Abstract — Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier multiplexing technique, where data is transmitted through several parallel frequency sub channels at a lower data rate. Multiple transmitter and receiver can form Multiple-Input Multiple-Output (MIMO) channels to increase the capacity and data rate. However, OFDM faces the Peak-to-Average Power Ratio (PAPR) problem that is a major drawback of multi-carrier transmission system, which leads to power inefficiency in RF section of the transmitter. In the proposed scheme, a novel technique for reducing PAPR in the MIMO-OFDM system by Discrete Cosine Transform (DCT) based grouping the adjacent sub carriers with companding technique is introduced. The performance of the proposed PAPR is evaluated using a computer simulation. The simulation results show that the proposed scheme can yield good tradeoff between PAPR reduction performance and computational complexity.

Keywords — Orthogonal Frequency Division Multiplexing (OFDM), Multiple Input Multiple Output (MIMO), Inter Carrier Interference (ICI), Peak-to-Average Power Ratio (PAPR), Discrete Cosine Transform (DCT), Complementary Cumulative Distribution Function (CCDF).

I. INTRODUCTION

Multiple-Input Multiple-Output Orthogonal Frequency Division Multiplexing is one of the most attractive standard technologies for fourth generation (4G) wireless communication. MIMO communication uses multiple antennas at both the transmitter and receiver to exploit the spatial domain for spatial multiplexing and/or spatial diversity [1,2]. Normally, OFDM signals have the problem of high Peak-to-Average Power Ratio (PAPR), means that when the signals of all sub-carriers are added constructively, the peak power can be the number of sub-carriers times the average power. The power consumption of a power amplifier depends largely on the peak power than the average power. Thus, handling occasional large peaks leads to low power efficiency [3-5].

Recently, researchers have proposed many technologies to tackle the PAPR problem including clipping companding [3], selected mapping (SLM) [5], non linear companding transforms [6] and DCT [7-9]. Clipping reduces the signal power but degrading bit error rate (BER) performance and causing non linear phenomena such as spectral spreading. Spectral spread causes degradation of spectral efficiency. In the proposed scheme the PAPR can be reduced by DCT based sub carrier grouping with companding technique and it is also compared with the original system for the reduction of PAPR mechanism.

The rest of this paper is organized as follows: Section 2 discusses the basics of PAPR problem in OFDM system. The Section 3 explains the sub-carrier grouping. DCT and companding techniques are introduced in section 4 and section 5 respectively. Section 6 talks about the Proposed scheme. Measurement and simulation results are given in section 7, followed by the conclusions in section 8.

II. PAPR IN OFDM SYSTEMS

An OFDM signal consists of N data symbol X = \{X_n, n = 0,1,2,......N−1\} can be transmitted parallely at lower data rate and each symbol is modulated by one of a set of sub carriers \{f_n, n = 0,1,2,......N−1\}, where N is the number of sub carriers. The transmitted OFDM signal x(t) is expressed as,

\[ x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi f_n t}, \quad 0 \leq t \leq NT \]  

(1)

The PAPR of the transmitted OFDM signal x(t) can be defined as the ratio between the maximum instantaneous power and its average power during its OFDM symbol period.

\[ PAPR[x(t)] = \max_{0 \leq t \leq NT} \frac{|x(t)|^2}{P_{avg}} \]

(2)

P_{avg} is the average power of x(t) and it is expressed as,

\[ P_{avg} = \frac{1}{NT} \int_0^{NT} |x(t)|^2 dt \]

(3)
PAPR of continuous-time OFDM is generally defined as,

\[
PAPR[x(t)] = \frac{\max_{0 \leq t \leq NT} |x(t)|^2}{1/NT \int_0^{NT} |x(t)|^2 dt}
\]  

(4)

In equation (4), PAPR reduction of OFDM signals is mainly achieved by minimizing the maximum instantaneous signal power. If the bandwidth of the OFDM signal is \( B = N \times \Delta f \), the signal \( x(t) \) is sampled by the sampling time of \( \Delta t = \frac{1}{B} = \frac{1}{N \Delta f} \), then the discrete OFDM signal is in expressed as,

\[
x(n) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_k e^{\frac{2\pi i k n}{N}} , \quad k = 0, 1, 2, \ldots, N-1
\]  

(5)

where, \( n \) denotes the index in frequency domain and \( X_k \) is the complex symbol in frequency domain. The PAPR computed from OFDM signal sample can be defined as,

\[
PAPR \{x[n]\} = \frac{\max_{0 \leq n \leq N-1} |x(n)|^2}{E[\|x(n)\|^2]}
\]  

(6)

E[.] denotes the expectation operator and it will be taken over all OFDM symbols.

### III. PRELIMINARY INFORMATION

#### A. Discrete Cosine Transform

When the DCT is applied to the data sequence, the sequence structure will not be changed. At the same time, many zero elements will appear in the transformed sequence. That means, the lower order components will be dominated in the transform domain signal after converted by DCT. The peak value of the auto correlation of data sequence at the IFFT input, which is the average power of input sequence [6-9,14]. The one-dimensional DCT with length \( N \) is expressed as,

\[
x[k] = \alpha(k) \sum_{n=0}^{N-1} x(n) \cos \left( \frac{2\pi nk}{N} \right), \quad 0 \leq k \leq N-1
\]  

(7)

\[
\alpha(k) = \sqrt{\frac{1}{N}} \quad \text{if } k = 0
\]

\[
\alpha(k) = \sqrt{\frac{2}{N}} \quad \text{if } k = \neq 0
\]

The IDCT is expressed as

\[
x[n] = \alpha(k) \sum_{k=0}^{N-1} x[k] \cos \left( \frac{2\pi nk}{N} \right), \quad 0 \leq n \leq N-1
\]  

(8)

Let \( \rho(i) \) be the aperiodic autocorrelation function (ACF) of an input vector \( X \) as,

\[
\rho(i) = \sum_{n=0}^{N-1} X_n X_n^* \quad \text{for } i = 0, 1, 2, \ldots, N-1
\]  

(9)

where, the superscript * denotes the complex conjugate. Then, the relation between autocorrelation and PAPR of OFDM signals is bounded as

\[
PAPR \leq 1 + \frac{2}{N} \sum_{i=0}^{N-1} |\rho(i)|
\]  

(10)

where \( |\rho(i)| \) can be called the absolute aperiodic ACF of input vector \( X \). From equations (9) and (10), we can find that the input vector with lower PAPR in MIMO-OFDM systems.

#### B. Grouped Sub-carriers

The basic idea behind the grouping of adjacent sub-carriers is to increase the frequency separation between the sub-carriers that means only total half number of sub-carriers is being used for transmission, which potentially leads to the PAPR reduction and the power efficiency improvement. It also reduce the ICI due to increased separation among sub-carrier. In conventional MIMO-OFDM systems (fig.1), each modulated symbol \( X_e \) is mapped to one of sub-carriers.
However, in the grouping sub-carrier in MIMO-OFDM system (fig.2), each pair of modulated symbol is jointly mapped into one sub-carrier [9,10]. The modified complex base band OFDM signal can be expressed as,

\[
x(t) = \frac{1}{\sqrt{N}} \sum_{l=1}^{N/2} (X_{1l}e^{j2\pi(2l-1)Nt} + X_{2l}e^{j2\pi(2l)Nt})
\]

where, \( l \) is the total number of groups.

C. Companding Techniques
The samples of OFDM signal \( x(n) \) are companded before it is converted into analog waveforms. A compression is used at the transmitter end after the IFFT block and a expansion is used at the receiver end before the FFT block [6-8]. The companded signal at the transmitter end \( S(n) \) can be expressed as,

\[
S(n) = C\{x(n)\} = \frac{v x(n)}{\ln(1 + u|x(n)|)} \ln \left( 1 + \frac{u}{v}|x(n)| \right)
\]

where, \( V \) is the average amplitude of the signal, \( u \) is the companding parameter, \( x(n) \) is the input of the compressor and \( S(n) \) is the output of the compressor. The companding transform should satisfy the following two conditions:

\[
E[S(n)^2] \approx E[x(n)^2]
\]

\[
|S(n)| \geq |x(n)| \quad \text{for} \quad |x(n)| \leq v \quad (14a)
\]

\[
|S(n)| \leq |x(n)| \quad \text{for} \quad |x(n)| \geq v \quad (14b)
\]

On the receiver end, the received signal must be expanded by the inverse companding transform before it can be sent to the FFT. The expanded signal at the receiver is given as,

\[
Y(n) = C^{-1}\{r(n)\} = \frac{v r(n)}{u|\ln(1 + u)|} \left\{ \exp \left[ \frac{\ln(1 + u)}{v} \right] - 1 \right\}
\]

IV. PROPOSED SYSTEM
In the proposed system, the PAPR of a MIMO-OFDM system is reduced by using DCT based sub carrier grouping with companding technique. In OFDM systems, the PAPR increases approximately linearly with the number of sub-carriers. Therefore, in order to reduce the PAPR, one of the best ways is to decrease the number of used sub-carriers. In this system, the adjacent two input data symbols are jointly mapped to a single sub-carrier, and then the frequency separation between the sub-carrier is increased and Inter Carrier Interference (ICI) The largest instantaneous peak power will appear if all sub-carrier phases are accumulated to the same direction in the same phase. A block diagram of the proposed system is shown in fig.3.
The maximum PAPR of an OFDM signal can be expressed as
\[ \text{max} \ PAPR[x(t)] = N \] (16)

\( S_{\text{DCT}} \) is the most of peak power of OFDM after DCT [6] and expressed as
\[ \lim_{y \to \infty} |S_{\text{DCT}}| \leq 2.2825 + \frac{2}{N} \ln N \] (17)

Therefore, DCT can improve the performance of OFDM, After DCT many data values are very small and even some zero elements appear in the transformed sequence.

V. EXPERIMENTAL RESULTS

To show the overall effect of the PAPR reduction, randomly generated data are modulated into 16 QAM and \( N = 128 \). The number of sub-carrier is depending upon symbol grouping and finally DCT and companding technique is used for grouped sub carriers. We can evaluate the performance of the PAPR reduction scheme using complementary cumulative distribution (CCDF) of the PAPR of the OFDM system.

A. CCDF Performance

Complementary cumulative distribution function (CCDF = 1-CDF) is used to evaluate the performance in PAPR reduction which denotes the probability that the PAPR exceeds a certain threshold. CCDF values are obtained by checking how often PAPR exceeds the threshold values.

\[ \text{CCDF} = P(\text{PAPR} > \text{threshold}) = 1 - P(\text{PAPR} < \text{threshold}) \]

The fig. 4 clearly shows that the DCT transformed signal have a much lower PAPR than the original signals. For example, when CCDF = 10^{-3}, the PAPR of the original signal is 24 dB, whereas that of the transformed signals is 8.5 dB and transformed grouped sub-carrier signal is 3.5 dB and, thus, 21 dB improvement in performance is achieved by the grouped DCT scheme.

![Fig. 4 Comparisons of CCDF of different PAPR reduction techniques in OFDM system](image1)

The fig. 5, shows the CCDF performance of DCT transformed sub-carrier grouped signal of companding algorithm for PAPR reduction in OFDM system. The value of the companding factor \( u \) was fixed to 2, 3, and number of sub-carrier \( N = 128 \). In this proposed scheme method, the peak power at CCDF = 10^{-3}, is reduced to 1.9 dB, 1.75 dB and 1.5 dB when compared with the original system.

![Fig. 5 Comparisons of CCDF of proposed PAPR reduction scheme with different companding factor u in OFDM system](image2)
The fig. 6, shows the CCDF performance of DCT transformed sub-carrier grouped signal of companding algorithm for PAPR reduction in MIMO-OFDM. The value of the companding factor $u$ was fixed to 2, 3.5 and number of sub-carrier $N = 128$. In this proposed scheme method, the peak power at CCDF $= 10^{-3}$, is reduced to 1.8 dB, 1.7 dB and 1.4 dB when compared with the original system.

Figure 6: Comparisons of CCDF of proposed PAPR reduction scheme with different companding factor $u$ in MIMO-OFDM system

VI. CONCLUSIONS

In this paper, PAPR is reduced in MIMO-OFDM system by Discrete Cosine Transform based sub-carrier grouping with companding technique has been proposed. The simulation results show that the proposed scheme can yield good tradeoff between PAPR reduction performance and computational complexity. This paper will be extended by Bit error rate performance analysis and channel estimation and by using forward correction coding.

REFERENCES


