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# Study of Numerical Analysis – Differential Equation

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***Abstract: This paper is devoted to the numerical comparison of methods applied to solve an integro-differential Equation. Differential equations can describe nearly all systems undergoing change. They are ubiquitous is science and engineering as well as economics, social science, biology, business, health care, etc. Many mathematicians have studied the nature of these equations for hundreds of years and there are many well-developed solution techniques. Often, systems described by differential equations are so complex, or the systems that they describe are so large, that a purely analytical solution to the equations is not tractable. It is in these complex systems where computer simulations and numerical methods are useful. The techniques for solving differential equations based on numerical approximations were developed before programmable computers existed. During World War II, it was common to find rooms of people (usually women) working on mechanical calculators to numerically solve systems of differential equations for military calculations.***

***Keywords: PDE, RK, ODE***

## I. INTRODUCTION

A differential equation is an equation involving derivatives of an unknown function and possibly the function itself as well as the independent variable

$$y' = \sin(x), \quad (y')^4 - y^2 + 2xy - x^2 = 0, \quad y'' + y^3 + x = 0$$

1st order equations

1st order equations

The order of a differential equation is the highest order of the derivatives of the unknown function appearing in the equation

In the simplest cases, equations may be solved by direct integration

Examples

$$y' = \sin(x) \Rightarrow y = -\cos(x) + C$$

$$y'' = 6x + e^x \Rightarrow y' = 3x^2 + e^x + C_1 \Rightarrow y = x^3 + e^x + C_1 x + C_2$$

Observe that the set of solutions to the above $1^{st}$ order equation has 1 parameter, while the solutions to the above $2^{nd}$ order equation depend on two parameters.

## II. TYPES OF DIFFERENTIAL EQUATION

**2.1 Ordinary D.E:** Equations which are composed of an unknown function and its derivatives are called differential equations.Differential equations play a fundamental role in engineering because many physical phenomena are best formulated mathematically in terms of their rate of change.

$$\frac{dv}{dt} = g - \frac{c}{m} v$$

$v$- dependent variable

$t$- independent variable

When a function involves one dependent variable, the equation is called an *ordinary differential equation (or ODE).*

**2.2 Partial D.E.**

A partial differential equation (or PDE) involves two or more independent variables.

Differential equations are also classified as to their order. A first order equation includes a first derivative as its highest derivative. A second order equation includes a second derivative.

Higher order equations can be reduced to a system of first order equations, by redefining a variable.

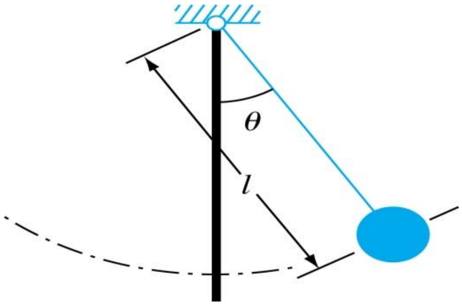



Fig (1) ODEs and Engineering Practice

## III. RUNGA-KUTTA METHODS

Runge-Kutta method here after called as RK method is the generalization of the concept used in Modified Euler's method.In Modified Eulers method the slope of the solution curve has been approximated with the slopes of the curve at the end points of the each sub interval in computing the solution. The natural generalization of this concept is computing the slope by taking a weighted average of the slopes taken at more number of points in each sub interval. However, the implementation of the scheme differes from Modified Eulers method so that the developed algorithm is explicit in nature. The final form of the scheme is of the form

$\mathbf{y_{i+1} = y_i +}$ **(**weighted average of the slopes**)** for **i = 0, 1, 2 . . .**

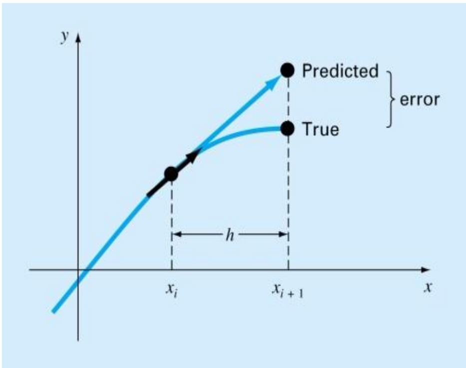
$$\frac{dy}{dx} = f(x, y)$$

## IV. EULER'S METHOD

Though in principle it is possible to use Taylor's method of any order for the given initial value problem to get good approximations, it has few draw backs like.

The scheme assumes the existence of all higher order derivatives for the given function**f(x,y)** which is not a requirement for the existence of the solution for any first order initial value problem.Even the existence of these higher derivatives is guaranteed it may not be easy to compute them for any given **f(x,y).** Because of the usage of higher order derivatives in the formula it is not convenient to write computer programs , that is the method is more suited for hand calculations. To overcome these difficulties, Euler developed scheme by approximating **y'** in the given ivp. The scheme is as follows:
The derivative term in the first order ivp

$\mathbf{y' = f(x, y) , y(x_0) = y_0}$




Fig(2) Euler's Method

The first derivative provides a direct estimate of the slope at $x_i$

$$\phi = f(x_i, y_i)$$

where $f(x_i,y_i)$ is the differential equation evaluated at $x_i$ and $y_i$. This estimate can be substituted into the equation:

$$y_{i+1} = y_i + f(x_i, y_i)h$$

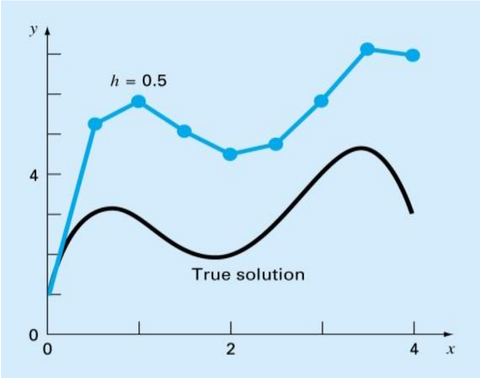
A new value of $y$ is predicted using the slope to extrapolate linearly over the step size $h$.

$$\frac{dy}{dx} = f(x, y) = -2x^3 + 12x^2 - 20x + 8.5$$

$$Starting \quad point \quad x_0 = 0, \quad y_0 = 1$$

$$y_{i+1} = y_i + f(x_i, y_i)h = 1 + 8.5 * 0.5 = 5.25$$




Not good

## V. ERROR ANALYSIS FOR EULER'S METHOD

**Numerical solutions of ODEs involves two types of error:**

**Truncation error:** In numerical analysis and scientific computing, truncation error is theerror made by truncating an infinite sum and approximating it by a finite sum. For instance, if we approximate the sine function by the first two non-zero term of its Taylor series, as in for small , the resulting error is a truncation error.

**Local truncation error:**

The local truncation error is the error that our increment function, , causes during a single iteration, assuming perfect knowledge of the true solution at the previous iteration.

$$E_a = \frac{f'(x_i, y_i)}{2!} h^2$$

$$E_a = O(h^2)$$

Propagated truncation error

The sum of the two is the total or global truncation error

Round-off errors

The Taylor series provides a means of quantifying the error in Euler's method. The Taylor series provides only an estimate of the local truncation error-that is, the error created during a single step of the method.In actual problems, the functions are more complicated than simple polynomials. Consequently, the derivatives needed to evaluate the Taylor series expansion would not always be easy to obtain.

## VI. CONCLUSION

The error can be reduced by reducing the step size. If the solution to the differential equation is linear, the method will provide error free predictions as for a straight line the 2$^{nd}$ derivative would be zero. Because we can choose an infinite number of values for $a_2$, there are an infinite number of second-order RK methods. Every version would yield exactly the same results if the solution to ODE were quadratic, linear, or a constant. However, they yield different results if the solution is more complicated (typically the case).

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**PROFILE**

I **(Prof. Rakesh Jaiswal)** have more than 15 Years of Experience in academics and administration. I have attended many conference and Seminar at National and International Level. I would like to thank my colleagues of the Christian Eminent College, Indore, M.P. , INDIA for their contributions, insights, and support.

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