Abstract---Travelling salesman problem (TSP) is a combinatorial optimization problem. It is NP hard problem and TSP is the most intensively studied problem in the area of optimization. But with the increase in the number of cities, the complexity of the problem goes on increasing. In this paper, we have solved Travelling Salesman Problem using Genetic algorithm approach. System starts from a matrix of the calculated Euclidean distances between the cities to be visited by the travelling salesman and randomly chosen city order as the initial population. Then new generations are created repeatedly until the proper path is reached upon reaching a stopping criterion.

Keywords-TSP, GA, Fitness value, Selection, 2-point Crossover, Interchange Mutation.

I. INTRODUCTION
The travelling salesman problem (TSP) is the most well-known combinatorial optimization problem. TSP is used to find a routing of a salesman who starts from a home location, visits a prescribed set of cities and returns to the original location in such a way that the total distance travelled is minimized and each city is visited exactly once [1]. This problem is known to be NP-hard, and cannot be solved exactly in polynomial time. Many exact and heuristic algorithms have been developed in the field of operations research (OR) to solve this problem [2]. TSP is solved very easily when there is less number of cities, but as the number of cities increases it is very hard to solve, as large amount of computation time is required. The numbers of fields where TSP can be used very effectively are military and traffic. Another approach is to use genetic algorithm to solve TSP because of its robustness and flexibility [3]. Some typical applications of TSP include vehicle routing, computer wiring, cutting wallpaper and job sequencing. The main application in statistics is combinatorial data analysis, e.g., reordering rows and columns of data matrices or identifying clusters [4].

II. TRAVELLING SALESMAN PROBLEM
The Travelling Salesman Problem is one of the best known NP-hard problems, which means that there is no exact algorithm to solve it in polynomial time. The minimal expected time to obtain optimal solution is exponential [5]. TSP is defined as a permutation problem with the objective of finding the path of the shortest length (or the minimum cost). TSP can be modelled as an undirected weighted graph, such that cities are the graph’s vertices, paths are the graph’s edges, and a path’s distance is the edge’s length. It is a minimization problem starting and finishing at a specified vertex after having visited each other vertex only once. Often, the model is a complete graph. If no path exists between two cities, adding an arbitrarily long edge will complete the graph without affecting the optimal tour. Mathematically, it can be defined as given a set of n cities, named \{c1, c2, ..., cn\}, and permutations, σ1, ..., σn!, the objective is to choose σi such that the sum of all Euclidean distances between each node and its successor is minimized. The successor of the last node in the permutation is the first one. The Euclidean distance d, between any two cities with coordinate (x1, y1) and (x2, y2) is calculated by equation 1 [6].

\[ d = \sqrt{(x1-x2)^2+(y1-y2)^2} \]  

The most popular practical application of TSP are regular distribution of goods or resources, finding of the shortest of customer servicing route, planning bus lines etc., but also in the areas that have nothing to do with travel routes[5].

III. Various Approaches Used For Solving TSP
In 1997, Rong Yang introduce several knowledge-augmented genetic operators which guide the genetic algorithm more directly towards better quality of the population but are not trapped in local optima prematurely. The algorithm applies a greedy crossover and two advanced mutation operations based on the 2-opt and 3-opt heuristics [7]. In 2001, Ching Moon introduces the concept of topological sort (TS), which is defined as an ordering of vertices in a directed graph. Also, a new crossover operation is developed for the proposed GA [8]. In 2004, new knowledge based multiple inversion operators and a neighbourhood swapping operator is proposed by Shubhra Sankar Ray [9]. In 2005, Lawrence V. Snyder presents a heuristic to solve the generalized traveling salesman problem. The procedure incorporates a local tour improvement heuristic into a random-key genetic algorithm. The algorithm performed quite well when tested on a set of 41 standard problems with known optimal objective values [10]. In 2005, Milena Karova introduces the solution, which includes a genetic algorithm implementation in order to give a maximal approximation of the problem, modifying
a generated solution with genetic operators [11]. In 2006, PlamenkaBorovska investigates the efficiency of the parallel computation of the travelling salesman problem using the genetic approach on a slack multicomputer cluster [12]. In 2007, A two-level genetic algorithm (TLGA) was developed for the problem, which favours neither intra-cluster paths nor inter-cluster paths, thus realized integrated evolutionary optimization for both levels of the CTSP [13]. In 2007, A novel particle swarm optimization (PSO)-based algorithm for the travelling salesman problem (TSP) is presented, and is compared with the existing algorithms for solving TSP using swarm intelligence [14]. In 2008, A software system is proposed to determine the optimum route for a Travelling Salesman Problem using Genetic Algorithm technique [6]. In 2009, S.N. Sivanandam presents two approaches i.e Genetic Algorithms and Particle swarm optimisation to find solution to a given objective function employing different procedures and computational techniques; as a result their performance can be evaluated and compared [15].

In 2010, a novel hybrid discrete PSO algorithm has been presented by add heuristic factor, crossover operator and adaptive disturbance factor into the approach. Numerical results show that the proposed algorithms are effective [16]. In 2011, Ivan Brezina Jr. discusses the Ant Colony Optimization (ACO), which belongs to the group of evolutionary techniques and presents the approach used in the application of ACO to the TSP [5]. In 2011, a comparative performance of roulette wheel, Elitism and tournament selection method is presented to solve the Travelling Salesman problem [18]. In 2012, different authors present a critical survey to solve TSP problem using genetic algorithm methods [17].

IV. PROPOSED GENETIC ALGORITHM

John Holland proposed Genetic Algorithm in 1975. In the field of artificial intelligence genetic algorithm is a search heuristic that mimics the process of natural evolution. Genetic Algorithm belongs to class of evolutionary algorithm. GA begin with various problem solution which are encoded into population, a fitness function is applied for evaluating the fitness of each individual, after that a new generation is created through the process of selection, crossover and mutation. After the termination of genetic algorithm, an optimal solution is obtained. If the termination condition is not satisfied then algorithm continues with new population [3]. The flowchart for proposed GA is described in Fig 1.

![Flowchart of the proposed technique](image)

V. IMPLEMENTATION OF PROPOSED ALGORITHM

The algorithm is applied to a set of total five problems taken from the literature. We have explained the working of the algorithm on a problem of 15 cities in the following section. The coordinates of 15 cities is shown in fig.2.
The distance matrix of the problem is given in Table 1. We have assumed the distance matrix to be symmetric; therefore, the part above the main diagonal contains all necessary information. The distance between the cities is considered to be symmetric i.e. if the salesman moves from city 1 to city 5 than the distance is same if he moves from city 5 to city 1. Due to this half of the matrix is empty. The information above the diagonal is enough to find the distance between different cities. The first row and column denotes the city.

TABLE I  DISTANCE MATRIX OF 15 CITIES PROBLEM [18]

<table>
<thead>
<tr>
<th>City</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</table>

A. Initial Population

Initial population of chromosomes is created randomly by using unique random number generator function in Matlab. The initial population created is shown in following section. The population consist of ten chromosomes, where each chromosome denotes the sequence in which cities have to be traversed and each gene represent the number assign to a city. Section A to E describes the various genetic operators applied to the initial population to obtain the optimal results.

Chromosome1 1 4 13 3 8 2 5 15 7 10 14 12 6 9 11
Chromosome 2 3 14 13 2 9 10 5 7 1 15 6 12 8 11 4
Chromosome 3 1 15 3 7 14 11 9 2 13 5 12 4 8 10 6
Chromosome 4 4 12 14 13 5 9 11 8 1 3 10 2 6 7 15
Chromosome 5 11 2 9 5 13 14 3 12 8 1 15 6 4 10 7
Chromosome 6 3 10 7 13 11 2 9 4 15 12 6 5 14 1 8
Chromosome 7  11  5  2  9  15  13  7  8  4  1  3  12  6  10  14  
Chromosome 8  3  4  13  14  11  7  10  2  8  15  1  5  9  12  6  
Chromosome 9  10  11  7  8  15  1  5  9  12  4  14  6  2  13  3  
Chromosome 10  10  11  4  7  12  1  6  3  9  5  15  14  13  8  2  

B. Fitness Value
The Purpose of the fitness function is to decide if a chromosome is good then how good it is? In the travelling salesman problem the criteria for good chromosome is its length. Calculation takes place during the creation of the chromosomes as given in equation 2. Each chromosome is created and then its fitness function is calculated. The length of the chromosome is measured in pixels by the scheme of the tour [11].

\[ \text{fitness}_{\text{chromosome}} = \sum_{i=1}^{\text{towncount}} t_i \]  

towncount = total number of cities 
ti = distance between two cities.

C. Selection
Selection is used to select the chromosome whose fitness value is small. We have used the tournament selection by using Sorting method.

D. Crossover
2-point crossover is applied to the pair of chromosomes so that new chromosomes will be generated which might have better fitness value. In 2-point crossover, randomly two positions in the chromosomes are chosen and then replace the gene with each other in both chromosomes.

E. Mutation
Mutation is applied to form a new generation. We apply interchange mutation. In interchange mutation, randomly select two genes from a chromosome and then swap them.

F. Termination and Result
After completing the number of iterations the best tour will be obtained and the process will be terminated. The tour obtained with minimum distance is 291 as shown in figure 3 for the problem of 15 cities.

![Figure 3: Graph Denoting Best Path Obtained for 15 Cities Problem](image)

VI. Results and Analysis
We have solved a total of 5 problems using the proposed technique. The best and worst results obtained are shown in table 2. The same table also denotes the best results of the problems present in literature. Looking at the results we can show that GA is a fast technique to obtain the optimal results.
TABLE III
RESULTS OBTAINED BY PROPOSED TECHNIQUE FOR DIFFERENT PROBLEMS

<table>
<thead>
<tr>
<th>No. of Cities</th>
<th>Best path</th>
<th>Worse path</th>
<th>Best known result</th>
</tr>
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<tbody>
<tr>
<td>15</td>
<td>291</td>
<td>341</td>
<td>291</td>
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<tr>
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<td>792</td>
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<td>699</td>
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</table>

VII. CONCLUSION AND FUTURE SCOPE

Combining the knowledge from heuristic methods and genetic algorithms is a promising approach for solving the TSP. Genetic algorithms appear to find good solutions for the travelling salesman problem, however it depends very much on the way the problem is encoded and which crossover and mutation methods are used. A number of genetic algorithm techniques have been analysed and surveyed for solving TSP. The research work can be extended for different hybrid selection, crossover and mutation operators. The proposed approach can be applied for various advanced network models like logistic network, task scheduling models, vehicle navigation routing models etc. The same approach can also be used for allocation of frequencies in cells of cellular network.

REFERENCES