraction, respectively. $f_n(d_{ij})$ is the stochastic term of force (noise) whose characteristics can be a function of $d_{ij}$. Usually, the mean of $f_n(d_{ij})$ is zero for all distances and there exists an equilibrium distance between two individuals, in which, attraction and repulsion forces are equal. If distance is less than this value, $F$ is repulsive ($f_r > f_a$), and repulsion is increased by decreasing the distance. If distance is larger than equilibrium distance, $F$ is attractive ($f_r < f_a$), and attraction is increased by increasing the distance (until reaching the threshold distance).

B. Inter-individual Interactions in Presence of Uncertainty

It is very important to note that in most of the previous researches, it is assumed that the deterministic terms of force in Eq. 2 (i.e. $f_r$ and $f_a$) can be mathematically modeled by deterministic functions including crisp functions (in most cases) and fuzzy modeling (in rare cases). Also, the non-deterministic term of force (i.e. $f_n$) is supposed to be modeled via one of the familiar probability distribution functions (PDFs). To obtain accurate model for inter-individual interactions, the model of forces should be very close to actual forces. But there exist two crucial problems for mathematical modeling of forces in natural swarms especially in micro and nano scales. First, taking sampling data by measuring the available forces among individuals is difficult and costly; therefore, system identification techniques cannot be used for finding crisp mathematical models. Instead, usually experts can linguistically describe forces based on the acquired knowledge from observations. Second, due to the presence of stochastic uncertainty, forces among individuals cannot be described deterministically; thus, the stochastic behavior of forces should be considered in mathematical modeling.

As mentioned in Section 1, only a few of previous works have used fuzzy logic to address the first problem, but their proposed approaches are not powerful enough to deal with the second problem. It should be mentioned that fuzzy logic is a very good technique for handling deterministic uncertainty. But real world is full of the both types of uncertainty including deterministic (possibilistic) and non-deterministic (probabilistic or stochastic). So, fuzzy logic cannot individually deal with non-deterministic uncertainty. On the other hand, probability theory is a well-known classical method for working on this type of uncertainty. But as a fundamental feature of probability theory, it is based on conventional mathematics which uses crisp functions for describing probability distribution functions. Therefore, finding accurate approximation for the crisp functions of PDFs still has all difficulties of the first problem mentioned above, and probability theory is not individually desirable and applicable for our purpose. As an alternative, we have been inspired
by the sentence of Prof. Zadeh, the father of fuzzy logic and computing with words, which says the fuzzy logic and probability theory are complementary rather than competitive. So, we must look for a synergic combination of fuzzy logic and probability theory. In this paper, we benefit from Probabilistic Fuzzy Logic, as a powerful method in soft computing ([31]-[36]), to simultaneously handle deterministic and non-deterministic uncertainty in mathematical modeling of forces. In the proposed method, force is modeled by a probabilistic fuzzy system. In contrast to the previous works, the proposed approach could efficiently address both of the above-mentioned problems.

C. Probabilistic Fuzzy Approach

Regarding the important property of universal function approximation, fuzzy systems are very efficient approach for handling deterministic uncertainty which plays a central role in approximating functions using expert’s linguistic knowledge. In fuzzy systems, the consequent part of a fuzzy if-then rule consists of only one fuzzy set:

\[ \text{Rule } l: \text{ if } d \text{ is } A^l \text{ then } \tilde{F}_f \text{ is } B^l \]

And the relation between input \((d)\) and output \((\tilde{F}_f)\) is:

\[ \tilde{F}_f(d) = \frac{\sum_{l=1}^{L} P_f(d) C_{Bl}}{\sum_{l=1}^{L} P_f(d)} \quad (3) \]

where \(\mu_{A^l}(.)\) is the membership function (MF) of \(A^l\) (the antecedent part of the \(l^{th}\) rule), \(C_{Bl}\) is the center of \(B^l\) (the consequent part of the \(l^{th}\) rule), and \(M\) is the number of rules in fuzzy rule base. Comparing the above equation with Eq.2 demonstrates that a fuzzy system is able to approximate the deterministic part of force \((f_i(d) - f_a(d))\), but cannot handle stochastic part \((f_n(d))\). In contrast to fuzzy systems, in probabilistic fuzzy systems, the consequent part of a probabilistic fuzzy rule contains different fuzzy sets \((B_{1l}, B_{2l}, \ldots, B_{Kl})\) with different corresponding probabilities \((p_{1l}, p_{2l}, \ldots, p_{Kl})\):

\[ \text{Rule } l: \text{ if } d \text{ is } A^l \text{ then } \tilde{F}_{pf} \text{ is } \]

\[ B_{1l} \text{ with probability } p_{1l} \text{ and } B_{2l} \text{ with probability } p_{2l} \text{ and } \ldots \]

\[ B_{Kl} \text{ with probability } p_{Kl} \]

where \(p_{1l} + p_{2l} + \ldots + p_{Kl} = 1\).

And the relation between input \((d)\) and output \((\tilde{F}_{pf})\) is:

\[ \tilde{F}_{pf}(d) = \frac{\sum_{l=1}^{L} P_{pf}(d) C_{Bl}}{\sum_{l=1}^{L} P_{pf}(d)} \quad (4) \]

where \(C_{Bl}^*\) is the center of \(B_{Kl}^*\) that is selected among \(B_{1l}, B_{2l}, \ldots, B_{Kl}\) by a random selection mechanism according to the corresponding probability values \(p_{1l}, p_{2l}, \ldots, p_{Kl}\). It should be mentioned that in this study, we use Mamdani inference engine, centers average defuzzifier and roulette wheel selection mechanism. Fig.2 schematically shows the roulette wheel selection mechanism as well as the probability vector that have been used in the simulation study of the present paper. More details about probabilistic fuzzy systems are available in ([31]-[36]). Comparing Eq.4 with Eq.2 shows that probabilistic fuzzy system can simultaneously approximate both deterministic and non-deterministic parts of force.

It is very important to note that the proposed approach is not restricted to familiar PDFs such as uniform and normal distributions, when any arbitrary probability distribution can be easily approximated by probabilistic fuzzy logic. Therefore, probabilistic fuzzy systems can be considered as efficient approach for mathematical modeling of the force of Eq.2. Finally, after approximating the actual force of Eq.2 by the probabilistic fuzzy system of Eq.4, the governing equation of swarm in Eq.1 is converted to:

\[ \dot{x}^i = \sum_{j=1, j \neq i}^{N} P_{pf}(\|x^i - x^j\|)(x^i - x^j) \quad , \quad i = 1, \ldots, N \quad (5) \]

Simulation results in the next section demonstrate that the proposed approach can approximate the dynamics of the given actual swarm accurately in the presence of both types of uncertainty.
In this section, we aim to demonstrate the efficiency of the proposed approach in approximating the dynamics of actual swarms in comparison with fuzzy approach. Without loss of generality, swarms have been simulated in two-dimensional space in MATLAB. For this purpose, we have simulated three different swarms. First, the actual swarm of Eq.1 with the actual force of Eq.2, where $f_r(d_{ij}) = \frac{6}{d_{ij}^{2}}$, $f_a(d_{ij}) = \frac{4}{d_{ij}}$, and $f_n(d_{ij})$ is a zero-mean Gaussian noise whose standard deviation is proportional to $d_{ij}$. Fig.3 represents this force. Second, the approximated model of Eq.5 with fuzzy force of Eq.3, where the fuzzy system has 11 rules. Fig.4 shows the antecedent MFs and the centers of the consequent MFs of the fuzzy system. Third, the approximated model of Eq.5 with probabilistic-fuzzy force of Eq.4, where the probabilistic fuzzy system has 11 rules and its antecedent MFs are as same as the fuzzy system of Fig.4. The consequent part of each probabilistic fuzzy rule contains 14 MFs and 14 probability values which are displayed in Fig.5. In this simulation, each swarm has 9 individuals.

Fig.6 depicts the trajectory of individuals from three random initial positions for the above-mentioned three swarms. This figure compares the generated spatial patterns by the actual swarm with fuzzy model and the proposed probabilistic fuzzy model. Also, Fig.7 illustrates the temporal changes in the position of one of the individuals (for example) for each test of Fig.6. The spatial and temporal patterns of Figures 6 and 7 clearly demonstrate that the generated pattern by fuzzy model is deterministic and could not approximate the stochastic behaviour of the actual swarm. In contrast, the proposed probabilistic fuzzy model could efficiently approximate the deterministic and non-deterministic dynamics of the actual swarm and generate similar patterns. These results show that the proposed approach can be considered as a powerful modeling tool for approximating the global spatial-temporal patterns generated by actual swarms, especially in micro and nano scales which are full of deterministic and non-deterministic uncertainty.
Fig. 6. Trajectory of individuals from 3 random initial positions (denoted by cross ‘x’). This figure compares the actual swarm with fuzzy model and the proposed probabilistic fuzzy model.
Fig 7. The temporal changes in the position of one of the individuals for Tests a-c of Fig.6. In each case, the top, middle and bottom plots are related to actual, fuzzy and probabilistic fuzzy swarms, respectively.

IV. CONCLUSIONS

In many natural and artificial swarms, especially in micro and nano scales, interactions among individuals can be described by attraction-repulsion forces. In most of the previous works, these forces are mathematically modeled by deterministic functions, either crisp or fuzzy, that cannot efficiently handle non-deterministic uncertainty in modeling. In this paper, we benefits from probabilistic fuzzy logic to simultaneously handle deterministic and non-deterministic uncertainty in mathematical modeling of forces. The performance of the proposed approach was compared with fuzzy method in approximating the dynamics of a given actual swarm thorough computer simulation. The results demonstrated that the proposed probabilistic fuzzy model could effectively model both deterministic and non-deterministic dynamics of the actual swarm and generate similar patterns, while the generated pattern by fuzzy model was deterministic and could not approximate the stochastic behavior of the actual swarm. Generally, the new approach is a promising modeling technique for accurate approximation of the global spatial-temporal patterns generated by actual swarms, especially in micro and nano scales, where the dynamics of swarms are highly affected by both deterministic and non-deterministic uncertainty.

REFERENCES


