Design and Analysis of Optimized Octave-Band Filter Bank with Low Complexity

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Abstract—This paper introduces an optimized tree-structure design of an octave-band filter bank with low complexity and near perfect reconstruction (NPR) with small amount of distortion at the output. A linear iterative optimization technique is used to minimize the distortion. Variable window functions are used to design prototype finite impulse response (FIR) lowpass filter. The proposed design has the property of linear phase and provides simplicity.

Keywords—Tree-structure, filter bank, NPR, octave-band, optimization.

I. INTRODUCTION

Multirate filter banks have found various applications in many different areas such as speech coding, scrambling, adaptive signal processing, image compression as well as transmission of several signals through the same channel, digital audio industry and many other fields [1-4]. Depending upon pass band width, filter banks mainly classified in to two categories i.e. uniform-band filter bank and octave-band filter bank. Uniform filter banks has equal pass band width and normally used for processing the signal of same nature [1, 5]. However in audio analysis and coding and broadband array signal processing applications octave-band filter banks are required for processing the signals of different natures [6-8] because they have unequal pass band width. Octave-band filter bank provide any kind of integer and rational decimation in each band, low computational complexity, any extent of time frequency resolution as per requirement of application and less quantization error.

In last few years several design methods and optimization algorithm [2, 10-14] have been proposed by different authors for designing multirate filter bank. Tree-structure is a simple and effective technique for designing filter bank with linear phase property. Depending on the filter bank used in design [15] it can be either perfect reconstruction (PR) or near perfect reconstruction (NPR) type. In this paper tree-structured technique in which two-band filter bank [18] is work as a basic building block as shown in Fig. 1, is used for design octave-band filter bank and is compared with earlier reported work [13, 15, 16]. Proposed design provides near perfect reconstruction (NPR) with small amount of distortion at the output. A linear iterative optimization technique is used to minimize the distortion. Different window functions are used for design of prototype filter [17]. Two-band filter bank consist of an analysis section formed by analysis filters \( H_L(z) \) and \( H_U(z) \) and a synthesis section formed by synthesis filters \( G_L(z) \) and \( G_U(z) \). In order to reduce the computational workload down-samplers and up-samplers are present between the analysis and synthesis filter bank. In this paper finite impulse response filter (FIR) is used in both analysis and synthesis sections.

II. OCTAVE-BAND FILTER BANK

An octave-band filter bank is designed with integer decimation and linear phase using tree-structure approach. In its first level of decomposition either upper or lower band of two-band filter bank is decomposed into another two-band filter bank with half of pass band and double decimation factor. To increase the number of bands this decomposition process is further repeated. Decomposition for four-band octave-band filter bank with decimation factors \( (8, 8, 4, 2) \) is shown in Fig. 2(a). It is convenient to make equivalent structure as shown in Fig. 2(b). For M-band octave filter bank decimation factors for each band satisfy the following condition,

\[
\sum_{k=0}^{M-1} \frac{1}{M_k} = 1
\]
In this paper we used two-band filter bank which is NPR in which aliasing error is completely eliminated by proper selection of synthesis filters in terms of analysis filters. In two-band aliasing can be eliminated with,
\[ G_1(z) = -2H_0(-z), \quad G_0(z) = 2H_1(-z) \]  

\[ H_0(z) = H_1(z)H_2(z^2)H_4(z^4) \]  
\[ H_1(z) = H_1(z)H_2(z^2)H_4(z^4) \]  
\[ H_3(z) = H_1(z)H_4(z^2) \]  
\[ G_0(z) = G_1(z)G_2(z^2)G_4(z^4) \]  
\[ G_1(z) = G_1(z)G_2(z^2)G_4(z^4) \]  
\[ G_2(z) = G_1(z)G_2(z^2) \]  
\[ G_3(z) = G_4(z) \]  

Figure 2. (a) Four-band octave-band filter bank (b) Equivalent structure of octave-band filter bank

Hence designed octave-band filter bank is also alias free. The analysis and synthesis filters transfer functions can be expressed as,

\[ H_0(z) = H_1(z)H_2(z^2)H_4(z^4) \]  
\[ H_1(z) = H_1(z)H_2(z^2)H_4(z^4) \]  
\[ H_3(z) = H_1(z)H_4(z^2) \]  
\[ G_0(z) = G_1(z)G_2(z^2)G_4(z^4) \]  
\[ G_1(z) = G_1(z)G_2(z^2)G_4(z^4) \]  
\[ G_2(z) = G_1(z)G_2(z^2) \]  
\[ G_3(z) = G_4(z) \]  

Since the basic element in this decomposition process is linear phase FIR filters. Therefore, the designed octave-band filter bank also posses the linear phase property [3, 16].

### III. WINDOW APPROACH

The impulse response coefficients of a causal Nth-order linear phase FIR filter \( p(n) \) using window technique is given by [3, 5],

\[ p(n) = h_i(n)w(n) \]  

Where, \( h_i(n) \) is the impulse response of ideal low pass filter expressed as,

\[ h_i(n) = \frac{\sin(\omega_c(n-0.5N))}{\pi(n-0.5N)} \]  

Where \( \omega_c \) the cutoff frequency of the ideal low is pass filter and \( w(n) \) is the impulse response of used window function. Different window functions are as follows:

**Kaiser Window:** The window equation is given as,

\[ w[n] = \frac{I_0[\beta \sqrt{1-(1-(2n/N-1))^2}]}{I_0[\beta]}, \quad 0 \leq n \leq (N-1) \]  

Where \( I_0[\cdot] \) is the modified zeroth-order Bessel function, and \( \beta \) is a parameter which depends on filter order (N) and can be varied. The empirical design equations developed by Kaiser [1, 5] is given by,
\[ \beta = \begin{cases} 0, & \text{for } A_s \leq 21 \vspace{0.5em} \\ 0.5842(A_s - 21)^{0.4} + 0.07886(A_s - 21), & \text{for } 21 < A_s < 50 \vspace{0.5em} \\ 0.1102(A_s - 8.7), & \text{for } A_s \geq 50 \end{cases} \]  
\[ D = \begin{cases} 0.9222, & A_s \leq 21 \\ \frac{(A_s - 7.95)}{14.36}, & A_s \geq 21 \end{cases} \]  
\[ \omega = \frac{2n\pi}{N} \]  
\[ H_n(e^{i\omega}) = \begin{cases} 1, & |\omega| \leq \frac{N-1}{2} \\ 0, & \text{otherwise} \end{cases} \]

Where \( A_s \) is the stop band attenuation.

**DC Window:** The implementation equation for the window function is given as:
\[ w[n] = \frac{1}{N+1} \left( \frac{1}{r} + \sum_{n=2}^{N/2} C_n \left[ x_0 \cdot \cos \left( \frac{2n\pi}{N+1} \right) \right] \right) \]
\[ r = \frac{A_s}{A_p}, \]
\[ x_0 = \cosh \left[ \frac{1}{N} \cdot \cosh^{-1} \left( \frac{1}{r} \right) \right], \]
\[ C_n(x) = \begin{cases} \cos \left( N \cdot \cosh^{-1}(x) \right), & \text{for } |x| \leq 1 \\ \cosh \left( N \cdot \cosh^{-1}(x) \right), & \text{for } |x| > 1 \end{cases} \]

**Cosh Window:** This window function is defined as,
\[ w[n] = \begin{cases} \frac{\cosh \left( \frac{\alpha_r}{\cosh^{-1} \left( \frac{2n\pi}{N} \right)} \right)}{\cosh(\alpha_r)}, & |n| \leq \frac{N-1}{2} \\ 0, & \text{otherwise} \end{cases} \]

Where
\[ \alpha_r = \begin{cases} 0, & A_s \leq 20.8 \\ 0.2445(A_s - 20.8)^{0.4} + 0.1169(A_s - 20.8), & 20.8 < A_s < 50 \\ -8.722 \times 10^{-5}A_s^2 + 0.1335A_s - 1.929, & 50 \leq A_s \leq 120 \end{cases} \]

The normalized transition width is given by,
\[ D_\ell = \begin{cases} 0, & A_s \leq 20.8 \\ 3.03 \times 10^{-4}A_s^2 + 0.05246A_s - 0.2397, & 20.8 < A_s < 50 \\ -7.771 \times 10^{-5}A_s^2 + 0.07432A_s - 0.5402, & 50 \leq A_s \leq 120 \end{cases} \]

**IV. Optimization Technique**

In NPR type filter bank output suffers from three type of distortions i.e. aliasing, phase and amplitude distortions. Aliasing is eliminated completely by care full design of synthesis filters as in eq. 2. Amplitude distortion in NPR type filter bank can not be eliminated completely but it can be minimize using optimization technique. In this paper an linear iterative optimization technique is used and objective function given by in eq. (20) is used in optimization algorithm. Initially input parameters i.e. number of bands, pass band and stop band frequencies, sampling rate, pass band ripple and stop band attenuation of prototype filter are defined and determine the transition width, cutoff frequency and filter length. Also initialize, different optimization pointers like step size, search direction (dir), flag, initial error (ierror) as well as minimum (tol) possible value of objective function. Design the prototype low pass filter and band pass filters for analysis and synthesis sections using tree-structure technique inside the loop. In Optimization technique cutoff frequency is gradually changed as per the direction and calculates the corresponding value of the objective function. Algorithm halts when it attains minimum value of objective function. The flowchart of optimization algorithm is given in appendix section and implemented on MATLAB 7.0.

\[ \varphi = \max \left( \left| \sum_{n=0}^{M-1} \left( \left| H_n(e^{i\omega}) \right| \right)^2 - 1 \right| \right) \]

for \( 0 < \omega < \frac{\pi}{M} \)

where \( M \) is number of bands in the octave-band filter bank.
V. DESIGN EXAMPLE

In this section we illustrate an example and comparison has been made with earlier reported work [16, 19].

Example: A four-band octave-band filter bank is proposed with (8, 8, 4, 2) decimation factors. The design filter specifications are $N=21$, $\omega_p = 0.5\pi$ and $\omega_s = 0.563\pi$ as taken in [16, 19]. The obtained values are summarized in table 1 and compared the proposed results with previous work. Figures show the graphical results for proposed designs. Figure 3 shows the magnitude responses of prototype filters. Figure 4 shows magnitude responses of analysis filters and Figure 5 shows the amplitude distortion plots.

![Magnitude response of prototype filters](image)

**Figure 3** Magnitude response of prototype filters

![Magnitude response of analysis filters](image)

**Figure 4** Magnitude response of analysis filters

![Amplitude distortion plots](image)

**Figure 5** Amplitude distortion plots
Figure 4 Magnitude responses of octave-band filter bank for decimation factor (8, 8, 4, 2) (a) Kaiser window (b) DC window (c) Cosh window

Figure 5 Plot of amplitude distortions for octave-band filter bank with decimation (8, 8, 4, 2)

<table>
<thead>
<tr>
<th>Work</th>
<th>Decimation factor</th>
<th>Technique/Window</th>
<th>As (dB)</th>
<th>N</th>
<th>Reconstruction error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elias et al.[16]</td>
<td>(8, 8, 2)</td>
<td>Tree-structure</td>
<td>-40</td>
<td>57</td>
<td>4.6×10^{-2}</td>
</tr>
<tr>
<td>Soni et al.[19]</td>
<td>(8, 8, 4, 2)</td>
<td>Optimized tree-structure  Kaiser</td>
<td>-40</td>
<td>21</td>
<td>2.5×10^{-2}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DC</td>
<td>-40</td>
<td>21</td>
<td>1.3×10^{-2}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PC6</td>
<td>-40</td>
<td>29</td>
<td>2.49×10^{-2}</td>
</tr>
<tr>
<td>Proposed</td>
<td>(8, 8, 4, 2)</td>
<td>Optimized tree-structure  Kaiser</td>
<td>-75</td>
<td>21</td>
<td>6.5×10^{-3}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DC</td>
<td>-80</td>
<td>21</td>
<td>6.9×10^{-3}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cosh</td>
<td>-40</td>
<td>21</td>
<td>2.7×10^{-2}</td>
</tr>
</tbody>
</table>
For octave-band filter bank using variable window functions. The obtained performance parameters shown in Table 1. It is evident from Table 1 Kaiser window gives the minimum distortion $6.5 \times 10^{-3}$ among all windows and DC window gives high stop band attenuation (-80dB) it also gives the minimum distortion $6.9 \times 10^{-3}$. For this case computational requirements in proposed design are minimum.

VI. CONCLUSION

In this proposed work a simple an efficient design of octave-band filter bank is presented and proposed results are compared with earlier reported work and provide superior results in terms of amplitude distortion and stop band attenuation. Since octave-band filter bank provide sharp channel selectivity hence it is used in various applications of speech and telecommunication, where channel selectivity is important. It can also be used in acoustic filtering and graphical equalizer.

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REFERENCES

APPENDIX

Specify stop band attenuation ($A_s$), number of bands ($M$)

Initialize: pass band ($\omega_p$), stop band freq ($\omega_s$), ierror, tol, step, dir and flag

Calculate cutoff frequency ($\omega_c$), filter order ($N$) and design the prototype filter. Obtained filters of analysis section using tree-structure

Is $|\text{error}| \leq |\text{tol}|$ or $|\text{ierror}| = |\text{error}|$

Stop

$|\text{ierror}| = |\text{error}|$
($\omega_c = \omega_c + \text{dir}.\text{step}$) and determine reconstruction error at new cutoff frequency

Is error < ierror

Step = Step/2
dir = -dir

No

Yes