Holistic Method in Biological System Modeling and the Rule of Lyapunov Exponent in Chaotic Assessment of a System

Gholamreza Attarodi
Biomedical Engineering, Science and Research Branch, Iran

Zahra Abbavandi
Biomedical Engineering, Amirkabir University, Iran

Hamid Mohammad Beigi,
Environment Management, Science and Research Branch, Iran

Abstract— In this paper first we have a look at holistic Method in Biological System Modeling and its difference from the single criterion like Lyapunov Exponent. We will show how to use Lyapunov Exponent in Biological Systems for Chaotic assessment. We will have a discussion on Logistic Equation. We see if the parameter in this equation is 3.7 then the system has chaotic behavior and the Lyapunov Exponent is approximately 3. But if we change this parameter then the system may be not chaotic but the Lyapunov Exponent is still positive. In deterministic view all the interaction in the system are not considered. We use a point processing method. We obtained each point from previous point by using functions of derivatives. In Continuous Logistic equation all the connections have continuous manner and we have no creation of the information and no new events are possible to change the behavior of the system. In Bifurcation we have discreet states and these cannot be expressed by these deterministic continuous equation. However the discreet Logistic equation can show these phenomena easily by low order equation. We show that we cannot use criterions like Lyapunov Exponent to show Chaos in Systems however some researchers use.

Keywords— holistic Methods, Discreet Systems, Lyapunov Exponent, Chaotic behavior

I. INTRODUCTION

A dynamical system is made of some variables which describe the state of the system and rules which express how these variables change in time. The new state of the system depends on the previous state and the inputs. In dynamical systems which is open system and have interaction with its environments we have creation of information and new events affect the behavior of the system. The advantage of holistic methods is our ability to assess the system without knowing all the details in the inner of the systems.

II. DISCREET SYSTEM

Discrete equations are widely used to model real systems. There are advantages in using discreet equations rather than continuous equations in describing systems[1]:

In general, discreet equations help us to model the complex systems in a very simple way rather than continuous. In discreet systems we do not need the condition t→0, so the trajectories can jump and can have options and we can describe complex chaotic systems with equations with lower order. However if we use continuous equations for such systems we should use high order of equations. If we say t→0 it means nothing is created in the world by itself but creation depends on previous events. Remember chess play of Kasparov and the Computer. The computer acted only according to a predefined algorithm but Kasparov acted upon his options. He used the previous data but can create new events.

A very famous discreet model is Logistic equations [2].

\[ X_{n+1} = AX_n(1 - X_n) \]

The Bifurcation diagram of this equation is shown in Fig. 1.

![Bifurcation diagram of Logistic equation](image-url)
In Fig. 2 we see output of the Logistic equation.

![Fig. 2- State space of Logistic equation](image)

In Table 1 we see all the behavior of Logistic equation for different values of the parameter. None of these behavior can be obtained from continuous equation.

<table>
<thead>
<tr>
<th>System behavior</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>System will converge to zero</td>
<td>( 0 &lt; r &lt; 1 )</td>
</tr>
<tr>
<td>System will converge to ((r-1)/r)</td>
<td>( 1 &lt; r &lt; 3 )</td>
</tr>
<tr>
<td>System oscillate between two Constant</td>
<td>( 3 &lt; r &lt; 3.45 )</td>
</tr>
<tr>
<td>System oscillate between four Constant</td>
<td>( 3.45 &lt; r &lt; 3.54)</td>
</tr>
<tr>
<td>System oscillate between eight Constant</td>
<td>( 3.54 &lt; r &lt; 3.57)</td>
</tr>
<tr>
<td>System has Chaotic behavior</td>
<td>( 3.57 &lt; r &lt; 4 )</td>
</tr>
<tr>
<td>System will converge to infinity</td>
<td>( r &gt; 4 )</td>
</tr>
</tbody>
</table>

In addition to all these advantages, some of the systems are originally discreet and we cannot use continuous models for them e.g. growth of population and profit of a factory.

### III. TWO SCOPES OF MODELING

In Biological system modeling we cannot say that there is only superposition based methods and we should decompose the system in sub-systems and use the deterministic rules based on Newton and Lipchitz Mathematics and then add the outputs to each other. In Biological systems when we take a part out of the system its behavior is not the same as it is in the system. The behavior of in vitro experiments is not the same as in vivo experiments. If we do not consider the interaction in the system, our model will not compare with the reality. For example the information of interaction of the muscles is the most important factor which defines the proper model and the system behavior. So modeling based only on material and energy is not sufficient for biological systems and the modeling should be based on the triple factor triangle: Material, Energy and Information.[3] The modeling of biological systems should have a look at the information of interaction. Modeling based on Poincare Mathematics is one of these proper methods which helps us to model the system in a better manner without knowing all the details of the inner system. In other words holistic Methods have a look at interaction and use open systems with self-organizing view. We use Global concepts such as regularity, self-organizing, complexity, hierarchy, structure and information. These concepts have no position in Newton Based Science. For example One small mapping functions Based on a simple function called Poincaré section is logistic mapping. logistic could model Heart Rate Variable.[4] we see result of simulink HRV with logistic chaotic :

![Fig 3. The IPFM model diagram block](image)
Using this holistic methods lead to new theories such as Chaotic dynamical system, Fuzzy theory, Fractal geometry, Fuzzy differential equations, intelligent system and so on. When we control the temperature of a room we can overcome all the unpredicted noise if we know the point of working. But in human body the point of working is not fixed. But it varies from 35-38 degree of centigrade in a strange attractor trajectory. The human body has a self-organized temperature controller and the skin, blood pressure and trembling of the skin and many others control the temperature. Artificial temperature control system like thermostat and autopilot are not self-organized.

IV. LYAPUNOV EXPONENT

One of the most popular index to show that a system has potential to be chaotic is Lyapunov Exponent. It is said that if this index is positive then the system is chaotic [5,6]. It can be easily shown that if only the Lyapunov Exponent is positive, we cannot be sure that the behavior of the system is chaotic. The chaotic systems always have positive Lyapunov Exponent but the reverse statement is not correct i.e. we cannot consider all the systems with positive Lyapunov Exponent be chaotic, but it may have random behavior. We did simulations for some random signals and showed the Lyapunov Exponents be positive and it is obvious that these are not chaotic signals.

See Fig. 5,6 and 7.

Fig. 4-Power spectrum of the output HRV

![Power spectrum of the output HRV](image)

Fig. 5- Lyapunov Exponent for a random signal 1

![Lyapunov Exponent for a random signal 1](image)

Fig. 6- Lyapunov Exponent for a random signal 2

![Lyapunov Exponent for a random signal 2](image)
In fact the Lyapunov Exponent is not sufficient index for chaotic assessment of a system. In Fig. 8 we see the Lyapunov Exponent of Logistic equation.

If we draw a horizontal line from zero in the vertical axe we see for some quantity of the parameter we have positive Lyapunov Exponent and for some other values we have NEGATIVE Lyapunov Exponent. If the bifurcation parameter is 3.7 the system shows chaotic behavior, here the Lyapunov Exponent is positive but if we change a little bit this parameter then the system in not any longer chaotic but the Lyapunov Exponent is still positive approximately 3. So only the Lyapunov Exponent cannot describes the system complexity and we need other tools to say that the system is chaotic or not.

So we can not trust the numerical results, very. [7]
V. CONCLUSIONS

In this paper we tried to show that in Biological systems we need holistic methods and the discreet modeling is more efficient to describe the systems rather than continuous one. We then can model these systems with lower order. In addition we showed using only indexes like Lyapunov Exponent cannot show the real behavior of the system, however in some papers the authors deducted chaotic behavior from positive Lyapunov Exponent. Finally we can say Lyapunov Exponent is only a necessary condition but not a sufficient one for chaotic assessment of a system.

REFERENCES